PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 129, Number 9, Pages 2777–2782
S 0002-9939(01)06204-9
Article electronically published on April 24, 2001

CHARACTERIZATIONS OF PARACOMPACT-LIKE PROPERTIES BY MEANS OF SET-VALUED SEMI-CONTINUOUS SELECTIONS

KAZUMI MIYAZAKI

(Communicated by Alan Dow)

Abstract. We give a characterization of countably paracompact and collectionwise normal spaces by means of set-valued semi-continuous selections. This provides a positive answer to a problem of V. Gutev.

1. Main results

Let X and Y be topological spaces, and let 2^Y be the family of non-empty subsets of Y. Let

$$\mathcal{F}(Y) = \{F \in 2^Y : F \text{ is closed}\},\ \mathcal{C}(Y) = \{F \in \mathcal{F}(Y) : F \text{ is compact}\},\ \text{and}$$

$$\mathcal{C}'(Y) = \mathcal{C}(Y) \cup \{Y\}.$$

A set-valued mapping $\Phi : X \to 2^Y$ is *lower semi-continuous* (*l.s.c.*) if the set

$$\Phi^{-1}(U) = \{x \in X : \Phi(x) \cap U \neq \emptyset\}$$

is open in X for every open subset U of Y, and Φ is *upper semi-continuous* (*u.s.c.*) if the set

$$\Phi^\sharp(U) = \{x \in X : \Phi(x) \subset U\}$$

is open in X for every open subset U of Y. A mapping $\varphi : X \to 2^Y$ is a *selection for* Φ if $\varphi(x) \subset \Phi(x)$ for every point $x \in X$.

There is a series of results which characterize separation and covering properties (like paracompactness, collectionwise normality, normality and so on) by means of the existence of continuous selections for l.s.c. mappings. We mention two of them.

Theorem 1.1 (Michael, Choban [12, 2]). *A T_1-space X is paracompact if and only if for every completely metrizable space Y, every l.s.c. mapping $\Phi : X \to \mathcal{F}(Y)$ admits a u.s.c. selection $\varphi : X \to \mathcal{C}(Y)$.*

Theorem 1.2 (Choban and Nedev [3]). *A T_1-space X is collectionwise normal if and only if for every completely metrizable space Y, every l.s.c. mapping $\Phi : X \to \mathcal{C}'(Y)$ admits a u.s.c. selection $\varphi : X \to \mathcal{C}(Y)$.*

Received by the editors May 12, 1999 and, in revised form, December 12, 1999.
2000 *Mathematics Subject Classification.* Primary 54B20, 54C65.
Key words and phrases. Hyperspace, selection, semi-continuous.

In the present paper we extend the above two characterizations by the following two theorems.

Theorem 1.3. *For a T_1-space X the following conditions are equivalent:*

(*a*) *X is countably paracompact and collectionwise normal.*
(*b*) *For every completely metrizable space Y, every l.s.c. mapping $\Phi : X \to \mathcal{C}'(Y)$, and every u.s.c. selection $\theta : X \to \mathcal{C}(Y)$ for Φ, there exist an l.s.c. $\varphi : X \to \mathcal{C}(Y)$ and a u.s.c. $\psi : X \to \mathcal{C}(Y)$ such that $\theta(x) \subset \varphi(x) \subset \psi(x) \subset \Phi(x)$ for every $x \in X$.*
(*c*) *X is a normal space and for every completely metrizable space Y and each u.s.c. mapping $\theta : X \to \mathcal{C}(Y)$, there exist an l.s.c. $\varphi : X \to \mathcal{C}(Y)$ and a u.s.c. $\psi : X \to \mathcal{C}(Y)$ such that $\theta(x) \subset \varphi(x) \subset \psi(x)$ for every $x \in X$.*
(*d*) *X is a normal space and for every Hilbert space Y and each u.s.c. mapping $\theta : X \to 2^Y$ all of whose values are finite-dimensional simplexes (i.e. convex hulls of finite subsets of Y), there exist an l.s.c. $\varphi : X \to \mathcal{C}(Y)$ and a u.s.c. $\psi : X \to \mathcal{C}(Y)$ such that $\theta(x) \subset \varphi(x) \subset \psi(x)$ for every $x \in X$.*

The above theorem gives an answer to a problem posed by V. Gutev [9].

