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ON POINTED HOPF ALGEBRAS OF DIMENSION p^n

M. BEATTIE, S. DĂSCĂLESCU, AND L. GRÜNENFELDER

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ABSTRACT. In this note we describe nonsemisimple Hopf algebras of dimension p^n with coradical isomorphic to kC, C abelian of order p^{n-1}, over an algebraically closed field k of characteristic zero. If C is cyclic or $C = (C_p)^{n-1}$, then we also determine the number of isomorphism classes of such Hopf algebras.

0. INTRODUCTION AND PRELIMINARIES

In recent years considerable effort has been made to classify finite dimensional Hopf algebras over an algebraically closed field k of characteristic 0. In [17], Zhu proved that a Hopf algebra of prime dimension p is isomorphic to kC_p. For the semisimple case, a series of results has appeared. Masuoka has classified semisimple Hopf algebras of dimensions $6, 8, p^2$, p^3 and $2p$ for p an odd prime (see [8], [9], [10], [11]). Larson and Radford [7] showed that a semisimple Hopf algebra of dimension ≤ 19 is commutative and cocommutative, and thus isomorphic to a group algebra. Also, in [4], Gelaki constructs interesting examples of semisimple Hopf algebras of dimension pq^2, p, q distinct primes. The nonsemisimple case seems to be more difficult. In dimension p^2, apart from kC_{p^2} and $k(C_p \times C_p)$ which are semisimple, $p-1$ types of nonsemisimple Hopf algebras are known. These are the Taft Hopf algebras, which we denote by T_λ, λ a primitive pth root of 1. The algebras T_λ are pointed with coradical kC_p. Conversely, if H is a pointed nonsemisimple Hopf algebra of dimension p^2, H is a Taft Hopf algebra.

Here, we describe Hopf algebras of dimension p^n, $n \geq 3$, which have coradical kC, C abelian of order p^{n-1}, and note that the situation is somewhat different from the dimension p^2 case. The Hopf algebras which occur can be obtained by an Ore extension construction as in [2] or [3]; if C is cyclic, there are $p^{[\frac{n}{2}]} + p^{[\frac{n-1}{2}]} + p - 3$ nonisomorphic such Hopf algebras; if $C = (C_p)^{n-1}$, then there are $p-1$ isomorphism classes.

Throughout, k is an algebraically closed field of characteristic 0. We follow the standard notation in [12]. For H a Hopf algebra, $G(H)$ will denote the group of grouplike elements and $H_0, H_1, H_2, \ldots$ will denote the coradical filtration of H. H is called pointed if $H_0 = kG(H)$. If $g, h \in G(H)$ then the set of (g,h)-primitive elements is $P_{g,h} = \{x \in H | \triangle(x) = x \otimes g + h \otimes x\}$. Since $g - h \in P_{g,h}$ we can choose a subspace $P'_{g,h}$ of $P_{g,h}$ such that $P_{g,h} = k(g-h) \oplus P'_{g,h}$. We will need the

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version of the Taft-Wilson theorem proved in [12, Theorem 5.4.1], which states that $H_1 = H_0 \oplus (\bigoplus_{g,h\in G(H)} P'_{g,h})$ for a finite dimensional pointed Hopf algebra H. If H is finite dimensional, $P_{1,1} = 0$. This implies that if H is also pointed of dimension > 1, then $G(H)$ is not trivial.

The notation $\binom{t}{j}_q$ denotes a q-binomial coefficient. Details on the definitions of these coefficients can be found in any introduction to quantum groups, for example, [6, IV.2] or the introductory section of [15].

An efficient method for constructing nonsemisimple finite dimensional pointed Hopf algebras by means of iterated Ore extensions was developed in [3]. If a single Ore extension of the group algebra of a finitely generated abelian group C is involved, then, simply put, the method consists of taking an automorphism φ of the group algebra kC of the form $\varphi(c) = \lambda^{-1}c$, $\lambda \in k$, for each generator c of C, and forming the Ore extension $kC[X, \varphi]$. Comultiplication is defined on X by $\triangle(X) = g \otimes X + X \otimes 1$, for some $g \in C$. This defines a Hopf algebra structure on the infinite dimensional space $kC[X, \varphi]$. Then we factor out by a Hopf ideal of finite codimension. For more details about the general construction, see [2, §4] or [3]. The specific Hopf algebras used in this note are described below.

