

ANY 3-MANIFOLD 1-DOMINATES AT MOST FINITELY MANY 3-MANIFOLDS OF S^3 -GEOMETRY

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ABSTRACT. Any 3-manifold 1-dominates at most finitely many 3-manifolds supporting S^3 geometry.

§1. INTRODUCTION

A fundamental topic in topology is to determine whether there exists a map of nonzero degree between given closed orientable manifolds of the same dimension. A natural question in the topic is that for given manifold M , are there at most finite N such that there is a degree one map $M \rightarrow N$?

For dimension 2, the answer is YES, since there is a map $f : F \rightarrow G$ of degree one between closed orientable surfaces if and only if the genus of G is at most of the genus of F and there are only finitely many surfaces with the bounded genus. For dimension at least 4, it is still hopeless to get some general answer. For dimension 3 it becomes especially interesting after Thurston's revolutionary work, and has been listed as the 100th problem of 3-manifold theory in the Kirby's list.

Question 1 ([3, 3.100, (Y. Rong)]). *Let M be a closed orientable 3-manifold. Are there only finitely many orientable closed 3-manifolds N such that there exists a degree one map $M \rightarrow N$?*

A closed orientable 3-manifold is called *geometric* if it admits one of the following geometries: H^3 , $\widetilde{PLS_2R}$, $H^2 \times E^1$, Sol, Nil, E^3 , $S^2 \times E^1$, S^3 . Thurston's geometrization conjecture claims that any closed orientable 3-manifold is either geometric or can be decomposed by the Kneser-Milnor sphere decomposition and the Jaco-Shalan-Johanson torus decomposition so that each piece is geometric. (For details see [11].)

All geometric 3-manifolds with finite fundamental group are exactly those with S^3 -geometry. All geometric 3-manifolds are Seifert manifolds except those that carry hyperbolic geometry or Sol geometry.

Many partial answers to Question 1 have been obtained: Any sequence $M_1 \rightarrow M_2 \rightarrow M_3 \rightarrow \dots$ of degree one maps between Haken 3-manifolds must be stable when n is large ([8], [14]). The answer to Question 1 is affirmative if both, the

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domain and the target, are Seifert manifolds with infinite fundamental group ([9], [10]) or the domain is non-Haken and the target is hyperbolic [7].

All partial results above need some conditions posed on the domain. A more recent result is that Question 1 has a positive answer when the targets are either hyperbolic [13], Sol manifolds, or Seifert manifolds with infinite fundamental group [16], where no conditions are posed on the domain. A main result of this article is that Question 1 has a positive answer when the targets support S^3 geometry.

For simplicity, we use the following terminology introduced in [1]. For an integer $d > 0$, a closed orientable 3-manifold M *d-dominates* a closed orientable 3-manifold N if there is a continuous map $f : M \rightarrow N$ of degree d . We also assume that all integers d, n, q involved in the results of this paper are positive. The modification for the negative case can be made easily by reversing the orientations.

Theorem 1. *An orientable closed 3-manifold M 1-dominates at most finitely many 3-manifolds supporting S^3 -geometry.*

Combining Theorem 1 with the results of [13] and [16], we know that Question 1 has a positive answer when the targets are geometric 3-manifolds.

To prove Theorem 1 we need the following result, which is independently interesting.

Theorem 2. *A closed orientable 3-manifold M d-dominates the lens space $L(n, q)$ if and only if there is an element $\alpha \in \text{Tor}H_1(M, \mathbb{Z})$ such that $\alpha \odot \alpha = \frac{dq}{n}$ in \mathbb{Q}/\mathbb{Z} , where $\alpha \odot \alpha$ is the self-linking number of α .*

Theorem 2 is an extension of the main result in [4]. To prove Theorem 2, some careful arguments for the orders of (co-)homology classes are needed. For this purpose, we present a more direct and geometric proof than the one given in [4].

The paper is organized as follows. Theorem 2 is proved in Section 2 and Theorem 1 is proved in Section 3. The necessary preparations are also given in each section, and therefore the paper is almost self-contained. We assume that all objects below are oriented with compatible orientations.

§2. PROOF OF THEOREM 2

1. Lens spaces $L(n, q)$ and $L^\infty(n) = K(Z_n, 1)$. Let $S^{2l+1} \subset \mathbb{C}^{l+1}$ be the unit sphere in $(l+1)$ complex space. Then $\tau : C^{l+1} \rightarrow C^{l+1}$ given by

$$\tau(z_1, z_2, \dots, z_{l+1}) = (e^{2\pi q/n} z_1, e^{2\pi/n} z_2, \dots, e^{2\pi/n} z_{l+1})$$

defines a free action on S^{2l+1} . Denote the quotient S^{2l+1}/τ by $L^{2l+1}(n, q)$, and the classical lens space $L^3(n, q)$ is still denoted as $L(n, q)$.

