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ON HEREDITARILY INDECOMPOSABLE CONTINUA, HENDERSON COMPACTA AND A QUESTION OF YOHE

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ABSTRACT. We answer a question of Yohe by showing that there exists a family of continuum many topologically different hereditarily indecomposable Cantor manifolds without any non-trivial weakly infinite-dimensional subcontinua. This family may consist either of compacta containing one-dimensional subsets or of compacta containing no weakly infinite-dimensional subsets of positive dimension.

1. INTRODUCTION

The first continuum without any non-trivial finite-dimensional subcontinua was constructed by Henderson (see [9] and [10]), solving a well-known problem which had been open for 40 years. Since then, many other constructions of such continua appeared (see [5], [27], [25], [21], [22], [23], [14], [15]; cf. also [7], page 269); in particular, there exist hereditarily strongly infinite-dimensional continua (i.e. non-trivial continua without any weakly infinite-dimensional subsets of positive dimension). Several years earlier, Bing [3] had shown, solving another outstanding problem in continuum theory, that every continuum can be separated by a closed set all of whose components are hereditarily indecomposable. In effect, there exist Henderson continua which are hereditarily indecomposable. Moreover, by a theorem of Tumarkin [26], there exist such continua which are Cantor manifolds, i.e., which cannot be separated by any finite-dimensional set.

This prompted Yohe [28] to ask if there exist uncountably many topologically different hereditarily indecomposable Henderson continua that are Cantor manifolds. The aim of this paper is to construct two such collections: the first one will consist of Henderson continua which contain 1-dimensional subsets, and the second one will consist of hereditarily strongly infinite-dimensional continua. The first version of this paper contained only a construction of the first collection. The referee suggested that the second collection would be also of interest, and provided a clever idea of such a construction (outlined, with his/her kind permission, in Remark 5.2). We decided to describe in the paper in detail another construction to that effect, based on our earlier results.

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Let us notice that in [19] we have constructed a family of continuum many Cantor manifolds without non-trivial finite-dimensional subcontinua that are not embeddable into each other, and Chatyrko and the author constructed in [6] such a family of hereditarily strongly infinite-dimensional Cantor manifolds. However, these continua are not hereditarily indecomposable.

2. Preliminaries

Our terminology follows [7] and [12]. All spaces are metrizable and separable. A continuum X is indecomposable, if it is not the union of two proper subcontinua. A continuum X is hereditarily indecomposable, abbreviated HI, if every subcontinuum of X is indecomposable. By P we will denote the pseudoarc, i.e., the hereditarily indecomposable 1-dimensional chainable continuum (unique, up to a homeomorphism); cf. [12], §48, X.

A space X is weakly infinite-dimensional (abbreviated WID) if for each infinite sequence $(A_1, B_1), (A_2, B_2), \ldots$ of pairs of disjoint closed subsets of X there are partitions L_i between A_i and B_i in X such that $\bigcap_{i=1}^{\infty} L_i = \emptyset$. A space is strongly infinite-dimensional, abbreviated SID, if it is not WID.

By a Henderson compactum we mean an infinite-dimensional compactum, every non-trivial subcontinuum of which is infinite-dimensional. An SID space X is hereditarily SID, if every subset of X of positive dimension is SID. The first hereditarily SID compactum was constructed by Rubin [25] (cf. [7], Problem 6.1.G).

An infinite-dimensional continuum X is a Cantor manifold if all closed sets which disconnect X are infinite-dimensional. As shown by Tumarkin [26], every Henderson compactum contains an infinite-dimensional Cantor manifold. Yohe [28] proved, using the result of Bing cited in the introduction, that every Henderson compactum contains uncountably many mutually exclusive HI Cantor manifolds.

In particular, there exist HI hereditarily SID Cantor manifolds.

A subcontinuum Y of a continuum X is terminal, if every subcontinuum of X which intersects both Y and its complement must contain Y. A mapping p of a continuum X onto a continuum Y is atomic if every fiber of p is a terminal continuum in X.

Our constructions depend essentially on a method of condensation of singularities, which goes back to Anderson and Choquet [1]. More precisely, we will need the following special case of Theorem 3.2 of [20], which in turn summarizes some results of Maćkowiak [17], [18].