1. The classification results

Before proving our results, we define Hopf algebras $H(\lambda, u)$ and $\tilde{H}(\lambda, u)$ of dimension p^n. These Hopf algebras are exactly Ore extension constructions as in [2, §4] or [3].

Fix a prime p and an integer $n \geq 2$. Let $C = C_1 \times \ldots \times C_m$ be a finite abelian group of order p^{n-1}, where $C_i = \langle c_i \rangle$ is cyclic of order p^{n_i}, and let ϕ be an automorphism of kC defined by $\phi(c_i) = \lambda_i^{-1} c_i$. Then clearly $\lambda_i^{p^{n_i}} = 1$. We write c^u to denote $c_1^{u_1} \ldots c_m^{u_m}$ and λ^u to denote $\lambda_1^{u_1} \ldots \lambda_m^{u_m}$ if $u = (u_1, \ldots, u_m)$ is an m-tuple of integers. As outlined in the introduction, we define a Hopf algebra structure on the Ore extension $kC[X, \phi]$ by setting $\Delta(X) = c^u \otimes X + X \otimes 1$ for some element $c^u \neq 1$ of C, and $\epsilon(X) = 0$. The Hopf algebra $kC[X, \phi]$ has generators c_i, $1 \leq i \leq m$, and X, relations

$$c_i^{p^{n_i}} = 1, \quad X c_i = \lambda_i^{-1} c_i X,$$

and coalgebra structure induced by

$$\triangle(c_i) = c_i \otimes c_i, \quad \triangle(X) = c^u \otimes X + X \otimes 1, \quad \epsilon(c_i) = 1, \quad \epsilon(X) = 0,$$

where $u = (u_1, u_2, \ldots, u_m) \in \mathbf{Z}^m$ and $c^u = c_1^{u_1} c_2^{u_2} \ldots c_m^{u_m}$. The antipode is given by $S(c_i) = c_i^{-1}$, $S(X) = -c^{-u}X$. Then $kC[X, \phi]$ is an infinite dimensional pointed Hopf algebra with $G(kC[X, \phi]) = C$.

Now let $\lambda^u = \lambda_1^{u_1} \ldots \lambda_m^{u_m}$ be a primitive pth root of unity. Write c^{up} for $(c^u)^p = c_1^{u_1 p} \ldots c_m^{u_m p}$. Then $\Delta(X^p) = c^{up} \otimes X^p + X^p \otimes 1$, so that the ideal generated by X^p is a Hopf ideal in $kC[X, \phi]$. If λ^u is a primitive pth root of unity and if also c^u has order different from p, the ideal generated by $X^p - c^{up} + 1$ is a different Hopf ideal. The quotient Hopf algebras

$$H(\lambda, u) = kC[X, \phi]/\langle X^p \rangle, \quad \tilde{H}(\lambda, u) = kC[X, \phi]/\langle X^p - c^{up} + 1 \rangle$$

are pointed Hopf algebras of dimension p^n and $G(H(\lambda, u)) \cong C \cong G(\tilde{H}(\lambda, u))$ [14, Lemma 1]. Let x denote the coset of X.

Note that in both cases $P'_{1,g} \neq 0$ only if $g = c^u$ and that $\dim P'_{1,c^u} = 1$. We see below that the cases H and $\tilde{H}$ are distinct.

Lemma 1. *For C an abelian group of order p^{n-1} and $H(\lambda,u)$, $\tilde{H}(\xi,v)$ as above, then $H(\lambda,u)$ cannot be isomorphic to $\tilde{H}(\xi,v)$.*

Proof. Suppose $\phi : \tilde{H}(\xi,v) \to H(\lambda,u)$ is a Hopf algebra isomorphism. Then ϕ induces an automorphism of the group C. Now $\dim P_{1,g} = 2$ in $\tilde{H}(\xi,v)$ if and only if $\dim P_{1,\phi(g)} = 2$ in $H(\lambda,u)$. So $\phi(c^v) = c^u$ and $0 \neq x \in P_{1,c^v} - k(c^v - 1) \subset \tilde{H}(\xi,v)$, so that $\phi(x) = \alpha(\phi(c^v)-1) + y$ for some $\alpha \in k$ and some $0 \neq y \in P_{1,c^u} - k(c^u - 1) \subset H(\lambda,u)$ with $y^p = 0$.