Theorem 1.4. *A normal space X is collectionwise normal if and only if for every finite dimensional completely metrizable space Y and every u.s.c. mapping $\theta : X \to \mathcal{C}_m(Y)$ for some natural number m, there exist an l.s.c. mapping $\varphi : X \to \mathcal{C}(Y)$ and a u.s.c. mapping $\psi : X \to \mathcal{C}(Y)$ such that $\theta(x) \subset \varphi(x) \subset \psi(x)$ for every $x \in X$ where $\mathcal{C}_m(Y) = \{F \in \mathcal{F}(Y) : |F| \leq m\}$.*

2. A construction of Michael's pairs between set-valued mappings

We will say that a mapping $\theta : X \to 2^Y$ has the *locally finite lifting property* if for every locally finite family $\mathcal{M}$ of closed subsets of Y there exists a locally finite family $\mathcal{U} = \{U_M : M \in \mathcal{M}\}$ of open subsets of X such that $\theta^{-1}(M) \subset U_M$ for every $M \in \mathcal{M}$.

Recall that a *sieve* $\mathcal{S} = (\{\mathcal{B}_n, n \in \mathbb{N}\}, \pi_n)$ on a space X is a sequence of covers $\mathcal{B}_n = \{B_\alpha : \alpha \in A_n\}_{n\in\mathbb{N}}$ of X (with disjoint A_n's) together with maps $\pi_n : A_{n+1} \to A_n$ such that $B_\alpha = \bigcup\{B_\gamma : \gamma \in \pi_n^{-1}(\alpha)\}$ for all $n \in \mathbb{N}$ and $\alpha \in A_n$ [1, 14]. A sieve $\mathcal{S}$ is *closed* (*open*) if each B_α is closed (open) in X, and is *locally finite* if $\mathcal{B}_n$ is index locally finite in X for every $n \in \mathbb{N}$.

For a subset $A \subset X$, we use $\overline{A}$ to denote the closure of A in X.

Lemma 2.1. *Let X be a collectionwise normal space, let Y be a space, let $\Phi : X \to \mathcal{C}'(Y)$ be an l.s.c. mapping, and let $\theta : X \to 2^Y$ be a u.s.c. selection for Φ with the locally finite lifting property. Also, let $(\{V_\alpha : \alpha \in A_n\}, \pi_n)$ be a locally finite open sieve on Y and let $(\{F_\alpha : \alpha \in A_n\}, \pi_n)$ be a closed sieve on Y such that $F_\alpha \subset V_\alpha$ for every $\alpha \in A_n$ and every n. Then there exists a locally finite open sieve $(\{U_\alpha : \alpha \in A_n\}, \pi_n)$ on X such that*

$$\theta^{-1}(F_\alpha) \subset U_\alpha \subset \overline{U_\alpha} \subset \Phi^{-1}(V_\alpha) \text{ whenever } n \in \mathbb{N} \text{ and } \alpha \in A_n.$$

Proof. We prove our lemma by induction on $n \in \mathbb{N}$. Since $\theta^{-1}(F_\alpha)$ is closed and $\Phi^{-1}(V_\alpha)$ is open in X with $\theta^{-1}(F_\alpha) \subset \Phi^{-1}(V_\alpha)$, using normality of X and the locally finite lifting property of θ we can find a locally finite open cover $\{U_\alpha : \alpha \in$

$A_1\}$ of X such that

$$\theta^{-1}(F_\alpha) \subset U_\alpha \subset \overline{U_\alpha} \subset \Phi^{-1}(V_\alpha) \text{ for every } \alpha \in A_1.$$

Assume that we have already constructed $\{U_\alpha : \alpha \in A_n\}$ for some n. For $\alpha \in A_n$, by the locally finite lifting property of θ we can construct a locally finite open family $\{U^1_\beta : \beta \in \pi_n^{-1}(\alpha)\}$ in X such that

$$U^1_\beta \subset U_\alpha \text{ and } \theta^{-1}(F_\beta) \subset U^1_\beta \subset \overline{U^1_\beta} \subset \Phi^{-1}(V_\beta) \text{ for every } \beta \in \pi_n^{-1}(\alpha).$$

Next, for every $\alpha \in A_n$, we construct a locally finite open family $\{U^2_\beta : \beta \in \pi_n^{-1}(\alpha)\}$ in X such that

$$U_\alpha \backslash \bigcup\{U^1_\beta : \beta \in \pi^{-1}(\alpha)\} \subset \bigcup\{U^2_\beta : \beta \in \pi_n^{-1}(\alpha)\}$$

and

$$U^2_\beta \subset U_\alpha \text{ and } \overline{U^2_\beta} \subset \Phi^{-1}(V_\beta) \text{ for every } \beta \in \pi_n^{-1}(\alpha).$$

