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WEAK AMENABILITY OF SEGAL ALGEBRAS

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ABSTRACT. Let G be a locally compact abelian group, and let $p \in [1, \infty)$. We show that the Segal algebra $S_p(G)$ is always weakly amenable, but that it is amenable only if G is discrete.

1. Introduction

Let A be an algebra, and let E be an A-bimodule. Then a linear map $D : A \to E$ is a *derivation* if

$$D(ab) = a \cdot Db + Da \cdot b \quad (a, b \in A)\,.$$

For example, let $x \in E$, and set

$$\delta_x(a) = a \cdot x - x \cdot a \quad (a \in A)\,.$$

Then δ_x is a derivation; these derivations are *inner derivations*.

Now let A be a Banach algebra, and let E be a Banach A-bimodule. Then the space of continuous derivations from A into E is denoted by $\mathcal{Z}^1(A, E)$, and the subspace consisting of the inner derivations is $\mathcal{N}^1(A, E)$; the *first (Banach) cohomology group* of A with coefficients in E is

$$\mathcal{H}^1(A, E) = \mathcal{Z}^1(A, E)/\mathcal{N}^1(A, E)\,.$$

(For the general theory of the Banach cohomology groups $\mathcal{H}^n(A, E)$, where $n \in \mathbb{N}$, see [3] and [7].) Let E' be the dual Banach space of E. Then E' is also a Banach A-bimodule for the operations defined by

$$\langle x,\, a \cdot \lambda\rangle = \langle x \cdot a,\, \lambda\rangle\,, \quad \langle x,\, \lambda \cdot a\rangle = \langle a \cdot x,\, \lambda\rangle \quad (a \in A,\, x \in E,\, \lambda \in E')\,.$$

The Banach algebra A is *amenable* if $\mathcal{H}^1(A, E') = \{0\}$ for each Banach A-bimodule E; this important concept was introduced by Johnson in [9], where it is proved that the group algebra $L^1(G)$ of a locally compact group G is amenable if and only if G is an amenable group. (See also [7, VII, §2.5].) In particular, $L^1(G)$ is amenable for each locally compact abelian (LCA) group G. Amenable Banach algebras have certain rather strong properties. For example, each closed ideal I of finite codimension in an amenable Banach algebra A has a bounded approximate identity ([7, VII, 2.31]), and so $I = I^2$.

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Let A be a Banach algebra. A particular example of a Banach A-bimodule is A itself; now A' is the *dual module* of A. The algebra A is *weakly amenable* if $\mathcal{H}^1(A, A') = \{0\}$. Of course, every amenable Banach algebra is weakly amenable; however the class of weakly amenable Banach algebras is considerably larger than that of amenable Banach algebras. For example, the group algebra $L^1(G)$ is weakly amenable for each locally compact group G; a short proof of this result is given in [4]. A commutative, semisimple Banach algebra, regarded as being defined on its character space Φ_A, is termed a *Banach function algebra*. Examples of weakly amenable, but not amenable, Banach function algebras are given in [1], where it is noted that a commutative Banach algebra is weakly amenable if and only if $\mathcal{H}^1(A, E) = \{0\}$ for each Banach A-module E.

Let G be an LCA group, and let $(L^1(G), \|\cdot\|_1)$ be the group algebra on G, so that $L^1(G)$ is a Banach algebra for the convolution product $(f, g) \mapsto f \star g$, where

$$(f \star g)(y) = \int_G f(x)g(y - x)\,\mathrm{d}x \quad (y \in G),$$

for $f, g \in L^1(G)$. The dual group of G is denoted by Γ or Γ_G, and we write $\mathrm{d}x$ and $\mathrm{d}\gamma$ for Haar measures on G and Γ, respectively. The Fourier transform is denoted by $\mathcal{F} : f \mapsto \widehat{f}$, where

$$\widehat{f}(\gamma) = \int_G f(x)\,\langle -x, \gamma\rangle\,\mathrm{d}x \quad (\gamma \in \Gamma)\,,$$

and $\mathcal{F}$ identifies $L^1(G)$ with the Banach function algebra $A(\Gamma)$ on Γ ([3], [8], [13], [15]).