Since $\phi(xc^v) = \phi(x)\phi(c^v) = \phi(\xi^{-v}c^v x) = \xi^{-v}\phi(c^v)\phi(x)$, we see that $\alpha = 0$, and $\phi(x) = y$. But then $x^p = 0$, in contradiction to the definition of $\tilde{H}(\xi,v)$. □

If $C = C_p \times \ldots \times C_p = C_p^{n-1}$ then no $\tilde{H}(\lambda,u)$ occur, since every element of C has order p. In particular, when $n = 2$ then $C = C_p$, and the Hopf algebras we get are isomorphic to the Taft Hopf algebras of dimension p^2, denoted T_λ, λ a primitive pth root of 1. There are $p-1$ nonisomorphic T_λ.

In $\tilde{H}(\lambda,u)$, if $C = C_{p^{n-1}}$, then $x^p = c^{up} - 1$ commutes with c, so that $\lambda^p = 1$. Thus λ is a primitive pth root of unity, and since λ^u is also a primitive pth root, p does not divide u and c^u is a generator of C.

We can now describe Hopf algebras of dimension p^n with coradical kC, where C is an abelian group of order p^{n-1}.

Theorem 2. *A Hopf algebra H of dimension p^n whose coradical H_0 is the group algebra of an abelian group of order p^{n-1} is isomorphic to $H(\lambda,u)$ or $\tilde{H}(\lambda,u)$ for some (λ,u).*

Proof. Let $C = \times_{i=1}^m C_i$ be a cyclic decomposition with generators $c_1, c_2, \ldots, c_m$. The Taft-Wilson theorem says that $H_1 = H_0 \oplus (\oplus_{g,h\in C} P'_{g,h})$. Observe that $aP_{g,h}b = P_{agb,ahb}$ for any $a,b \in C$. Consider the action of C on H_1 given by conjugation, i.e. for each $g \in C$ the automorphism $T_g : H_1 \to H_1$ is defined by $T_g(y) = gyg^{-1}$. Then $T_g^{p^{n-1}}$ is the identity and the eigenvalues of T_g satisfy the equation $\lambda^{p^{n-1}} = 1$. Since C is abelian, each $P_{g,h}$ is invariant under this action and has a basis of joint eigenvectors, i.e. vectors w such that $T_a(w) = awa^{-1} = \lambda_a w$ for every $a \in C$. Since H is not commutative there is a nonzero joint eigenvector $w \notin H_0$ contained in some $P_{g,h}$ and in fact in $P_{1,r}$ for some $1 \neq r = c_1^{s_1}c_2^{s_2}\ldots c_m^{s_m} = c^s$, $s = (s_1,\ldots,s_m) \in \mathbf{Z}^m$, whose eigenvalue $\lambda_i \neq 1$ for at least one generator c_i. Thus, we have

$$\Delta(w) = r \otimes w + w \otimes 1, \quad c_i w = \lambda_i w c_i.$$

The Hopf subalgebra of H generated by $\{c_1, c_2, \ldots, c_m, w\}$ is the subspace spanned by the elements $c^i w^j$ with $i \in \mathbf{N}^m$ and $j \in \mathbf{N}$. By the Nichols-Zoeller Theorem [12, 3.1.5] this Hopf subalgebra must be all of H. Note that $(w\otimes 1)(r\otimes w) = \lambda^{-s}(r \otimes w)(w \otimes 1)$. Also $\lambda^{-s} \neq 1$ since otherwise r and w commute and so generate a commutative nonsemisimple Hopf subalgebra of H, which then must be infinite dimensional. Thus λ^{-s} is a primitive root of unity of order p^l for some $l > 0$, so that for $t \leq p^l$

$$\Delta(w^t) = \sum_{j=0}^{t} \binom{t}{j}_{\lambda^{-s}} r^{t-j} w^j \otimes w^{t-j}.$$

We prove by induction on t, $1 \leq t \leq p-1$, that $w^t \in H_t - H_{t-1}$. For $t = 1$ the statement is obvious. Assume that $w^v \in H_v - H_{v-1}$ for $v < t$. Then $r^t \otimes w^t \in H_0 \otimes H$, $r^{t-j}w^j \otimes w^{t-j} \in H \otimes H_{t-2}$ for $j \geq 2$, and $r^{t-1}w \otimes w^{t-1} \in H \otimes H_{t-1}$.

