

LOCAL AUTOMORPHISMS

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ABSTRACT. We show that any linear map on a finite dimensional CSL algebra \mathcal{A} which at each point is equal to the value of some automorphism of \mathcal{A} is either an automorphism or can be factored as an automorphism and the transpose of a self-adjoint summand of \mathcal{A} . New examples of local mappings are constructed.

1. INTRODUCTION AND PRELIMINARIES

A problem which several authors have considered recently is that of finding sufficient conditions on a linear map to ensure it preserves a particular algebraic or function property. One aspect of this is the notion of a *local mapping*—a function that agrees at each point with some map (the map possibly changing from point to point) that has the desired property. In this note, we examine those functions which agree with automorphisms at each point or multiple points. Formally, a (*linear*) *local automorphism* is a linear mapping α from an algebra \mathcal{A} into itself such that, for each $A \in \mathcal{A}$, there is an automorphism β_A of \mathcal{A} such that $\alpha(A) = \beta_A(A)$. It is immediate from this definition that such a map must be a linear isomorphism of \mathcal{A} into itself. A *2-local automorphism* θ is a map, not necessarily linear, such that if $A, B \in \mathcal{A}$, then there is an automorphism $\beta_{A,B}$ with the property that $\theta(A) = \beta_{A,B}(A)$ and $\theta(B) = \beta_{A,B}(B)$.

Local automorphisms and other local maps have been studied in a variety of contexts. Larson initially considered local maps in his examination of reflexivity and interpolation for subspaces of $\mathcal{B}(\mathcal{H})$, where \mathcal{H} is a Hilbert space [L]. Kadison showed that any local derivation on a von Neumann algebra is actually a derivation [K]. He also constructs an example due to C. Jensen of an algebra, but not an operator algebra, which has nontrivial local derivations. That all local derivations are actually global derivations has been proven to be the case for more general types of derivations of von Neumann and on C*-algebras [B, Sh], for $\mathcal{B}(X)$ and standard operator algebras [BS2, LS], and for many nonselfadjoint operator algebras [HW, C]. A nontrivial local derivation on an operator algebra was found in [C]. Other local maps such as local Jordan derivations and homomorphisms have also been considered (e.g., see [BS2]). Using a slightly different definition of local map, locally contractive maps on finite dimensional CSL algebras have been studied by Davidson [D].

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Local automorphisms are quite natural to consider in several situations. For a finite dimensional full matrix algebra, inner local automorphisms (those implemented by inner automorphisms) preserve the spectrum of an operator. Spectrum preserving maps have been studied for over a century. For example, Frobenius [F] showed that a linear map on $M_n(\mathbf{C})$ into itself that preserves the determinant must be the composition of an automorphism or antiautomorphism with left multiplication by a matrix of determinant 1. In [JS], spectrum preserving maps were characterized for maps from $\mathcal{B}(\mathcal{X})$ into $\mathcal{B}(\mathcal{Y})$, where \mathcal{X} and \mathcal{Y} are Banach spaces. Larson and Sourour took this further:

Theorem 1.1 ([LS, Theorem 2.2]). *If $M_n(\mathbf{C})$ is the algebra of $n \times n$ complex matrices, then $\alpha : M_n(\mathbf{C}) \rightarrow M_n(\mathbf{C})$ is a local automorphism iff α is an automorphism or an anti-automorphism, i.e., either α takes the form $\alpha(A) = TAT^{-1}$ for a fixed T and for all $A \in M_n(\mathbf{C})$ or $\alpha(A) = TA^tT^{-1}$ for a fixed T and for all $A \in M_n(\mathbf{C})$.*

We study finite dimensional CSL algebras. We say that \mathcal{A} is a *finite dimensional CSL algebra* if there is a set of matrix units $\{E_{ij}\}_{i,j}$, which contain all diagonal matrix units $\{E_{ii}\}_i$ and are closed under multiplication, whose linear span is equal to \mathcal{A} . These algebras, also referred to as digraph algebras, have been studied over the last 15 years (e.g., see [C, D, DP, GM, P]) and interesting behavior has been observed. This class, for example, contains algebras that have nonzero Hochschild cohomology groups and automorphisms that do not preserve rank.