2.1. Theorem. *Let X and K be non-trivial continua, and let A be a 0-dimensional F_σ-subset of X. Then there exist a continuum $S(X,K,A)$ and an atomic mapping $p : S(X,K,A) \longrightarrow X$ such that*

(i) $p^{-1}(a)$ is a copy of K if $a \in A$,

(ii) $p \mid p^{-1}(X \setminus A)$ is a homeomorphism onto $X \setminus A$, and $p^{-1}(X \setminus A)$ is dense in $S(X,K,A)$,

(iii) if A is dense in X, then every open subset of $S(X,K,A)$ contains a copy of K,

(iv) if both X and K are HI, then $S(X,K,A)$ is HI,

(v) if $X \setminus A$ and K do not contain any non-trivial WID subcontinuum, then $S(X,K,A)$ contains no non-trivial WID subcontinuum,

(vi) if X is an infinite-dimensional Cantor manifold, then so is $S(X,K,A)$,

(vii) if X and K are hereditarily SID and A is countable, then $S(X, K, A)$ is hereditarily SID.

Proof. Let us decompose A into a sequence of disjoint compact subsets $A_1, A_2, \dots$ (if A is countable then we assume that every A_i is a one-point set) and let $S(X, K, A)$ be the space $L(X, K \times A_i, A_i)$ described in Theorem 3.2 of [20]. Note that in the case when A is countable, the space $L(X, K \times A_i, A_i)$ is the same as the space $S(X, K, A)$ described in [6], sec.2. The properties (i) - (iii) of $S(X, K, A)$ follow from conditions (ii) and (iv) of the cited Theorem 3.2. The property (iv) follows from a theorem of Maćkowiak (see [17], Proposition 11,(i)) stating that a continuum which is the preimage of an HI continuum under an atomic mapping with HI fibers is itself HI.

To show property (v), suppose that Y is a non-trivial continuum in $S(X, K, A)$. Then either Y embeds in $X \setminus A$, or $Y \subset p^{-1}(a)$ for some $a \in A$, or else there exists $a' \in A$ such that $Y \cap p^{-1}(a') \neq \emptyset \neq Y \setminus p^{-1}(a')$. In the first two cases Y is SID, since $X \setminus A$ and $p^{-1}(a)$ do not contain any non-trival WID subcontinuum. In the third case, Y contains the SID continuum $p^{-1}(a')$, since the fiber is a terminal continuum.

The property (vi) is a generalization of Lemma 2.5 in [6]. Indeed, by (ii) and the monotonicity of p, for every partition F in X the set $p(F)$ is a partition in Z; hence $p(F)$, and thus $p^{-1}(p(F) \setminus A) = F \setminus p^{-1}(A)$, is infinite-dimensional.

Finally, the property (vii) follows from Lemma 2.7 in [6]. □

3. An HI Henderson continuum containing a 1-dimensional subset

An example given in sec.4 of [6] shows that there exists a continuum Z all of whose non-trivial subcontinua are strongly infinite-dimensional, but which contains a 1-dimensional subset. We will construct now two continua having the above mentioned properties which are also hereditarily indecomposable. The second example will be vital in answering the question of Yohe.

3.1. Example. There exists a hereditarily indecomposable continuum $\tilde{P}$ all of whose non-trivial subcontinua are strongly infinite-dimensional, but which contains a 1-dimensional subset.

There exists a 1-dimensional G_δ-subset G of the pseudoarc P without non-trivial subcontinua such that the set $P \setminus G$ is 0-dimensional. The detailed construction of such a set, based on a theorem of Lelek [13] stating that there exists an embedding i of the product $C \times P$ of the Cantor set C and the pseudoarc P into P, is given in [20]. For the sake of completeness, let us sketch a variant of such a construction. First consider in the hyperspace of $C \times P$ the set $\mathcal{F}$ of all partitions between $C \times \{a\}$ and $C \times \{b\}$, where a and b are different points of P. Since $\mathcal{F}$ is analytic, then there exists a map f of a G_δ dense subset D of C onto $\mathcal{F}$. Let $\pi : D \times P \longrightarrow D$ be the projection; then $\pi^{-1}(x) \cap f(x)$ is non-empty for every $x \in D$. Since the set $Y = \bigcup\{\pi^{-1}(x) \cap f(x) : x \in D\}$ is closed in $D \times P$, there exists a G_δ-subset H of Y intersecting every $\pi^{-1}(x) \cap Y$ in exactly one point (see [8], Theorem 4; cf. [2], Ch.IX, §6, Exercise 9). Let Z be a 0-dimensional G_δ dense subset of P such that $P \setminus Z$ is 0-dimensional. Then the set $G = i(H \cup (C \times Z)) \cup (Z \cap (P \setminus i(C \times P)))$ has all the required properties.