Thus, $\Delta(w^t) \in H_0 \otimes H + H \otimes H_{t-1}$, and it follows that $w^t \in H_t$. If $w^t \in H_{t-1}$, then $\Delta(w^t) \in H_0 \otimes H + H \otimes H_{t-2}$; hence $\binom{t}{1}_{\lambda^{-s}} r^{t-1} w \otimes w^{t-1} \in H_0 \otimes H + H \otimes H_{t-2}$. Since $t < p$, we must have $\binom{t}{1}_{\lambda^{-s}} \neq 0$. Now $r^{t-1}w \in H_1 - H_0$ and $w^{t-1} \notin H_{t-2}$ gives a contradiction, and thus $w^t \in H_t - H_{t-1}$.

By [14, Corollary 2.3], H_t is a free H_0-module so that $\dim H_t \geq (t+1)p^{n-1}$ for $0 \leq t \leq p-1$ and in particular $\dim H_{p-1} \geq p^n$. This shows that $H = H_{p-1}$ and $\dim H_t = (t+1)p^{n-1}$ for $t \leq p-1$. Thus, $\dim H_1 = 2p^{n-1}$ and by the Taft-Wilson Theorem the subset $\{hw | h \in C\}$ of $H_1 - H_0$ is linearly independent in H_1, so that $\dim P_{h,hr} = 2$ for all $h \in C$.

We claim that the subset $\{hw^j | h \in C, 0 \leq j \leq p-1\}$ of H is linearly independent. If not, then there is some $t \leq p-1$ such that

$$\sum_{h \in C} \alpha_h h w^t = \sum_{h \in C, 0 \leq q < t} \beta_{h,q} h w^q$$

for some scalars α_h and $\beta_{h,q}$ with $\alpha_g \neq 0$ for some $g \in G$. The image of the comultiplication map Δ applied to the right hand side of the equation above lies in $H \otimes H_{t-1}$. Let $\psi \in H^*$ be such that $\psi(H_{t-1}) = 0$ and $\psi(gw^t) = 1$, and also let $\phi \in H^*$ be such that $\phi(gr^t) = 1$ and $\phi(hr^t) = 0$ for $h \neq g$. Then $(\phi \otimes \psi)(H \otimes H_{t-1}) = 0$ but $\phi \otimes \psi$ applied to the left hand side of the equation above yields $(\phi \otimes \psi)(\sum_{h \in C} \alpha_h hr^t \otimes hw^t) = \alpha_g \neq 0$, a contradiction.

Since $H_{p-1} = H$, we have that $w^p \in H_{p-1}$, so that $\Delta(w^p) \in H_0 \otimes H + H \otimes H_{p-2}$. Expanding as above, we find that $\binom{p}{1}_{\lambda^{-s}} r^{p-1} w \otimes w^{p-1} \in H_0 \otimes H + H \otimes H_{p-2}$. Since $r^{p-1}w \in H_1 - H_0$ and $w^{p-1} \notin H_{p-2}$ we conclude that $\binom{p}{1}_{\lambda^{-s}} = 0$, so that λ^{-s} is a primitive pth root of 1. Then $\binom{p}{j}_{\lambda^{-s}} = 0$ for $1 \leq j \leq p-1$ and

$$\Delta(w^p) = r^p \otimes w^p + w^p \otimes 1$$

so that $w^p \in P_{1,r^p} = k(r^p - 1) \oplus P'_{1,r^p}$. Since $r^p \neq r$ we have $P_{1,r^p} = k(r^p - 1)$ and $w^p = \gamma(r^p - 1)$ for some scalar γ.

If $w^p = 0$, then it is clear that $H \cong H(\lambda, s)$. If $w^p \neq 0$, then replacing w by $\gamma^{-1/p} w$, we see that $H \cong \tilde{H}(\lambda, s)$. ☐

Corollary 3. *The only nonsemisimple pointed Hopf algebras of dimension p^2 are the Taft-Hopf algebras.* ☐

Now we show that if $C = (C_p)^{n-1} = \langle c_1 \rangle \times \langle c_2 \rangle \times \ldots \times \langle c_{n-1} \rangle$, then a result similar to Corollary 3 holds.