There has been some study of certain subalgebras of the group algebras $L^1(G)$; these are the Segal algebras ([13, Chapter 6, §2.1], [14]). Indeed, a subalgebra S of $L^1(G)$ is a *Segal algebra* if the following conditions are satisfied:

(i) S is dense in $(L^1(G), \|\cdot\|_1)$;

(ii) S is translation-invariant (i.e., $\tau_x f \in S$ for each $f \in S$ and $x \in G$, where $(\tau_x f)(y) = f(y - x)\ (y \in G)$);

(iii) S is a Banach algebra with respect to a norm $\|\cdot\|_S$, and

$$\|\tau_x f\|_S = \|f\|_S \ \ (f \in A,\, x \in G)\,;$$

(iv) the map $x \mapsto \tau_x f$, $G \to (S, \|\cdot\|_S)$, is continuous.

It is noted in [13, Chap. 6, §2.3] that the subalgebra of S consisting of functions f such that $\operatorname{supp}\widehat{f}$ is compact is dense in $(S, \|\cdot\|_S)$.

Particular examples of Segal algebras are the algebras $S_p(G)$, which we now define.

Definition 1.1. Let G be an LCA group, and let $p \in [1, \infty)$. Then

$$S_p(G) = \{f \in L^1(G) : \widehat{f} \in L^p(\Gamma)\}\,,$$

and

$$|||f|||_p = \|f\|_1 + \left\|\widehat{f}\right\|_p \quad (f \in S_p(G))\,.$$

The algebras $(S_p(G), |||\cdot|||_p)$ are Segal algebras on S, and they may be identified with Banach function algebras on Γ by using the Fourier transform.

Our purpose here is to consider when the algebras $S_p(G)$ are amenable, and when they are weakly amenable.

Basic properties of the algebras $S_p(G)$ are given in [13] and [14]. For example, the character space of $S_p(G)$ is naturally identified with Γ (and indeed it is shown in [2] that the closed ideal theory of $S_p(G)$ coincides with that of $L^1(G)$). It is clear that $S_p(G) \subset S_q(G)$ when $1 \leq p \leq q < \infty$ and that $S_2(G) = L^1(G) \cap L^2(G)$. In the case where G is discrete (and Γ is compact), the algebras $S_p(G)$ and $L^1(G)$ coincide, and so $S_p(G)$ is an amenable Banach algebra. In the case where G is not discrete, it is proved in [11] that

$$S_p(G)^2 \subset S_q(G) \subsetneq S_p(G) \quad (p > 1)\,, \tag{1}$$

where $q = \max\{1, p-1\}$, and that $S_1(G)^2 \subsetneq S_1(G)$. It follows that $S_p(G)$ does not have a bounded approximate identity, and so $S_p(G)$ is not amenable in this case. However, $S_p(G)$ does have an approximate identity. We shall use the following fact. Let $f \in S_p(G)$, and take $\varepsilon > 0$. Then there exists $u \in L^1(G)$ such that $\|u\|_1 = 1$, $\operatorname{supp} \widehat{u}$ is compact, and $|||f - u \star f|||_p < \varepsilon$; in particular, $S_p(G)^2$ is dense in $(S_p(G), |||\cdot|||_p)$.

It remains to prove that $S_p(G)$ is always weakly amenable. For this, we shall establish some preliminary results in §2 and conclude the proof in §3.

2. Preliminaries

Let G be an LCA group, and let $p \in [1, \infty)$. We shall often write L^p for $L^p(G)$ and S_p for $S_p(G)$.

Lemma 2.1. *The subalgebra of S_p consisting of functions with compact support is dense in $(S_p, |||\cdot|||_p)$.*

Proof. The result is immediate in the case where $p = 2$, for in this case $S_2 = L^1 \cap L^2$.

Now consider the case where $p = 1$. Let $f \in S_1$, and take $\varepsilon > 0$. Then there exist $f_1, f_2 \in S_1$ with $|||f - f_1 \star f_2|||_1 < \varepsilon$. We have $f_1, f_2 \in S_2$, and so there exist $g_1, g_2 \in S_2$ such that $\operatorname{supp} g_j$ is compact and

$$|||f_j - g_j|||_2 < \varepsilon/m$$

for $j = 1, 2$, where $m = \max_{j=1,2}\{|||f_j|||_1 + 1\}$. Clearly $\operatorname{supp}(g_1 \star g_2)$ is compact. Set $h = f_1 \star f_2 - g_1 \star g_2$. Then $\|h\|_1 < 2\varepsilon$ and

$$\begin{aligned} \left\|\widehat{h}\right\|_1 &\leq \left\|\widehat{f_1} \cdot (\widehat{f_2} - \widehat{g_2})\right\|_1 + \left\|(\widehat{f_1} - \widehat{g_1}) \cdot \widehat{g_2}\right\|_1 \\ &\leq \left\|\widehat{f_1}\right\|_2 \left\|\widehat{f_2} - \widehat{g_2}\right\|_2 + \left\|\widehat{f_1} - \widehat{g_1}\right\|_2 \|\widehat{g_2}\|_2 \end{aligned}$$

by Hölder's inequality, and so $\|\widehat{h}\|_1 < 2\varepsilon$. Thus $|||h|||_1 < 4\varepsilon$, giving the result in this case.

