PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 130, Number 2, Pages 577–578
S 0002-9939(01)06070-1
Article electronically published on May 7, 2001

A NOTE ON THE COHOMOLOGICAL BELOW 1/4-PINCHING THEOREM

ZIZHOU TANG

(Communicated by Wolfgang Ziller)

ABSTRACT. By making use of a theorem of Toda, we establish a sharper version of the below 1/4-pinching theorem of Abresch and Meyer.

By a δ-pinched manifold, we mean a complete Riemannian manifold with sectional curvature $0 < \delta \leq K \leq 1$. The celebrated Berger's Rigidity Theorem (see e.g. [AM2]) deals with the 1/4-pinching problem. As for the case of $\delta < 1/4$, Abresch and Meyer [AM1] proved the following result.

Theorem ([AM1, AM2]). *There exists a constant $\delta_{ev} \in (0, \frac{1}{4})$, such that, for any even-dimensional, compact, simply connected Riemannian manifold M^n with δ_{ev}-pinched sectional curvature, the cohomology rings $H^*(M^n; F)$ with coefficients $F \in \{Q, Z_2\}$ are isomorphic to the corresponding cohomology rings of one of the compact, rank-one, symmetric spaces S^n, $CP^{n/2}$, $HP^{n/4}$, or CaP^2, or to the rings $H^*(M^n; F)$ which are truncated polynomial rings generated by an element of degree 8.*

Recall that $H^*(CaP^2; F) = F[\xi]/(\xi^3)$, where $deg\ \xi = 8$. Then, it is a natural problem to decide whether there exists a manifold M^n such that $H^*(M; F) = F[\xi]/(\xi^{m+1})$, where $deg\ \xi = 8$ and $m > 2$.

The purpose of this note is to point out that the exceptional alternative "or the rings $H^n(M^n; F)$ which are truncated polynomial rings generated by an element of degree 8" can be eliminated due to the following Theorem of Toda.

Theorem ([To]). *If a simply connected space X has the truncated polynomial ring $Z_p[\xi]/(\xi^{m+1})$ as its mod p cohomology ring with $p = 2$ or 3 and $m > 2$, then $deg\ \xi = 2$ or 4.*

It should be pointed out that the condition of simple-connectivity in the preceding theorem can be eliminated [Hu, p. 304] by using K-theory $K(X) \otimes Z_{(2)}$ and the Adams operations.

Received by the editors October 14, 1999 and, in revised form, June 19, 2000.

2000 *Mathematics Subject Classification.* Primary 53C20; Secondary 57R19.

Key words and phrases. Pinching constant, rank-one symmetric spaces, cohomology rings.

The author's research was partially supported by the Hong Kong Qiu-Shi Foundation (1998), the Outstanding Youth Foundation of NSF in China (No.19925103) and the Education Foundation of Tsinghua University, as well as the Grants-in-Aid for Science Research of the Japanese Ministry of Education (No.09440039).

Acknowledgements

It is with great pleasure that I express my gratitude to the referee for useful comments and to Professors R. Miyaoka and Y. Ohnita for their invitation to visit Sophia University and TMU in Tokyo.

References

[AM1] U. Abresch and W.T. Meyer, *A sphere theorem with a pinching constant below* 1/4, J. Diff. Geom. 44(1996), 214-261. MR **97i:**53036

[AM2] U. Abresch and W.T. Meyer, *Injectivity radius estimates and sphere theorems,* In Comparison geometry, MSRI Pub. Cambridge University Press, 1997. MR **98e:**53052

[Hu] D. Husemoller, *Fibre bundles,* GTM Vol.20, Springer-Verlag, New York, 1975. MR **51:**6805

[To] H. Toda, *Note on cohomology ring of certain spaces,* Proc. Amer. Math. Soc. 14(1963), 89-95. MR **27:**750

Department of Mathematical Sciences, Tsinghua University, Beijing, 100084, People's Republic of China

E-mail address: zztang@mx.cei.gov.cn