Let $L^\infty(n) = \bigcup_{l=1}^\infty L^{2l+1}(n, q)$ be a $K(Z_n, 1)$ space, that is, $\pi_1 = Z_n$ and the higher homotopy groups are vanished. Note that $L(n, q)$ is the 3-skeleton of $L^\infty(n)$.

An alternative description of $L(n, q)$ is identifying the boundary of two solid tori V_1 and V_2 such that the meridian of V_1 is matched with (q, n) curve in ∂V_2 , where on the boundary ∂V_2 a meridian-longitude system is chosen.

2. The linking pair of 3-manifolds. Suppose M is a closed oriented 3-manifold. Suppose $\alpha \in \text{Tor}H_1(M, \mathbb{Z})$ is an element of order dividing n . Let a be a 1-cycle representing α , then na bounds a 2-chain A . Push a slightly so that A and a are in general position. Then the self-linking number of α , denoted as $\alpha \odot \alpha$, is defined by $\frac{1}{n}A \cdot a$, where “ \cdot ” indicates the algebraic intersection number. (See [12, Section 77].)

3. The Bockstein homomorphism. For the chain complex $C_* = C_*(M)$, the short exact sequence

$$0 \rightarrow \mathbb{Z} \otimes C_* \rightarrow \mathbb{Z} \otimes C_* \rightarrow \mathbb{Z}_n \otimes C_* \rightarrow 0$$

provides the long exact homology sequence and cohomology sequence, and the ∂ -operators are the Bockstein homomorphisms $\beta^* : H^1(M, \mathbb{Z}_n) \rightarrow H^2(M, \mathbb{Z})$ and $\beta_* : H_2(M, \mathbb{Z}_n) \rightarrow H_1(M, \mathbb{Z})$. We have

$$(1) \quad \beta^* \circ D = D \circ \beta_*$$

up to sign, where D is the isomorphism of the Poincaré duality [5, Lemma 69.2].

4. Relationship between linking pair and Bockstein homomorphism. A geometric feature of β_* is that (check [5, Step 1, p. 137])

$$(2) \quad \beta_*(c) = [\partial C/n]$$

for any $c \in H_2(M, \mathbb{Z}_n)$, where ∂ is the boundary operator on 2-chain complex $C_2(M)$ and $C \in C_2(M)$ is a mod n cycle representing c . Clearly the order of $\beta_*c \in \text{Tor}H_1(M, \mathbb{Z})$ divides n . Moreover, we have

$$(3) \quad \frac{1}{n}c \cdot \beta_*(c) = \beta_*(c) \odot \beta_*(c)$$

and

$$(4) \quad D^{-1}(c) \cup \beta^* \circ D^{-1}(c) = c \cdot \beta_*(c)[M],$$

where $[M] \in H^3(M, \mathbb{Z}_n)$ is the fundamental class. Equality (3) follows from (2) and equality (4) follows diagram (1).

Lemma 0. *If there is $\alpha \in \text{Tor}H_1(M, \mathbb{Z})$ such that $\alpha \odot \alpha = \frac{dq}{n}$, where q and n are coprime, then the order of α divides n and is divided by $\frac{n}{\gcd(n,d)}$.*

Proof. The order of α dividing n follows from the definition of the linking pair, and the order of α is divided by $\frac{n}{\gcd(n,d)}$ follows from that the order of $\alpha \odot \alpha$ is $\frac{n}{\gcd(n,d)}$ in \mathbb{Q}/\mathbb{Z} and the order of $\alpha \odot \alpha$ divides the order of α . \square

Proof of Theorem 2. Suppose that there is a map $f : M \rightarrow L(n, q)$ of degree d .

Recall $L(n, q)$ is obtained by identifying two solid tori V_1 and V_2 such that the meridian of V_1 is matched with the (q, n) curve in ∂V_2 , where in the boundary V_2 a meridian-longitude system is chosen. Let Y be the centerline of V_2 . Then note first nY and (q, n) -curve are homological in V_2 and therefore they bound a 2-complex B' . Since the (q, n) -curve on ∂V_2 bounds the meridian disc B of V_1 , nY bounds a 2-complex $C = B' + B$, which is a mod n cycle.