Let K be an HI hereditarily SID continuum (see sec.2) and let $\tilde{P} = S(X, K, A)$ and $p : \tilde{P} \longrightarrow P$ be respectively the continuum and the atomic mapping described in Theorem 2.1, where $X = P$ and $A = P \setminus G$. From conditions (iv) and (v) of Theorem 2.1 it follows that $\tilde{P}$ is an HI continuum without any non-trivial WID subcontinua. Since, by (ii), the set $p^{-1}(G)$ is homeomorphic to G, then $\tilde{P}$ contains a one-dimensional subset.

3.2. Example. There exists a hereditarily indecomposable Cantor manifold X without non-trivial weakly infinite dimensional subcontinua, every open subset of which contains a 1-dimensional subset.

Let Z be an HI hereditarily SID Cantor manifold and let A be a countable dense subset of Z. Let $X = S(Z, \tilde{P}, A)$, where $\tilde{P}$ is the continuum constructed in Example 3.1. Then X is an HI Cantor manifold without non-trivial WID subcontinua by conditions (iv), (v) and (vi) of Theorem 2.1. Moreover, by (iii) of Theorem 2.1, every open subset of X contains a copy of $\tilde{P}$, hence also a 1-dimensional subset.

4. No Henderson compactum contains topologically all HI hereditarily SID Cantor manifolds

The following lemma is closely related to Lemma 6.3 in [19].

4.1. Lemma. *Let L be a non-trivial continuum. Then there exist compacta K_α, $\alpha < \omega_1$, such that*

(a) K_α is the union of countably many disjoint topological copies of L, and

(b) if a compactum K contains topologically uncountably many K_α, then K contains a 1-dimensional subcontinuum.

Proof. Given a compactum X, we denote by $\mathcal{K}(X)$ the hyperspace of X, i.e. the space of all non-empty closed subsets of X with the topology induced by the Hausdorff metric (see [12], §42,I,II). Let $\mathcal{K}(I^\infty)$ be the hyperspace of the Hilbert cube I^∞, $\mathcal{C}(I^\infty) = \{X \in \mathcal{K}(I^\infty) : X \text{ is a non-trivial continuum}\}$, $\mathcal{D} = \{X \in \mathcal{C}(I^\infty) : X \text{ is 1-dimensional}\}$, and $\mathcal{F} = \{X \in \mathcal{C}(I^\infty) : X \text{ is homeomorphic to } L\}$.

Then both $\mathcal{D}$ and $\mathcal{F}$ are dense in $\mathcal{C}(I^\infty)$. Indeed, each open set in $\mathcal{C}(I^\infty)$ contains a broken line and each neighbourhood of a broken line contains a copy of L.

Let C be the Cantor set, let Q be a countable dense set in C, and let $\varphi(x) = \mathcal{D}$ for $x \in C \setminus Q$ and $\varphi(x) = \mathcal{F}$ for $x \in Q$. Since $\mathcal{C}(I^\infty)$ is topologically complete and $\mathcal{D}$ is a G_δ-set in $\mathcal{C}(I^\infty)$ (see [12], §45,IV,Th.4), a theorem of Michael (see [16], Corollary 1.6) yields a continuous map $s : C \to \mathcal{C}(I^\infty)$ with $s(x) \in \varphi(x)$ for $x \in C$.

Let $\mathcal{K}(C)$ be the hyperspace of C and let, for $A \in \mathcal{K}(C)$,

$$M(A) = \bigcup_{x \in A} \{(x, y) : y \in s(x)\} \subset C \times I^\infty.$$

Then $M(A)$ is compact, s being continuous.