Indeed, choose an open subset W of $\overline{U_\alpha}$ such that

$$\overline{U_\alpha} \backslash \bigcup\{U^1_\beta : \beta \in \pi_n^{-1}(\alpha)\} \subset W \subset \overline{W} \subset \overline{U_\alpha} \backslash \theta^{-1}(F_\alpha).$$

Set $Z = \overline{W}$ and $\Psi = \Phi|Z$. Then Z is collectionwise normal and $\Psi : Z \to \mathcal{C}'(Y)$ is an l.s.c. mapping. Since $\{V_\beta : \beta \in \pi_n^{-1}(\alpha)\}$ is a locally finite open family of Y, using results from [7, 15] we can find a locally finite open cover $\{W_\beta : \beta \in \pi_n^{-1}(\alpha)\}$ of Z (i.e. locally finite in X) such that $\overline{W_\beta} \subset \Psi^{-1}(V_\beta)$. Set $U^2_\beta = U_\alpha \cap W_\beta$ and $U_\beta = U^1_\beta \cup U^2_\beta$ for every $\beta \in \pi_n^{-1}(\alpha)$. □

Theorem 2.2. *Let X be a collectionwise normal space, let Y be a completely metrizable space, let $\Phi : X \to \mathcal{C}'(Y)$ be an l.s.c. mapping, and let $\theta : X \to 2^Y$ be a u.s.c. selection for Φ having the locally finite lifting property. Then there exist an l.s.c. $\varphi : X \to \mathcal{C}(Y)$ and a u.s.c. $\psi : X \to \mathcal{C}(Y)$ such that*

$$\theta(x) \subset \varphi(x) \subset \psi(x) \subset \Phi(x) \textit{ for every } x \in X.$$

The above pair (φ, ψ) of set-valued mappings is known as Michael's pair for Φ (since Michael was the first to consider them in [12]).

Proof. Let d be a compatible complete metric on Y bounded by 1. There exist a locally finite open sieve $(\{V_\alpha : \alpha \in A_n\}, \pi_n)$ and a locally finite closed sieve $(\{F_\alpha : \alpha \in A_n\}, \pi_n)$ on Y such that $F_\alpha \subset V_\alpha$, $diam(V_\alpha) \leq 2^{-n}$ for $n \in \mathbb{N}$ and $\alpha \in A_n$. Then, by Lemma 2.1, there exists a locally finite open sieve $(\{U_\alpha : \alpha \in A_n\}, \pi_n)$ satisfying $\theta^{-1}(F_\alpha) \subset U_\alpha \subset \overline{U_\alpha} \subset \Phi^{-1}(V_\alpha)$ for all $n \in \mathbb{N}$ and $\alpha \in A_n$. Choose $y_\alpha \in F_\alpha$ for every $\alpha \in A_n$. Define $\varphi_n(x) = \{y_\alpha : x \in U_\alpha, \alpha \in A_n\}$ for every $x \in X$. Then $\varphi_n : X \to \mathcal{C}(Y)$ is l.s.c.

Let $B^d_\varepsilon(A) = \{y \in Y : d(y, A) < \varepsilon\}$ for every $\varepsilon > 0$ and $A \in 2^Y$. We consider $\mathcal{C}(Y)$ as a topological space with the topology generated by the Hausdorff distance d_H on $\mathcal{C}(Y)$ as follows [11]:

$$d_H(K, Q) = inf\{\varepsilon > 0 : K \subset B^d_\varepsilon(Q) \text{ and } Q \subset B^d_\varepsilon(K)\} \text{ for every } K, Q \in \mathcal{C}(Y).$$

For each $x \in X$ the sequence $\{\varphi_n(x) : n \in \mathbb{N}\}$ is Cauchy in $\mathcal{C}(Y)$ with respect to Hausdorff metric d_H. So we can define a mapping $\varphi : X \to \mathcal{C}(Y)$ by $\varphi(x) = \lim_n \varphi_n(x)$ [11]. Clearly θ is a selection for φ. The map φ is l.s.c. by the following lemma.