We generalize Larson and Sourour's theorem to the much broader class of finite dimensional CSL algebras. The structure of local automorphisms is more rigid here: if there are no self-adjoint summands, then the map must be an automorphism, providing a sufficient condition to ensure a map is an automorphism. This advances Larson's program of studying concepts related to reflexivity [L]. Also, this theorem could provide a tool to help understand the automorphisms and local automorphisms of direct limits of finite dimensional CSL algebras such as triangular subalgebras of AF C^* -algebras, a heavily studied class of operator algebras [P].

Examples of local mappings can be difficult to obtain, even in finite dimensions (see [K]). Nontrivial local automorphisms and 2-local automorphisms on matrix algebras are constructed in section 3. Example 3.1 strongly suggests that, for finite dimensional matrix algebras, Theorem 2.4 marks the line between algebras having nontrivial local automorphisms and those having none.

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2. LOCAL AUTOMORPHISMS

We first develop some lemmas that will allow us to calculate with local automorphisms. Let \mathcal{A} denote a finite dimensional CSL algebra and \mathcal{D} its diagonal.

Lemma 2.1. *If α is a local automorphism on finite dimensional CSL algebra \mathcal{A} , then $\alpha = \beta\alpha'$, where β is an automorphism of \mathcal{A} and α' is a local automorphism with $\alpha'(E_{ii}) = E_{ii}$ for all i .*

Proof. Let $E \in \mathcal{A}$ be an idempotent. Then $\alpha(E^2) = \alpha(E) = \alpha_E(E) = \alpha_E(E^2) = (\alpha_E(E))^2 = (\alpha(E))^2$, where α_E is an automorphism that agrees with α at E . So

for the idempotent $E_{ii} + E_{jj}$, we have

$$\begin{aligned} \alpha(E_{ii} + E_{jj}) &= \alpha((E_{ii} + E_{jj})^2) \\ &= (\alpha(E_{ii} + E_{jj}))^2 \\ &= \alpha(E_{ii}) + \alpha(E_{ii})\alpha(E_{jj}) + \alpha(E_{jj})\alpha(E_{ii}) + \alpha(E_{jj}). \end{aligned}$$

Thus

$$\alpha(E_{ii})\alpha(E_{jj}) + \alpha(E_{jj})\alpha(E_{ii}) = 0.$$

Since $\alpha(E_{ii}E_{jj} + E_{jj}E_{ii}) = 0$, we can conclude that $\alpha|_{\mathcal{D}}$ is a Jordan homomorphism on \mathcal{D} . This implies that $\alpha(D^k) = (\alpha(D))^k$ for every $D \in \mathcal{D}$. As \mathcal{D} is abelian, it is singly generated as a von Neumann algebra by some $T \in \mathcal{D}$. Let α_T be the automorphism of \mathcal{A} that agrees with α at T . Since $\alpha|_{\mathcal{D}}$ preserves powers, $\alpha|_{\mathcal{D}}(T^k) = (\alpha|_{\mathcal{D}}T)^k = (\alpha_T(T))^k = \alpha_T(T^k)$. $\alpha|_{\mathcal{D}}$ and $\alpha_T|_{\mathcal{D}}$ agree on polynomials of T . Using continuity of the maps involved and the functional calculus, we have that they agree on continuous functions of T , so agree on \mathcal{D} . Let $\beta = \alpha_T$ and $\alpha' = (\alpha_T)^{-1}\alpha$, which is a local automorphism that fixes the diagonal. Thus, $\alpha = \beta\alpha'$, as desired. \square

Lemma 2.2. *If α is a local automorphism on \mathcal{A} and $\{E_{ij}\}_{i,j}$ the partial system of matrix units that generate \mathcal{A} as an algebra, with $\alpha(E_{ii}) = E_{ii}$ for all i , then $\alpha(E_{ij}) = s_{ij}E_{ij}$ or $s_{ij}E_{ji}$.*

Proof. Consider α evaluated at the idempotent $E_{ii} + aE_{ij}$, where $a \neq 0$ is a complex scalar. Then,

$$\begin{aligned} \alpha(E_{ii} + aE_{ij}) &= \alpha((E_{ii} + aE_{ij})^2) = (\alpha(E_{ii} + aE_{ij}))^2 \\ &= \alpha(E_{ii}) + a\alpha(E_{ii})\alpha(E_{ij}) + a\alpha(E_{ij})\alpha(E_{ii}) + a^2\alpha(E_{ij}). \end{aligned}$$