Let $\gamma = [Y]$ in $H_1(L(n, q), \mathbb{Z})$, and $c = [C] \in H_2(L(n, q), \mathbb{Z}_n)$. Clearly γ is of order n , and $\beta_*(c) = \gamma$ by (2). By direct geometric observation we have $C \cdot Y = q$, and therefore $\gamma \odot \gamma = \frac{q}{n}$ by (3). Let $y = D^{-1}(c) \in H^1(L(n, q), \mathbb{Z}_n)$. Then y has order n and, by (4), $y \cup \beta^*(y) = q[L(n, q)]$. Let $x = f^*(y) \in H^1(M, \mathbb{Z}_n)$. Then the order of x divides n . Since f is of degree d we have

$$(5) \quad x \cup \beta^*(x) = f^*(y) \cup \beta^*(f^*(y)) = f^*(y \cup \beta^*(y)) = f^*(q[L(n, q)]) = dq[M].$$

Let $\alpha = \beta_* \circ D(x)$. Then α has order dividing n . By (3), (1), (5) and (4), we have

$$\alpha \odot \alpha = \frac{1}{n}(\beta_* \circ D(x)) \cdot D(x) = \frac{dq}{n}.$$

Next we prove the other direction. If $\alpha \in H_1(M, \mathbb{Z})$ has the properties that $\alpha \odot \alpha = \frac{dq}{n}$, then the order of α divides n (Lemma 0) and therefore there is $[c] \in H_2(M, \mathbb{Z}_n)$ such that $\beta_*([c]) = \alpha$. Let $x = D^{-1}(c) \in H^1(M, \mathbb{Z}_n)$. Then similarly, we have $x \cup \beta^*(x) = dq[M]$.

We consider the Eilenberg-MacLane space $K(\mathbb{Z}_n, 1) = L^\infty(n)$. Let $j: L(n, q) \rightarrow K(\mathbb{Z}_n, 1)$ be the inclusion. Then

$$j^*: H^1(K(\mathbb{Z}_n, 1), \mathbb{Z}_n) \rightarrow H^1(L(n, q), \mathbb{Z}_n)$$

is an isomorphism. Let y be a generator of $H^1(L(n, q), \mathbb{Z}_n)$ such that $y \cup \beta^*(y) = q[L(n, q)]$, and choose $z = j^{*-1}(y) \in H^1(K(\mathbb{Z}_n, 1), \mathbb{Z}_n)$ as a generator. By [15, Theorem 10 (p. 428)], x determines a map $g_x: M \rightarrow K(\mathbb{Z}_n, 1)$, uniquely up to homotopy, such that $g_x^*(z) = x$. Since M is of dimension 3, g_x can be deformed such that its image lies in $L(n, q)$, the 3-skeleton of $K(\mathbb{Z}_n, 1)$. Thus there is a map $g: M \rightarrow L(n, q)$ such that $j \circ g = g_x$. Hence, in cohomology level we have

$$g^*(y) = g^*j^*(z) = g_x^*(z),$$

and

$$\deg(g)q[M] = g^*(q[L(n, q)]) = g^*(y \cup \beta^*(y)) = x \cup \beta^*(x) = dq[M].$$

Since $\gcd(n, q) = 1$, we have $\deg(g) \equiv d \pmod{n}$, i.e., $\deg(g) = d + ln$ for some integer l . Starting with g and a map $h: S^3 \rightarrow L(n, q)$ of degree $-nl$, we can now construct a map $f = g\#h: M\#S^3 \rightarrow L(n, q)$ of degree d . \square

Corollary 1. *Any closed orientable 3-manifold M d -dominates at most finitely many lens spaces.*

Proof. $TorH_1(M, \mathbb{Z})$ is finite and therefore there are only finitely many integers l dividing the order of $TorH_1(M, \mathbb{Z})$. For each integer l the number of integers n such that $l = kn/\gcd(n, d)$ is finite since $\gcd(n, d) \leq d$. For each n , there exist only finitely many lens spaces $L(n, q)$. Corollary 1 now follows from Theorem 2 and Lemma 0. \square

§3. PROOF OF THEOREM 1

1. Presentations and standard forms of Seifert manifolds. Each oriented Seifert manifold M with base F_g , the closed orientable surfaces of genus g , has a presentation $M = (g; a_0, b_0; a_1, b_1; \dots; a_n, b_n)$, which indicates the construction of M as follows: Let F be a $n+1$ punctured F_g with boundary components c_0, c_2, \dots, c_n . Then on T_i , the i th torus boundary component of $F \times S^1$, the pair (c_i, h_i) forms a coordinate system on T_i , where h_i is a S^1 -fiber on T_i . Then M is obtained from $F \times S^1$ by identifying the boundary of a solid torus S_i and T_i such that the meridian of S^i is matched with the (a_i, b_i) -curve, $a_i > 0$, for each i . Clearly $\gcd(a_i, b_i) = 1$. Since $a_i > 0$, the circle fibration on $F \times S^1$ can be extended to each S_i , where the centerline of S^i is a regular fiber if $a_i = 1$, and a singular fiber if $a_i > 1$.