The case of general p follows immediately. □

Let A and B be algebras. Then the tensor product $A \otimes B$ is an algebra with respect to a product that satisfies the conditions

$$(a_1 \otimes a_2)(b_1 \otimes b_2) = a_1 b_1 \otimes a_2 b_2 \quad (a_1, a_2 \in A,\, b_1, b_2 \in B)\,;$$

the algebra $A \otimes B$ is commutative when A and B are commutative. Now suppose that $(A, \|\cdot\|)$ and $(B, \|\cdot\|)$ are Banach algebras. Then $A \otimes B$ is a normed algebra with respect to the projective norm $\|\cdot\|_\pi$, where

$$\|z\|_\pi = \inf\left\{\sum_{j=1}^n \|a_j\|\,\|b_j\| : z = \sum_{j=1}^n a_j \otimes b_j,\, n \in \mathbb{N}\right\};$$

the completion of $(A \otimes B, \|\cdot\|_\pi)$ is the projective tensor product $(A\widehat{\otimes}B, \|\cdot\|_\pi)$.

For example, let G and H be locally compact groups. For $f \in L^1(G)$ and $g \in L^1(H)$, identify $f \otimes g \in L^1(G) \otimes L^1(H)$ with the element of $L^1(G \times H)$ given by

$$(f \otimes g)(x, y) = f(x)g(y) \quad (x \in G,\, y \in H)\,.$$

Then the identification extends to an isometric isomorphism of $L^1(G)\widehat{\otimes}L^1(H)$ with $L^1(G \times H)$. The dual group of $G \times H$ is $\Gamma_G \times \Gamma_H$. We write

$$(k \otimes \ell)(\gamma, \delta) = k(\gamma)\ell(\delta) \quad (\gamma \in \Gamma_G,\, \delta \in \Gamma_H)$$

whenever k and ℓ are functions on Γ_G and Γ_H, respectively. Clearly we have $\widehat{f} \otimes \widehat{g} = \widehat{f \otimes g}$ for $f \in L^1(G)$ and $g \in L^1(H)$.

Let $p \in [1, \infty)$, and suppose that $f \in S_p(G)$ and $g \in S_p(H)$. Then

$$\left\|\widehat{f \otimes g}\right\|_p = \left\|\widehat{f}\right\|_p \|\widehat{g}\|_p\,,$$

and so

$$|||f \otimes g|||_p = \|f\|_1 \|g\|_1 + \left\|\widehat{f}\right\|_p \|\widehat{g}\|_p \le |||f|||_p\, |||g|||_p\,.$$

Thus $f \otimes g \in S_p(G \times H)$, and the map

$$(f, g) \mapsto f \otimes g, \quad S_p(G) \times S_p(H) \to S_p(G \times H)\,,$$

is continuous and bilinear. It follows that there is a continuous linear map

$$T : S_p(G) \,\widehat{\otimes}\, S_p(H) \to S_p(G \times H)$$

such that $T(f \otimes g) = f \otimes g$ for $f \in S_p(G)$ and $g \in S_p(H)$. Clearly T is a homomorphism. The image of T is denoted by $\mathfrak{A}_p$.

Lemma 2.2. *Let $p \in [1, \infty)$. Then $\mathfrak{A}_p$ is dense in $S_p(G \times H)$.*

Proof. First consider the case where $p = 2$. The algebra $\mathfrak{A}_2$ contains $\chi_{E\times F}$ for each rectangle $E \times F$ in $G \times H$ such that E and F are Borel subsets of G and H, respectively, and the linear span of the collection of these functions is dense in $S_2(G \times H) = (L^1 \cap L^2)(G \times H)$. Thus the result holds.