Repeating this with $E_{jj} + aE_{ij}$ and noting that the diagonal matrix units are fixed by α , we obtain the following set of equations:

$$(2.1) \quad E_{ii}\alpha(E_{ij}) + \alpha(E_{ij})E_{ii} = \alpha(E_{ij}),$$

$$(2.2) \quad E_{jj}\alpha(E_{ij}) + \alpha(E_{ij})E_{jj} = \alpha(E_{ij}),$$

$$(2.3) \quad (\alpha(E_{ij}))^2 = 0.$$

Equations (2.1) and (2.2) imply that $\alpha(E_{ij}) = sE_{ij} + tE_{ji}$ for some $s, t \in \mathbf{C}$. Using equation (2.3), we find that

$$0 = (\alpha(E_{ij}))^2 = (sE_{ij} + tE_{ji})^2 = st(E_{ii} + E_{jj}).$$

So we must have either $s = 0$ or $t = 0$, but not both, since α is a linear isomorphism. Set this constant equal to s_{ij} . \square

A finite dimensional CSL algebra is *wholly non-self adjoint* if it does not contain any self-adjoint direct summands; that is, there is no reducing projection $P \in \text{Lat}\mathcal{A}$ such that PAP is self-adjoint and $P^\perp\mathcal{A}P = PAP^\perp = 0$. It is an elementary exercise to see that any finite dimensional CSL algebra can be decomposed into a direct sum $\mathcal{R} \oplus \mathcal{B}$ of a finite dimensional C^* -algebra \mathcal{R} and a wholly non-self-adjoint CSL algebra \mathcal{B} acting on a Hilbert space $\mathcal{H}_k \oplus \mathcal{H}_l$. Moreover, there is a maximal decomposition in the sense that $k = \dim \mathcal{H}_k$ is as large as possible.

Lemma 2.3. *Let $\mathcal{A} = \mathcal{R} \oplus \mathcal{B}$, where \mathcal{R} is self-adjoint and \mathcal{B} is wholly non-self-adjoint. Then any local automorphism restricted to \mathcal{B} is Schur multiplication by a fixed matrix. $\alpha|_{\mathcal{B}}$ is an automorphism of \mathcal{B} iff $s_{ij} \neq 0$ when $E_{ij} \in \mathcal{A}$ and $s_{ij}s_{jk} = s_{ik}$ when $E_{ij}, E_{jk}, E_{ik} \in \mathcal{A}$ and $s_{ii} = 1$ for all i .*

Proof. By Lemmas 2.1 and 2.2, we can restrict our attention to $E_{ij} \in \mathcal{B}$. Suppose that $\alpha(E_{ij}) = s_{ij}E_{ji}$. Because $E_{ii} \in \mathcal{B}$, there is, without loss of generality, an $E_{ik} \in \mathcal{B}$ with $k \neq j$, such that $E_{ki} \notin \mathcal{A}$. Let $A = E_{ij} + E_{ik}$. We know that $\alpha(A) = s_{ij}E_{ji} + s_{ik}E_{ik} = \alpha_{(E_{ij}+E_{ik})}(E_{ij} + E_{ik})$, where $\alpha_{(E_{ij}+E_{ik})}$ is an automorphism of \mathcal{B} . By a theorem of Gilfeather and Moore [GM, Theorem 2.1], $\alpha_{(E_{ij}+E_{ik})}$ is the composition of a similarity $\text{Ad}T$ and a Schur automorphism ρ . A Schur automorphism of \mathcal{B} cannot kill any matrix unit, so $\rho(E_{ij} + E_{ik}) = a_{ij}E_{ij} + a_{ik}E_{ik}$, so $\rho(E_{ij} + E_{ik})$ remains a rank-1 operator. Applying $\text{Ad}T$, we have

$$\alpha(E_{ij} + E_{ik}) = \text{Ad}T(a_{ij}E_{ij} + a_{ik}E_{ik}) = s_{ij}E_{ji} + s_{ik}E_{ik},$$

a rank-2 operator. Since a similarity transformation preserves rank, we have a contradiction, so that $\alpha|_{\mathcal{B}}$ is just Schur multiplication by a fixed matrix.

To determine when the map is an automorphism, assume that the cocycle condition holds and that $s_{ij} \neq 0$ when $E_{ij} \in \mathcal{A}$. As α is a linear isomorphism and the matrix units of \mathcal{A} generate it as an algebra, it is enough to show α is multiplicative on matrix units. Using the cocycle equation shows that $\alpha(E_{ij}E_{jk}) = \alpha(E_{ik}) = \alpha(E_{ij})\alpha(E_{jk})$, as desired. The other direction is immediate from the same calculation. \square

We now have all of the necessary tools to prove the main theorem.