Lemma ([6]). *If $\varphi_n : X \to \mathcal{C}(Y)$ is l.s.c. (u.s.c.), $d_H(\varphi_n(x), \varphi_{n+1}(x)) \leq 2^{-n}$ for $n \in \mathbb{N}$ and $x \in X$, and*

$$\varphi(x) = \{y \in Y : y = \lim y_n \text{ where } y_n \in \varphi_n(x) \text{ and } d(y_n, y_{n+1}) \leq 2^{-n}\},$$

then $\varphi : X \to \mathcal{C}(Y)$ and φ is l.s.c. (u.s.c.).

Turning to the proof of the theorem, define $\psi_n(x) = \{y_\alpha : x \in \overline{U_\alpha}, \alpha \in A_n\}$ for $x \in X$ and $n \in \mathbb{N}$. Each $\psi_n(x)$ is a finite subset of Y and the map $\psi_n : X \to \mathcal{C}(Y)$ is u.s.c. For each $x \in X$ we can also define a mapping $\psi : X \to \mathcal{C}(Y)$ by $\psi(x) = \lim\limits_n \psi_n(x)$. Using the above lemma we can conclude that ψ is a u.s.c. mapping such that φ is a selection for ψ. Now the implication $\psi(x) \subset \Phi(x)$ follows directly from the definition of ψ. □

3. Proofs of Theorems 1.3 and 1.4

Proof of Theorem 1.3. $(a) \to (b)$. According to [7], θ has the locally finite lifting property. Now the conclusion of item (b) follows from Theorem 2.2.

$(b) \to (c)$ follows by taking $\Phi(x) = Y$ for all $x \in X$. We show the normality of X. Let F_1 and F_2 be two disjoint closed subsets of X. Let $Y = \{0, 1, 2\}$ with the usual discrete topology. Put $F_0 = X$. Define a u.s.c. mapping $\theta : X \to \mathcal{C}(Y)$ by $\theta(x) = \{y \in Y : x \in F_y\}$. Further, define an l.s.c. mapping $\Phi : X \to \mathcal{C}'(Y)$ by $\Phi(x) = \{0, 1\}$ if $x \in F_1, \Phi(x) = \{0, 2\}$ if $x \in F_2$, and $\Phi(x) = Y$ otherwise. Then by the condition of (b), there exist an l.s.c. $\varphi : X \to \mathcal{C}(Y)$ and a u.s.c. $\psi : X \to \mathcal{C}(Y)$ such that $\theta(x) \subset \varphi(x) \subset \psi(x) \subset \Phi(x)$ for every $x \in X$. For $i = 1, 2$, define the open sets

$$V_i = \varphi^{-1}(\{i\}) \text{ and } W_i = \psi^\sharp(\{0, i\}).$$

Then $F_i \subset V_i$ because θ is a selection for φ and $F_i \subset W_i$ because ψ is a selection for Φ. Set $U_i = V_i \cap W_i$. Then $U_i \subset \{x \in X : i \in \psi(x) \subset \{0, i\}\}$. Namely $U_1 \cap U_2 = \emptyset$.

$(c) \to (d)$ is obvious.

$(d) \to (a)$. Let $\mathcal{F} = \{F_y : y \in Y\}$ be a locally finite family consisting of closed subsets of X. By adding $\{X\}$ to $\mathcal{F}$ if necessary, we can assume that $\mathcal{F}$ is a cover of X. Let R be the set of reals. For a function $f : Y \to R$ we define $s(f) = \{y \in Y : f(y) \neq 0\}$. Define $l_2(Y) = \{f : Y \to R : s(f) \text{ is countable and } \sum_{y \in s(f)} f(y)^2 < +\infty\}$. Then $l_2(Y)$ is a Hilbert space [5]. For $y \in Y$ define $f_y \in l_2(Y)$ such that $f_y(y) = 1$ and $f_y(z) = 0$ for every $z \in Y - \{y\}$. We can identify Y with $\{f_y : y \in Y\}$ where Y is a closed discrete subset of $l_2(Y)$. Define $\theta : X \to 2^{l_2(Y)}$ by $\theta(x) = K_x$ where K_x is a convex hull of $\{f_y : y \in Y, x \in F_y\}$. Then θ is a u.s.c. mapping. According to (d), there exist an l.s.c. $\varphi : X \to \mathcal{C}(l_2(Y))$ and a u.s.c. $\psi : X \to \mathcal{C}(l_2(Y))$ such that $\theta(x) \subset \varphi(x) \subset \psi(x)$ for every $x \in X$. For every $y \in Y$ there exists an open neighborhood V_y of f_y in $l_2(Y)$ such that $\overline{V_y} \cap Y = \{f_y\}$ and $\{\overline{V_y} : y \in Y\}$ is a discrete collection of $l_2(Y)$. Define $\mathcal{U} = \{\varphi^{-1}(V_y) : y \in Y\}$ and $\mathcal{E} = \{\psi^{-1}(\overline{V_y}) : y \in Y\}$. Then $\mathcal{E}$ is a locally finite closed cover of X.