The presentations of a Seifert manifold M is not unique. However, the following fact is known:

Lemma 1 ([6]). *Suppose $M = (g; a_0, b_0; a_1, b_1; \dots; a_n, b_n)$, $M' = (g'; a'_0, b'_0; a'_1, b'_1; \dots; a'_m, b'_m)$ are oriented Seifert manifolds with oriented base, $n \leq m$. Then M and M' are orientation preserving homeomorphic by a fiber-preserving homeomorphism*

if and only if the following conditions hold after adding some pair “(1, 0)”s to $M = (g; a_0, b_0; a_1, b_1; \dots; a_n, b_n)$ and re-indexing those pairs if necessary.

1. $g = g'$,
2. $m = n$,
3. $a_i = a'_i, b_i = b'_i + k_i a_i$ for all i and $\sum_{i=1}^n k_i = 0$.

Recall that a presentation of the Seifert manifold is in standard form, if $a_0 = 1, a_i > b_i > 0$. A given Seifert manifold M has unique standard form. The standard form is often written as $(g; 1, l; a_1, b_1; \dots; a_n, b_n)$.

2. The order of the homology of Seifert manifolds supporting S^3 -geometry. If M admits S^3 -geometry, then M is either a lens space or M has the standard form $(0; 1, l; a_1, b_1; a_2, b_2; a_3, b_3)$, a Seifert fibered space with base S^2 and 3 singular fibers, where (a_1, a_2, a_3) is one of the following: $(2, 2, n), (2, 3, 3), (2, 3, 4), (2, 3, 5)$.

If $M = (0; 1, l; a_1, b_1; a_2, b_2; a_3, b_3)$, then

$$\pi_1(M) = \langle x_1, x_2, x_3, h \mid [x_j, h], x_j^{a_j} h^{b_j} \text{ for } 0 \leq j \leq 3, x_1 x_2 x_3 h^{-l} \rangle,$$

By abelianizing $\pi_1(M)$, one can calculate that the order of $H_1(M, \mathbb{Z})$ is

$$|a_1 a_2 a_3 (\frac{b_1}{a_1} + \frac{b_2}{a_2} + \frac{b_3}{a_3} + l)|.$$

An easy consequence of this fact is

Lemma 2. *For any given positive integers C, a_1, a_2 and a_3 , there are only finitely many Seifert manifolds M with standard form $(0; 1, l; a_1, b_1; a_2, b_2; a_3, b_3)$ such that the order of $H_1(M, \mathbb{Z})$ is smaller than C . □*

We also need the following known fact (see [2, I.2.5], for example).

Lemma 3. *Let M and N be orientable closed 3-manifolds. If $f: M \rightarrow N$ is a degree one map, then $Tor H_1(M, \mathbb{Z}) = A \oplus Tor H_1(N, \mathbb{Z})$. □*

3. \mathbb{Z}_2 action on Seifert manifolds from the fiber direction. Suppose $M = F \times S^1 \cup (\bigcup_0^n S_i) = (g; a_0, b_0; a_1, b_1; \dots; a_n, b_n)$, where F is the $(n + 1)$ -punctured F_g . Consider the \mathbb{Z}_2 action τ' on $F \times S^1$ which induces the identity on F and rotation on each S^1 fiber by π .

Lemma 4. *The above \mathbb{Z}_2 action can extend to a \mathbb{Z}_2 action τ on $M = (g; a_0, b_0; a_1, b_1; \dots; a_n, b_n)$. Moreover, the quotient space M/τ is a Seifert manifold with representation $(g; a'_0, b'_0; \dots; a'_n, b'_n)$, where $a'_i = \frac{a_i}{2}, b'_i = b_i$ if a_i is even and $a'_i = a_i, b'_i = 2b_i$ if a_i is odd.*