Now consider the case where $p = 1$. Let $F \in S_1(G \times H)$, and take $\varepsilon > 0$. Then there exist $F_1, F_2 \in S_1(G \times H)$ with $|||F - F_1 \star F_2|||_1 < \varepsilon$. Clearly, we have $F_1, F_2 \in S_2(G \times H)$, and so there exist $G_1, G_2 \in \mathfrak{A}_2$ with

$$|||F_j - G_j|||_2 < \varepsilon/m$$

for $j = 1, 2$, where $m = \max_{j=1,2}\{|||f_j|||_1 + 1\}$. Essentially as before, $G_1 \star G_2 \in \mathfrak{A}_2$ and $|||F - G_1 \star G_2|||_1 < 4\varepsilon$. Thus $\mathfrak{A}_2$ is dense in $(S_1(G \times H), |||\cdot|||_1)$.

Again, the case of general p follows immediately. □

The following results are proved in [6, §2]. Let $\mathfrak{A}$ and $\mathfrak{B}$ be commutative Banach algebras. (i) Suppose that $\mathfrak{A}$ is weakly amenable and that there is a continuous homomorphism $T : \mathfrak{A} \to \mathfrak{B}$ such that $T(\mathfrak{A})$ is dense in $\mathfrak{B}$. Then $\mathfrak{B}$ is weakly amenable. (ii) Suppose that $\mathfrak{A}$ and $\mathfrak{B}$ are weakly amenable. Then $\mathfrak{A}\widehat{\otimes}\mathfrak{B}$ is weakly amenable. Using these results, we immediately obtain the following proposition.

Proposition 2.3. *Let G and H be LCA groups, and let $p \in [1,\infty)$. Suppose that $S_p(G)$ and $S_p(H)$ are weakly amenable. Then $S_p(G \times H)$ is weakly amenable.* □

3. The result

We first examine some special cases. Throughout, $p \in [1,\infty)$.

Lemma 3.1. *The algebra $S_p(\mathbb{Z})$ is amenable.*

Proof. We have $S_p(\mathbb{Z}) = \ell^1(\mathbb{Z})$, which is amenable. □

Let $\mathfrak{A}$ be a commutative algebra, and let $D : \mathfrak{A} \to E$ be a derivation into an $\mathfrak{A}$-module E. Then $Da = 0$ for each idempotent $a \in \mathfrak{A}$.

Lemma 3.2. *Let K be a compact group. Then $S_p(K)$ is weakly amenable.*

Proof. The dual group of K is denoted by Γ. Since Γ is discrete, the linear span of functions $f \in S_p(K)$ such that $\widehat{f} = \chi_{\{\gamma\}}$ for some $\gamma \in \Gamma$ is dense in $S_p(K)$. Each such function is an idempotent in $S_p(K)$.

Let $D : S_p(K) \to E$ be a continuous derivation into a Banach $S_p(K)$-module E. Then $D(f) = 0$ for each such idempotent f, and so $D = 0$. □

Lemma 3.3. *The algebra $S_p(\mathbb{R})$ is weakly amenable.*

Proof. Set $\Pi = \{\zeta = \xi + \mathrm{i}\eta : \xi > 0\}$, the open right-hand half-plane. The Poisson semigroup $(P^\zeta : \zeta \in \Pi)$ is defined by the formula

$$P^\zeta(t) = \frac{1}{\pi} \cdot \frac{\zeta}{\zeta^2 + t^2} \quad (t \in \mathbb{R})\,.$$

It is proved in [16, 2.17] that $(P^\zeta : \zeta \in \Pi)$ is an analytic semigroup in $L^1(\mathbb{R})$ and that

$$\left\|P^{1+\mathrm{i}\eta}\right\|_1 = O(\log|\eta|) \quad \text{as} \quad |\eta| \to \infty\,.$$

Further, it is proved that

$$(\mathfrak{F}P^\zeta)(y) = \exp(-\zeta\,|y|) \quad (y \in \mathbb{R})\,,$$

and so

$$\left|(\mathfrak{F}P^\zeta)(y)\right| = \exp(-\xi\,|y|) \quad (y \in \mathbb{R})\,.$$

Thus $\mathfrak{F}P^\zeta \in L^p(\mathbb{R})$ for each $\zeta \in \Pi$, and, for each $\eta \in \mathbb{R}$, we have

$$\left\|\mathfrak{F}P^{1+\mathrm{i}\eta}\right\|_p = \left(\int_{-\infty}^{\infty} \exp(-p\,|y|)\mathrm{d}y\right)^{1/p}\,,$$

a constant independent of η. Thus $(P^\zeta : \zeta \in \Pi)$ is an analytic semigroup in $S_p(\mathbb{R})$ satisfying the growth condition that

$$\left|\left|\left|P^{1+\mathrm{i}\eta}\right|\right|\right|_p = O(\log|\eta|) \quad \text{as} \quad |\eta| \to \infty\,.$$

Now let $D : S_p(\mathbb{R}) \to S_p(\mathbb{R})'$ be a continuous derivation.