Indeed, closedness is clear. Notice that if $\xi : X \to \mathcal{C}(Y)$ is l.s.c. (u.s.c.) and $\mathcal{M}$ is a locally finite family of Y, then $\xi^{-1}(\mathcal{M})$ is point-finite (locally finite). So, $\mathcal{U}$ is also a locally finite open cover of X because φ is l.s.c. and a selection for ψ. Moreover $F_y \subset \varphi^{-1}(V_y)$ for every $y \in Y$. We have proved that for every locally finite collection $\{F_y : y \in Y\}$ of closed subsets of X there exists a locally finite collection $\{U_y : y \in Y\}$ of open subsets of X such that $F_y \subset U_y$ for every $y \in Y$.

Therefore X is countably paracompact and collectionwise normal by the well-known Dowker's theorem [4]. □

Proof of Theorem 1.4. (Sufficiency) Let X be collectionwise normal, and let Y be a finite dimensional completely metrizable space. Let $\theta : X \to \mathcal{C}_m(Y)$ be a u.s.c. mapping for some natural number m. Then, θ has the locally finite lifting property by [7]. Therefore the conclusion follows from Theorem 2.2.

(Necessity) Let $\mathcal{F} = \{F_\alpha : \alpha \in A\}$ be a locally finite closed collection of X of bounded order (i.e. every point of X belongs to at most m members of this family for some natural number m). Consider the set $Y = \{F_\alpha : \alpha \in A\} \cup \{X\}$ with the discrete topology and define $\theta(x) = \{F \in Y : x \in F\}$. Then θ becomes a mapping from X to $\mathcal{C}_m(Y)$ for some natural number m. Now, θ is u.s.c. because $\mathcal{F} \cup \{X\}$ is a locally finite closed cover of X. By the assumption of our theorem there exist an l.s.c. $\varphi : X \to \mathcal{C}(Y)$ and a u.s.c. $\psi : X \to \mathcal{C}(Y)$ such that $\theta(x) \subset \varphi(x) \subset \psi(x)$ for every $x \in X$. Arguing in the same way as in the proof of implication $(c) \to (a)$ of Theorem 1.3, we can show that there exist a locally finite open cover $\mathcal{U} = \{U_\alpha : \alpha \in A'\}$ of X, where $A' = A \cup \{\alpha_0\}$, $X = F_{\alpha_0}$ such that $F_\alpha \subset U_\alpha$ for every $\alpha \in A'$. Therefore by Katětov's result [5], X is collectionwise normal. □

4. Metacompactness

We prove a result about metacompactness in the same spirit as Theorems 1.3 and 1.4.

Theorem 4.1. *Let X be a metacompact space, let Y be a completely metrizable space and let $\theta : X \to \mathcal{C}(Y)$ be a u.s.c. mapping. Then there exists an l.s.c. $\varphi : X \to \mathcal{C}(Y)$ such that $\theta(x) \subset \varphi(x)$ for every $x \in X$.*

Proof. Let d be a complete metric on Y. Consider $\mathcal{C}(Y)$ endowed with the Hausdorff topology generated by the Hausdorff metric d_H associated to d. By the completeness of d, $(\mathcal{C}(Y), d_H)$ is a complete metric space [11]. Define a set-valued mapping

$$\Phi : X \to 2^{\mathcal{C}(Y)} \text{ by } \Phi(x) = \{K \in \mathcal{C}(Y) : \theta(x) \subset K\} \text{ for every } x \in X.$$

Then $\Phi(x)$ is a closed subset of $\mathcal{C}(Y)$. Indeed, for every point $K \in \overline{\Phi(x)}$ it suffices to show that $K \in \Phi(x)$, i.e. $\theta(x) \subset K$. Choose a sequence $\{K_i : i \in \mathbb{N}\} \subset \Phi(x)$ converging to K. If $\theta(x) \not\subset K$, then choose $y \in \theta(x) - K$. Put $2\varepsilon = d(y, K)$, and select some $j \in \mathbb{N}$ such that $d_H(K, K_j) < \varepsilon$. Then $y \notin K_j$, a contradiction with $K_j \in \Phi(x)$.