For each $\alpha < \omega_1$, let us pick a compactum $A_\alpha \subset Q$ with the Cantor-Bendixson index $\geq \alpha$ (see [7], 6.1.I(a)). A theorem of Hurewicz (see [11], sec.5) implies that any analytic set in $\mathcal{K}(C)$ that contains uncountably many A_α must contain also some A with $A \setminus Q \neq \emptyset$. Let $K_\alpha = M(A_\alpha)$ for $\alpha < \omega_1$. Then K_α is a countable disjoint union of topological copies of L. To prove (b), take a compactum K that contains topologically uncountably many K_α. Let $\mathcal{A} = \{A \in \mathcal{K}(C) : M(A) \text{ embeds in } K\}$. Standard arguments show that $\mathcal{A}$ is analytic (see [19], proof of Lemma 6.3). Since $A_\alpha \in \mathcal{A}$ for uncountably many α, one concludes that there exists $A_0 \in \mathcal{A}$ with $t \in A_0 \setminus Q$. Thus $M(A_0)$ contains a 1-dimensional compactum homeomorphic to $s(t)$. Since $M(A_0)$ embeds in K, there is a 1-dimensional continuum in K. ∎

4.2. Theorem. *There exists a family M_α, $\alpha < \omega_1$, of HI hereditarily SID Cantor manifolds such that no Henderson compactum contains topologically uncountably many of M_α.*

Proof. Let L be any HI hereditarily SID Cantor manifold and let K_α, $\alpha < \omega_1$, be a family of compacta which are countable disjoint unions of topological copies of L, constructed in Lemma 4.1. For every α, let $D_\alpha \subset L$ be a countable subset of L homeomorphic with the space A_α of components of K_α. By a theorem of Maćkowiak (see [17], Th.15, and [18],(1.14)), there exist a continuum M_α (a pseudosuspension of K_α over L at D_α) containing K_α as a boundary subset and an atomic map $r_\alpha : M_\alpha \to L$ such that $r_\alpha \mid M_\alpha \setminus K_\alpha$ is a homeomorphism onto $L \setminus D_\alpha$ and $r_\alpha^{-1}(a)$ is a component of K_α for every $a \in D_\alpha$. Since r_α is an atomic map onto an HI continuum with HI fibers, M_α is HI. Each M_α is hereditarily SID, being the union of $r_\alpha^{-1}(L \setminus D_\alpha)$ and countably many components of K_α. Also, since L is a Cantor manifold, the proof of property (vi) in Theorem 2.1 shows that each M_α is a Cantor manifold. Finally, recalling that M_α contains topologically K_α, we infer from (b) in Lemma 4.1 that every Henderson compactum contains only countably many M_α. □

Given M_α as in Theorem 4.2, one readily defines a strictly increasing function $\varphi : \omega_1 \to \omega_1$ such that $M_{\varphi(\alpha)}$ does not embed in $M_{\varphi(\beta)}$ if $\alpha > \beta$.

4.3. Corollary. *For every Henderson compactum K there exists an HI hereditarily SID Cantor manifold X no open subset of which embeds in K.*

Proof. By Theorem 4.2 there exists an HI hereditarily SID Cantor manifold M (equal to some M_α) which does not embed in K. Let A be a countable dense subset of M. Then $X = S(M, M, A)$ satisfies the required conditions (see Theorem 2.1). □

4.4. *Remark.* Let us note that the proof of Lemma 4.1 shows in fact that the set of all Henderson (resp., hereditarily SID) compacta in $\mathcal{K}(I^\infty)$ is not analytic in $\mathcal{K}(I^\infty)$. Indeed, in the notation of Lemma 4.1, since the map $A \to M(A)$ from $\mathcal{K}(C)$ to $\mathcal{K}(C \times I^\infty)$ is continuous, any analytic set $\mathcal{M}$ in $\mathcal{K}(C \times I^\infty)$ containing all K_α contains also a compactum with a 1-dimensional subcontinuum.

5. Continuum many types of HI Cantor manifolds without any non-trivial WID subcontinua

A composant of a point x in a continuum X is the union of all proper subcontinua of X containing x. Every composant of a continuum X is a dense connected F_σ-subset of X, and every indecomposable continuum has continuum many composants which are pairwise disjoint and co-dense in X (see [12], §48,VI).