A theorem of Galé [5, Theorem 2.3] asserts the following. Let A be a Banach algebra generated by an analytic semigroup $(a^\zeta : \zeta \in \Pi)$. Suppose that

$$\left\|a^{1+\mathrm{i}\eta}\right\| = O(|\eta|^\rho) \text{ as } |\eta| \to \infty,$$

where $0 \leq \rho < 1/2$. Then A is weakly amenable. By applying this theorem to the closed subalgebra A of $S_p(\mathbb{R})$ generated by the semigroup $(P^\zeta : \zeta \in \Pi)$, we see that $D(P^\zeta) = 0 \ (\zeta \in \Pi)$.

We extend this result by using a theorem of White [17, Theorem 2.4]. Let $t_0 \in \mathbb{R}$. For $n \in \mathbb{Z}$ and $\zeta \in \Pi$, define $a_n^\zeta = \tau_{nt_0} P^\zeta$. Then the function $\zeta \mapsto a_n^\zeta, \ \Pi \to S_p(\mathbb{R})$, is analytic, and

$$a_m^\zeta \cdot a_n^\lambda = a_{m+n}^{\zeta+\lambda} \quad (m, n \in \mathbb{Z},\, \zeta, \lambda \in \Pi)\,.$$

Further $|||a_n^1|||_p = |||P^1|||_p \ \ (n \in \mathbb{Z})$, and so

$$\lim_{n\to\infty} |||a_n^1|||_p \ |||a_{-n}^1|||_p / n = 0\,.$$

Thus the elements a_n^ζ satisfy the conditions in White's theorem, and so, by that theorem,

$$a_n^\zeta \cdot D(a_{-n}^\zeta) = P^\zeta \cdot D(P^\zeta) \quad (n \in \mathbb{Z},\, \zeta \in \Pi)\,.$$

In particular, $P^\zeta \cdot D(\tau_{t_0} P^\zeta) = 0 \ (\zeta \in \Pi)$, and so

$$D(\tau_{t_0} P^\zeta) = D(P^{\zeta/2} \star \tau_{t_0} P^{\zeta/2}) = P^{\zeta/2} \cdot D(\tau_{t_0} P^{\zeta/2}) = 0 \quad (\zeta \in \Pi)\,.$$

Let $f \in S_p(\mathbb{R})$. For each $\zeta \in \Pi$, we have

$$f \star P^\zeta = \int_{\mathbb{R}} f(t) \tau_t P^\zeta \,\mathrm{d}t$$

in $S_p(\mathbb{R})$, and so $D(f \star P^\zeta) = 0 \ (\zeta \in \Pi)$. Finally, $f \star P^{1/n} \to f$ in $S_p(\mathbb{R})$ as $n \to \infty$, and so $Df = 0$.

We have proved that $D = 0$, and so $S_p(\mathbb{R})$ is weakly amenable. □

We are grateful to the referee for a valuable remark about the above proof.

Theorem 3.4. *Let G be a locally compact abelian group, and let $p \in [1, \infty)$. Then the Segal algebra $(S_p(G), |||\cdot|||_p)$ is weakly amenable.*

Proof. Let E be a Banach $S_p(G)$-module, and let $D : S_p(G) \to E$ be a continuous derivation. We *claim* that $D = 0$.

By Lemma 2.1, it suffices to prove that $D(f) = 0$ whenever $f \in S_p(G)$ and $\operatorname{supp} f$ is compact. Let f be such a function. By [8, (5.14)], there is an open and closed, compactly generated subgroup, say H, of G such that $\operatorname{supp} f \subset H$. We may regard $S_p(H)$ as a closed subalgebra of $S_p(G)$ with $f \in S_p(H)$.

By the structure theorem for compactly generated LCA groups ([8, (9.8)]), the compactly generated group H is topologically isomorphic to a group of the form $\mathbb{R}^m \times \mathbb{Z}^n \times K$ for some $m, n \in \mathbb{Z}^+$ and some compact abelian group K. It follows from Lemmas 3.1, 3.2, and 3.3, and Proposition 2.3 that the algebra $S_p(H)$ is weakly amenable, and so $D \mid S_p(H) = 0$. In particular, $D(f) = 0$, as required. □

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