Proof. The \mathbb{Z}_2 -action τ' on T_i , the i th torus boundary component, is free and orientation preserving. Hence it provides a free and orientation preserving \mathbb{Z}_2 action on ∂S_i via the identification map. The latter can extend to a \mathbb{Z}_2 -action τ_i on S_i . Therefore τ' and τ_i together give an \mathbb{Z}_2 action on M which extends τ' . And we have

$$M/\tau = (F \times S^1/\tau') \cup (\bigcup_{i=0}^n S_i/\tau_i).$$

Note that (1) $S'_i = S_i/\tau_i$ is still a solid torus and the quotient map S_i sends the meridian disc to the meridian disc; (2) $F \times S^1/\tau' = F \times S^1$ and the quotient map $T_i \rightarrow T_i/\tau' = T'_i$ for each boundary torus sends the (a_i, b_i) curve on T to the

$(\frac{a_i}{2}, b_i)$ curve on T' if a_i is even and to the $(a_i, 2b_i)$ curve if a_i is odd. Then the lemma follows. \square

Proof of Theorem 1. Suppose M 1-dominates infinitely many Seifert manifolds N_j with finite fundamental group. These targets form the following two classes:

Class (1). N_j is a lens space $L(n, q)$ with $|H_1(N_j, \mathbb{Z})| = n$.

Class (2). One of the spaces with standard forms $(0; 1, l; 2, 1; 3, b_2; 3, b_3)$ with $|H_1(N_j, \mathbb{Z})| = |9 + 6b_2 + 6b_3 + 18l|$, or $(0; 1, l; 2, 1; 3, b_2; 4, b_3)$ with $|H_1(N_j, \mathbb{Z})| = |12 + 8b_2 + 6b_3 + 24l|$, or $(0; 1, l; 2, 1; 3, b_2; 5, b_3)$ with $|H_1(N_j, \mathbb{Z})| = |15 + 10b_2 + 6b_3 + 30l|$.

Class (3). N_j is a prism space $(0; 1, l; 2, 1; 2, 1; n, b_3)$ with $|H_1(N_j, \mathbb{Z})| = 4|n + k|$, where $k = b_3 + nl$.

By Lemma 3, $|H_1(N_j, \mathbb{Z})|$ is bounded by $|\text{Tor}H_1(M, \mathbb{Z})|$. By Lemma 2, there are at most finitely many N_j in classes (1) and (2).

We now handle the manifolds in class (3). The important difference to cases (1) and (2) is that there are infinitely many distinct prism spaces with the same first homology group because the conditions are only that $n > 0$ and $0 < b_3 < n$ while l , hence $k = b_3 + nl$, is arbitrary.

For convenience we represent the manifolds in class (3) by $(0; 1, 0; 2, 1; 2, 1; n, k)$ with first homology of order $4|k + n|$; here k can be any integer prime to n and l vanishes.

Assume that M 1-dominates infinitely many distinct prism manifolds. By Lemma 3, $|n_j + k_j|$ is bounded by the order of $H_1(M, \mathbb{Z})$. By passing to a subsequence, we may assume that $n_j + k_j = C$ is fixed. Moreover, we may assume that all n_j are different.

Consider the \mathbb{Z}_2 action on $(0; 1, 0; 2, 1; 2, 1; n_j, k_j)$ which induces the identity on the base orbifold and rotates on each regular S^1 fiber by π . Then the quotient space $(0; 1, 0; 2, 1; 2, 1; n_j, k_j)/\mathbb{Z}_2$ has presentation $(0; 1, 0; 1, 1; 1, 1; n_j, 2k_j)$ if n_j is odd and $(0; 1, 0; 1, 1; 1, 1; n_j/2, k_j)$ if n_j is even by Lemma 4. Now

$$(0; 1, 0; 1, 1; 1, 1; n_j, 2k_j) = (0; 1, 2; n_j, 2k_j) = L(2(k_j - n_j), *)$$

where $* \in \mathbb{Z}$. The first equality follows from Lemma 1, and the second follows by counting the intersection numbers of two meridians and

$$(0; 1, 0; 1, 1; 1, 1; n_j/2, k_j) = (0; 1, 2; n_j/2, k_j) = L(k_j - n_j, *).$$

This construction provides maps of M of degree 2 onto the above lens spaces. If infinitely many of these n_j are odd, then M 2-dominates the family of lens spaces $\{L(2(k_j - n_j), *)\}$. By Theorem 2, $2|k_j - n_j|$ must be bounded. By passing to a subsequence we may therefore assume that $2(k_j - n_j) = C'$ for some constant C' . Now substitute $2n_j = 2k_j - C'$ into $n_j + k_j = C$ to deduce that k_j and, hence, n_j are constant, which is a contradiction.

If infinitely many of these n_j are even, then M 2-dominates the family $\{L(k_j - n_j, *)\}$ of lens spaces. Now one obtains a contradiction as above by again showing that the n_j coincide for infinitely many j . \square

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