For every $\varepsilon > 0$ and $K \in \mathcal{C}(Y)$, $\{x \in X : \Phi(x) \cap B_\varepsilon^{H(d)}(K) \neq \emptyset\} = \{x \in X : \theta(x) \subset B_\varepsilon^d(K)\}$ is open in X because θ is u.s.c. Therefore Φ is l.s.c. By Choban [2], there exists an l.s.c. $\psi : X \to \mathcal{C}(\mathcal{C}(Y))$ such that $\psi(x) \subset \Phi(x)$ for every $x \in X$. Let $\varphi(x) = \bigcup \psi(x)$. Then $\varphi(x)$ is compact [11]. From the definition of Φ it follows that $\theta(x) \subset \varphi(x)$ for every $x \in X$. Finally, φ is l.s.c. by [7]. □

Remark. One may wonder if both mappings φ and ψ are necessary in Theorem 1.3. Without the map φ the conclusion of Theorem 1.3 becomes trivial (because $\psi = \theta$ will do). Theorem 4.1 shows that ψ cannot be omitted in item (b) of Theorem 1.3 (because otherwise metacompact normal spaces must be countably paracompact and collectionwise normal).

Remark. The converse of Theorem 4.1 is not true in general because there exists a collectionwise normal and countably paracompact space which is not metacompact.

The author expresses gratitude to Professors V. Gutev, T. Nogura and D. Shakhmatov for their valuable suggestions. Also deep appreciation goes to the referee for his/her comments, especially for the suggestion to add the condition (d) in Theorem 1.3.

References

[1] J. Chaber, M. Choban and K. Nagami, *On monotone generalizations of Moore spaces, Cech-complete spaces and p-spaces*, Fund. Math. **84** (1974), 107-119. MR **49:**7988

[2] M. Choban, *Many-valued mappings and Borel sets I, II*, Trans. Moscow Math. Soc. **22,23** (1970), 258-280, 286-310. MR **51:**9016

[3] M. Choban and S. Nedev, *Factorization theorems for set-valued mappings, set-valued selections and topological dimension*, Mat. Balkanica **4** (1974), 457-460 (in Russian). MR **52:**9165

[4] C. H. Dowker, *Homotopy extension theorems*, Proc. London. Math. Soc. **6** (1956), 100-116. MR **17:**518f

[5] R. Engelking, *General Topology*, Heldermann, Berlin, revised ed. 1989. MR **91c:**54001

[6] M. K. Fort, Jr., *A unified theory of semi-continuity*, Duke Math. J. **16** (1949), 237-246. MR **10:**716f

[7] V. Gutev, *Generic extensions of finite-valued u.s.c. selections*, Topology and Appl. **104** (2000), 101-118. CMP 2001:01

[8] ———, *Weak factorizations of continuous set-valued mappings*, Topology and Appl. **102** (2000), 33-51. MR **2000j:**54017

[9] ———, *Selections of set-valued mappings and hyperspace topologies*, (unpublished).

[10] V. Gutev and T. Nogura, *Selections for Vietoris-like hyperspace topologies*, Proc. London Math. Soc. **80** (2000), 235-256. MR **2000j:**54013

[11] E. Michael, *Topologies on spaces of subsets*, Trans. Amer. Math. Soc. **71** (1951), 152-182. MR **13:**54f

[12] ———, *A thorem on semi-continuous set-valued functions*, Duke. Math. J. **26** (1959), 647-656. MR **22:**229

[13] ———, *Continuous selection I*, Ann. Math. **63** (1956), 361-382. MR **17:**990e

[14] ———, *Complete spaces and tri-quotient maps*, Illinois J. Math. **21** (1977), 716-733. MR **57:**7543

[15] S. Nedev, *Selection and factorization theorems for set-valued mappings*, Serdica **6** (1980), 291-317. MR **83c:**54023

Department of Mathematical Sciences, Faculty of Sciences, Ehime University, Matsuyama 790-8577, Japan

E-mail address: BZQ22206@nifty.ne.jp