PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 128, Number 9, Pages 2833–2834
S 0002-9939(00)05857-3
Article electronically published on May 12, 2000

ERRATUM TO "CHERN NUMBERS OF CERTAIN LEFSCHETZ FIBRATIONS"

ANDRÁS I. STIPSICZ

(Communicated by Ronald A. Fintushel)

The proof of Proposition 2.5 in [S] is incorrect as stated. The argument given in that paper proves only the following more restrictive version:

Proposition 2.5′. *Suppose that* $f\colon X \to \Sigma$ *is a genus-k Lefschetz fibration which can be written as the fiber sum of a Lefschetz fibration over* S^2 *with the trivial genus-k fibration on* Σ. *If* $k \geq 2$ *and* $g(\Sigma) = l \geq 1$, *then* $b_1(X) \leq 2k + 2l$ *and* $b_2^-(X) \geq 2kl + 1$. □

Remark. The first conclusion ($b_1(X) \leq 2k+2l$) is valid in the general case presented in [S]; the proof of $b_2^-(X) \geq 2kl + 1$, however, uses the triviality of the fibration over a system of homotopically nontrivial loops of the base Σ. For this reason the extra assumption on the fibration $f\colon X \to \Sigma$ described above should be added to Corollary 2.6 and Theorems 1.3 and 1.5 in [S].

A different argument provides a proof for the following (weaker) bound on the first Chern number of a Lefschetz fibration $f\colon X \to \Sigma$.

Lemma. *If* $f\colon X \to \Sigma$ *is a genus-k Lefschetz fibration,* $k \geq 2$ *and* $l = g(\Sigma) \geq 1$, *then* $c_1^2(X) \leq 5c_2(X) + 6(2l - 2)$.

Proof. A covering argument given in [K] can be adapted to the present situation. Note first that for a genus-k Lefschetz fibration $X \to \Sigma$ we have $\sigma(X) \leq b_2(X) = \chi(X) - 2 + 2b_1(X) \leq \chi(X) + 4k + 4l - 2$, hence

$$c_1^2(X) \leq 5c_2(X) + 6(2l - 2) + 6(2k + 1). \tag{1}$$

Suppose that $\varphi_n\colon \Sigma(n) \to \Sigma$ is an (unramified) n-fold covering and define the genus-k Lefschetz fibration $X(n) \to \Sigma(n)$ as the pull-back of $f\colon X \to \Sigma$ via φ_n. Since both the Euler characteristic and the signature multiplies by n under an n-fold cover, we get that $c_1^2(X(n)) - 5c_2(X(n)) + 6\chi(\Sigma(n)) = n(c_1^2(X) - 5c_2(X) + 6\chi(\Sigma))$. Consequently if $c_1^2(X) - 5c_2(X) + 6\chi(\Sigma)$ is positive for $X \to \Sigma$, then by choosing $n > 6(2k+1)$ we get that $c_1^2(X(n)) - 5c_2(X(n)) + 6\chi(\Sigma(n)) > 6(2k+1)$, contradicting the trivial estimate (1). This observation shows that $c_1^2(X) - 5c_2(X) + 6\chi(\Sigma) \leq 0$, which proves the lemma. □

Remark. Note that the above lemma proves Proposition 2.5 of [S] in case $l = 1$, i.e., for fibrations over the torus T^2. We also would like to point out that the other result of [S] (stating that relative minimality of a Lefschetz fibration is equivalent

Received by the editors December 27, 1999.
1991 *Mathematics Subject Classification.* Primary 57R99, 57M12.

to minimality over a base with nonzero genus) is independent of Proposition 2.5, hence is unaffected by the mistake discussed above.

Acknowledgement

The author would like to thank Dieter Kotschick for pointing out the mistake in [S].

References

[S] A. Stipsicz, *Chern numbers of certain Lefschetz fibrations*, Proc. AMS **128** (2000), 1845–1851. CMP 99:01

[K] D. Kotschick, *Signatures, monopoles and mapping class groups*, Math. Res. Letters **5** (1998), 227–234. MR **99d:**57023

Department of Analysis, ELTE TTK, 1088. Múzeum krt. 6-8., Budapest, Hungary
E-mail address: stipsicz@cs.elte.hu