5.1. Example. There exists a family $\{X_s : s \in \mathcal{S}\}$, where $\mathcal{S}$ is a set of cardinality $2^{\aleph_0}$ of topologically different hereditarily indecomposable Cantor manifolds without non-trivial weakly infinite-dimensional subcontinua. Moreover, we can assume that either

(a) every X_s contains a 1-dimensional subset, or
(b) every X_s is hereditarily strongly infinite-dimensional.

We will use an idea of Bing [4], Theorem 5. Let $\mathcal{S}$ be the collection of all monotone increasing sequences of natural numbers greater than 1. Then $\mid \mathcal{S} \mid = 2^{\aleph_0}$.

For each sequence $s = (n_1, n_2, \dots) \in \mathcal{S}$ we construct the continuum X_s in the following way.

Let us fix a hereditarily indecomposable hereditarily SID Cantor manifold Z and choose a sequence $p_{11}, p_{12}, \dots, p_{1n_1}, p_{21}, \dots, p_{2n_2}, p_{31}, \dots$ of points of Z converging to a point p_{00} of Z such that p_{ij} belongs to the composant containing p_{rs} if and only if $i = r$.

Let $A = \bigcup_{i=1}^{\infty} \bigcup_{j=1}^{n_i} \{p_{ij}\} \cup \{p_{00}\}$. In the case (a) let X be the continuum constructed in Example 3.2 and in the case (b) let X be a continuum obtained in Corollary 4.3 for $K = Z$. Put $X_s = S(Z, X, A)$ and let $p_s : X_s \longrightarrow Z$ be the atomic mapping described in Theorem 2.1. Then X_s is an HI Cantor manifold all of whose non-trivial subcontinua are SID, by conditions (iv)-(vi) of Theorem 2.1. Moreover, in the case (a) every X_s contains a 1-dimensional subset and in the case (b) every X_s is hereditarily SID by condition (vii).

It is easily seen that the composants of X_s are identical with the preimages of the composants of Z under p_s (cf. [20], Lemma 2.8).

We will say that a subcontinuum L of some X_s has property $(\star)$ if no open subset of L embeds in Z.

Then $p_s^{-1}(Z \setminus A)$ is exactly the set of points of X_s which do not belong to any subcontinuum L of X_s having the property $(\star)$. Indeed, by the choice of X every $p_s^{-1}(a)$ for $a \in A$ has the property $(\star)$, and if $x \notin p_s^{-1}(A)$, then x has a neighbourhood U such that $\overline{U} \subset p_s^{-1}(Z \setminus A)$, so x does not belong to any continuum having property $(\star)$. Note that $p_s^{-1}(a)$, for $a \in A$, are pairwise disjoint subcontinua of X_s maximal with respect to the property $(\star)$.

Now, suppose that $s = (n_1, n_2, \dots), s' = (n'_1, n'_2, \dots) \in \mathcal{S}$ and there exists i such that $n_i \neq n'_j$ for every $j \in N$. Then X_s has a composant C which contains exactly n_i disjoint subcontinua maximal with respect to the property $(\star)$ (namely, C is the preimage under p_s of the composant of Z containing $p_{i1}, p_{i2}, \dots, p_{in_i}$), while $X_{s'}$ has no such composants. Hence X_s is not homeomorphic to $X_{s'}$.

5.2. *Remark.* We shall outline the idea of the referee, mentioned in the introduction. It is based on Rogers' approach [24]. Fix a Waraszkiewicz spiral W and let S be the circle in W. By Theorem 1 of [24] there is an HI hereditarily SID continuum A admitting a map f onto W. One can also assume that in the complement of the preimage of S there is an increasing sequence of subcontinua whose union is dense in A. Take any HI hereditarily SID Cantor manifold C, and let B be a subset of C whose complement is a singleton. It is possible to construct a continuum X that is a disjoint union of A and B such that A is terminal in X and the map f extends over X. Then X is an HI hereditarily SID Cantor manifold, and Theorem 2 of [24] guarantees that by using different Waraszkiewicz spirals one gets the desired collection of continua.

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