Theorem 2.4. *Let $\mathcal{A} = \mathcal{R} \oplus \mathcal{B}$ be a finite dimensional CSL algebra with self-adjoint summand \mathcal{R} and wholly non-self-adjoint summand \mathcal{B} . Let α be a local automorphism on \mathcal{A} . Then α is either an automorphism of \mathcal{A} or an automorphism composed with the map that is the transpose map on \mathcal{R} and the identity on \mathcal{B} .*

Proof. Using Theorem 1.1, Lemmas 2.1 – 2.3, it suffices to show that a local automorphism of the wholly non-self-adjoint summand \mathcal{B} is an automorphism. So we may assume that α is given by Schur multiplication by a fixed matrix with $s_{ij} \neq 0$ if $E_{ij} \in \mathcal{B}$ and $s_{ii} = 1$. Suppose that $E_{ij}, E_{jk}, E_{ik}, E_{jj} \in \mathcal{A}$. Let A be the rank-1 operator $E_{ij} + E_{jk} + E_{ik} + E_{jj}$. Now $\alpha(A) = s_{ij}E_{ij} + s_{jk}E_{jk} + s_{ik}E_{ik} + E_{jj}$, which is either rank-1 or rank-2. If it is rank-1, then it is immediate that $s_{ij}s_{jk} = s_{ik}$. Suppose that $\alpha(A)$ is rank-2. We know that $\alpha(A) = \alpha_A(A)$, where α_A is an automorphism of \mathcal{A} . Again, by [GM, Theorem 2.1], $\alpha_A = (\text{Ad}T)\rho$, where ρ is a Schur automorphism. $\text{Ad}T$ preserves the rank of a matrix, so we must have that $\rho(A)$ is rank-2. $\rho(A) = r_{ij}E_{ij} + r_{jk}E_{jk} + r_{ik}E_{ik} + E_{jj}$ being rank-2 implies that $r_{ij}r_{jk} \neq r_{ik}$, a contradiction to the fact that ρ is an automorphism. Thus α must be an automorphism by Lemma 2.3. \square

3. EXAMPLES

There are few examples in the literature of nontrivial local maps on operator algebras. 2-local automorphisms are introduced in [S2].

Example 3.1. A nontrivial local automorphism on a subalgebra of $M_3(\mathbf{C})$.

Let $\mathcal{A} \subset M_3(\mathbf{C})$ be the algebra of finite dimensional upper triangular matrices constant on each diagonal, i.e., $\mathcal{A} = \{a(\sum_{i=1}^3 E_{ii}) + b(E_{12} + E_{21}) + cE_{13} \mid a, b, c \in \mathbf{C}\}$, where $\{E_{ij}\}_{i,j}$ are the standard matrix units. Let $I = \sum_{i=1}^3 E_{ii}$ and $T = E_{12} + E_{21}$. Then $\{I, T, T^2\}$ is a basis for \mathcal{A} . Automorphisms on this algebra are characterized as linear maps γ of the following form: $\gamma(I) = I, \gamma(T) = xT + yT^2, \gamma(T^2) = xT^2, x, y \in \mathbf{C}, x \neq 0$. Define the function $\alpha : \mathcal{A} \rightarrow \mathcal{A}$ on the basis and extend linearly:

$\alpha(I) = I, \alpha(T) = e^{\frac{1}{2}}T, \alpha(T^2) = T^2$. Note that $\alpha(T^2) \neq (\alpha(T))^2$, so it is not an automorphism. To check that the map is local is straightforward except in the case of an element of the form $aT + bT^2$. It is easy to check that the automorphism β defined by $\beta(I) = I, \beta(T) = e^{\frac{1}{2}}T + \frac{b}{a}(1 - e)T^2, \beta(T^2) = T^2$, agrees with α on these elements.

Example 3.2. A nontrivial 2-local automorphism on a subalgebra of $M_3(\mathbf{C})$.