Proposition 4. *If $C = (C_p)^{n-1}$, then $H(\lambda, u) \cong k(C_p)^{n-2} \otimes T_{\lambda_i}$ for some i, and there are exactly $p-1$ isomorphism classes of such Hopf algebras.*

Proof. We argue by induction on n. If $n - 1 = 1$, then the result follows from Corollary 3. Recall that $c^u \neq 1$, since $\dim P_{1,1} = 0$ and also $\lambda^u \neq 1$, else c^u and x commute and $H(\lambda, u)$ could not be finite dimensional. Thus some $\lambda_i^{u_i} \neq 1$, and we may assume that $\lambda_2^{u_2} \neq 1$, in particular $\lambda_2 \neq 1$ and $u_2 \neq 0$. Now we distinguish two cases.

If $\lambda_1 = 1$, then let $u' = (u_2, u_3, \ldots, u_{n-1})$ and $\lambda' = (\lambda_2, \lambda_3, \ldots, \lambda_{n-1})$. Then the map

$$f : H(\lambda, u) \to H((1, \lambda'), (0, u')) \cong kC_p \otimes H(\lambda', u')$$

defined by

$$f(x) = x, \quad f(c_1) = c_1^{-u_2}, \quad f(c_2) = c_1^{u_1} c_2, \quad f(c_i) = c_i$$

for $i > 2$ is an isomorphism of Hopf algebras.

If $\lambda_1 \neq 1$, then $\lambda_1 = \lambda_2^h$ for some h. Now let $u' = (hu_1 + u_2, u_3, \dots, u_{n-1})$ and $\lambda' = (\lambda_2, \lambda_3, \dots, \lambda_{n-1})$. Then the map

$$f : H(\lambda, u) \to H((1, \lambda'), (0, u')) \cong kC_p \otimes H(\lambda', u')$$

defined by

$$f(x) = x, \quad f(c_1) = c_1^{hu_2} c_2^h, \quad f(c_2) = c_1^{-hu_1} c_2, \quad f(c_i) = c_i$$

for $i > 2$ is an isomorphism.

Thus, if the assertion holds for $C = (C_p)^{n-2}$, it holds for $C = (C_p)^{n-1}$ and we are done. $\square$

Finally, we count the number of isomorphism classes of Hopf algebras of dimension p^n with coradical $kC_{p^{n-1}}$. First we need two lemmas. From now on, since $C = \langle c \rangle$ is cyclic, the notation $H(\lambda, i)$ refers to $\lambda \in k, i \in \mathbf{Z}$. Also, since in the cyclic case in $\tilde{H}(\lambda, u)$ the element c^u is a generator of C we may assume that $u = 1$, and we abbreviate $\tilde{H}(\lambda, 1)$ by $H(\lambda)$.

Lemma 5. *$H(\lambda, i) \simeq H(\mu, j)$ if and only if there exists h relatively prime to p such that $\lambda = \mu^h$ and $j \equiv hi$ (mod p^{n-1}).*

Proof. Let $f : H(\lambda, i) \to H(\mu, j)$ be a Hopf algebra isomorphism. Then f induces an automorphism of the coradical $kC_{p^{n-1}}$, so that $f(c) = c^h$ for some h relatively prime to p.

Furthermore, let $x \in P'_{c^i,1}$ in $H(\lambda, i)$. Then $P_{c^{hi},1}$ in $H(\mu, j)$ is nonzero, so $c^{hi} = c^j$. Thus $hi \equiv j \bmod p^{n-1}$. Finally, since $cx = \lambda xc$, $c^h f(x) = \lambda f(x) c^h$. But $f(x) \in P_{c^j,1}$ and the above equation implies that $f(x) \in P'_{c^j,1}$ so that $cf(x) = \mu f(x) c$ in $H(\mu, j)$. Then $c^h f(x) = \mu^h f(x) c^h$, so $\lambda = \mu^h$.