Let $\mathcal{A} = \text{span}\{I, E_{12}, E_{13}\} \subset M_3(\mathbf{C})$. A generic automorphism of this algebra is the natural linear extension of the map γ acting on the basis as follows: $\gamma(I) = I, \gamma(E_{12}) = aE_{12} + bE_{13}, \gamma(E_{13}) = cE_{12} + dE_{13}$, where $ad - bc \neq 0$. Let $\theta(a_{11}I + a_{12}E_{12} + a_{13}E_{13}) = a_{11}I + a_{12}^3E_{12} + a_{13}^3E_{13}$. This is nonlinear, thus not an automorphism, but it is a 2-local automorphism. Let $A = a_{11}I + a_{12}E_{12} + a_{13}E_{13}$ and $B = b_{11}I + b_{12}E_{12} + b_{13}E_{13}$. If they are linearly dependent, it is easy to obtain the needed automorphism. If they are linearly independent, then $\theta(A)$ and $\theta(B)$ are linearly independent. In particular,

$$\left\{ \begin{pmatrix} a_{12} \\ a_{13} \end{pmatrix}, \begin{pmatrix} b_{12} \\ b_{13} \end{pmatrix} \right\}, \left\{ \begin{pmatrix} a_{12}^3 \\ a_{13}^3 \end{pmatrix}, \begin{pmatrix} b_{12}^3 \\ b_{13}^3 \end{pmatrix} \right\}$$

are bases for \mathbf{C}^2 . So there exists a nonsingular change of basis matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ such that

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} a_{12} \\ a_{13} \end{pmatrix} = \begin{pmatrix} a_{12}^3 \\ a_{13}^3 \end{pmatrix}, \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} b_{12} \\ b_{13} \end{pmatrix} = \begin{pmatrix} b_{12}^3 \\ b_{13}^3 \end{pmatrix}.$$

So there is an automorphism $\beta_{A,B}$ of \mathcal{A} , defined by a, b, c, d , so that $\beta_{A,B}(A) = \theta(A), \beta_{A,B}(B) = \theta(B)$.

For 2-local automorphisms, we have a lemma analogous to Lemma 2.1.

Proposition 3.3. *If θ is a 2-local automorphism of a finite dimensional CSL algebra \mathcal{A} , then there is an automorphism α of \mathcal{A} such that $\alpha^{-1}\theta$ is a 2-local automorphism which is the identity on the diagonal \mathcal{D} .*

Proof. Since the algebra is a finite dimensional matrix algebra and the map is injective, note that $\theta(I) = I$ and $\theta(0) = 0$. The following are immediate from the definition of a 2-local automorphism: (1) $\theta(A)\theta(B) = 0 \Leftrightarrow AB = 0$; (2) $\theta(A^2) = \theta(A)^2$; (3) $\theta(aA) = a\theta(A), a \in \mathbf{C}$. Consider the diagonal of $\mathcal{A}, \mathcal{D} = \text{span}\{E_{11}, E_{22}, \dots, E_{nn}\}$. We claim that $\theta|_{\mathcal{D}}$ is an automorphism of \mathcal{D} into \mathcal{A} . Let $F_{jj} = \theta(E_{jj})$. By (1) and (2), the F_{jj} are n orthogonal projections. By an elementary theorem of linear algebra, their sum is I . Let $A = \sum_{j=1}^n a_j E_{jj}$. Then there is an automorphism α of \mathcal{A} such that for a given $k, \theta(A) = \alpha(A), \theta(E_{kk}) = \alpha(E_{kk})$. This implies that

$$F_{kk}\theta(A) = \theta(E_{kk})\theta(A) = \theta(a_k E_{kk}) = a_k\theta(E_{kk}) = \theta(A)\theta(E_{kk}) = \theta(A)F_{kk}.$$

Consequently, for all $k,$

$$F_{kk} \left(\theta(A) - \sum_{j=1}^n a_j F_{jj} \right) = \left(\theta(A) - \sum_{j=1}^n a_j F_{jj} \right) F_{kk} = 0.$$

So θ is an isomorphism of the diagonal obtained by restricting an automorphism of the algebra, and $\alpha^{-1}\theta$ is the desired map. \square

Problem 3.4. For any finite dimensional CSL algebra, is any 2-local automorphism automatically an automorphism?

Šemrl showed that for $\mathcal{B}(\mathcal{H})$, \mathcal{H} a Hilbert space, a 2-local automorphism must be a global automorphism in [S2]. It is immediate from the previous proposition that a 2-local automorphism behaves on individual matrix units like a Schur product type map composed with an automorphism of the algebra. Another implication is that such a map, after being reduced by the proposition to one that fixes the diagonal, must map rows to rows. However, the lack of linearity prevents a proof similar to that of Theorem 2.4. The evidence points to a positive answer to the question.

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