Conversely, given such an integer h, define f by $f(c) = c^h$ and $f(x) = z \in P'_{g^j,1}$. $\square$

Lemma 6. *$H(\lambda) \simeq H(\mu)$ if and only if $\lambda = \mu$. Thus there are exactly $p - 1$ nonisomorphic $H(\lambda)$.*

Proof. As in Lemma 5, if $f : H(\lambda) \to H(\mu)$ is a Hopf algebra isomorphism, $f(c) = c^h$ for some h. Let $x \in P'_{1,c}$ in $H(\lambda)$. Then $f(x) \in P_{1,c^h}$ in $H(\mu)$ which implies that $h = 1$. The result then follows immediately. $\square$

Proposition 7. *For $n \geq 3$, there exist precisely $p^{[\frac{n}{2}]} + p^{[\frac{n-1}{2}]} + p - 3$ Hopf algebras of dimension p^n with coradical $kC_{p^{n-1}}$, where $[y]$ is the largest integer less than or equal to y.*

Proof. We first count the Hopf algebras of the form $H(\lambda, i)$. If $H(\lambda, i) \simeq H(\mu, j)$, then λ and μ must have the same order. Let us fix the order of λ, say p^b, $1 \leq b \leq n-1$. Then by Lemma 5, we can fix λ. Since λ^i is primitive of order p, $i = qp^{b-1}$ where $(p, q) = 1$ and $1 \leq q \leq p^{n-b}$. Thus there are $p^{n-b} - p^{n-b-1}$ choices for i. For a fixed such i we have $H(\lambda, i) \simeq H(\lambda, j)$ if and only if there exists an h not divisible by p with $\lambda^h = \lambda$ and $j \equiv hi$ (mod p^{n-1}). But $\lambda^h = \lambda$ implies that $h = \alpha p^b + 1$,

$0 \le \alpha \le p^{c-b}-1$. Since for $h = \alpha p^b + 1,\ h' = \beta p^b + 1$ we have $hi \equiv h'i \pmod{p^{n-1}}$ if and only if $p^{n-b} | (\alpha - \beta)p^b$, we distinguish two cases. If $b \ge \frac{n}{2}$, then $hi \equiv i \pmod{p^{n-1}}$ for all the indicated h's, and in this case $H(\lambda, i) \simeq H(\lambda, j)$ implies $i = j$. Therefore there exist $p^{n-b} - p^{n-b-1}$ types of such Hopf algebras. If $b < \frac{n}{2}$, then $hi \equiv h'i \pmod{p^{n-1}}$ is equivalent to $p^{n-2b} | \alpha - \beta$, which means that for a fixed h there are precisely $(p^{n-1-b})/(p^{n-2b}) = p^{b-1}$ elements h' for which $h'i \equiv hi \pmod{p^{n-1}}$. This implies that there are $(p^{n-1-b})/(p^{b-1}) = p^{n-2b}$ elements j such that $H(\lambda, i) \simeq H(\lambda, j)$. Therefore there exist $(p^{n-b} - p^{n-b-1})/(p^{n-2b}) = p^b - p^{b-1}$ types of such Hopf algebras. It follows that there are

$$\sum_{1 \le b \le \frac{n}{2}} (p^b - p^{b-1}) + \sum_{\frac{n}{2} < b \le n-1} (p^{n-b} - p^{n-b-1})$$

types of Hopf algebras of the form $H(\lambda, i)$. Adding the $p - 1$ types of the form $H(\lambda)$ (see Lemma 6), an easy computation proves the required formula. □

Remark 8. Note that Corollary 3 also follows from [5, 1.1.1] for dim $H = p^2$, p prime. Contrary to the assertion in [5, 1.1.1], if N is not prime, then Hopf algebras such as $H(\lambda)$ appear. (An addendum to [5] will appear.)

Added in proof. The fact that pointed nonsemisimple Hopf algebras of dimension p^2 are Taft Hopf algebras seems to have been known for some time. Proofs appear also in [1], where Nichols is quoted, and in [16] where Andruwskiewitsch and Chin are acknowledged.

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Department of Mathematics and Computer Science, Mount Allison University, Sackville, New Brunswick, Canada E4L 1E6

E-mail address: mbeattie@mta.ca

Faculty of Mathematics, University of Bucharest, Str. Academiei 14, RO-70109 Bucharest 1, Romania

E-mail address: sdascal@al.math.unibuc.ro

Department of Mathematics, Statistics and Computing Science, Dalhousie University, Halifax, Nova Scotia, Canada B3H 3J5

E-mail address: Luzius@mscs.dal.ca