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NONTRANSITIVE QUASI-UNIFORMITIES IN THE PERVIN QUASI-PROXIMITY CLASS

H.-P. A. KÜNZI

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Abstract. We show that each topological space that does not admit a unique quasi-uniformity possesses a Pervin quasi-proximity class containing at least 2^c nontransitive members.

1. Introduction

The construction of nontransitive totally bounded quasi-uniformities was studied in [2]. Among other things it was shown that each infinite completely regular Hausdorff space admits a nontransitive totally bounded quasi-uniformity. It is also known that each topological space that admits a nontransitive totally bounded quasi-uniformity, admits at least 2^c nontransitive totally bounded quasi-uniformities [7, Proposition 1].

In [9, Remark 2.12] Losonczi observed that the Pervin quasi-proximity class of a topological space that does not admit a unique quasi-uniformity possesses at least 2^c (transitive) members. Subsequently Künzi [4, Proposition 1] proved that a topological space admits a nontransitive quasi-uniformity if and only if it admits at least two quasi-uniformities. In a joint publication [6, Theorem 2.1] Künzi and Losonczi then showed that a topological space that admits at least two quasi-uniformities possesses at least 2^c nontransitive quasi-uniformities. They also verified [6, Theorem 3.6] that if a quasi-proximity class of a transitive quasi-uniformity contains at least two members, then it contains at least 2^c transitive members (compare with [10]). Künzi [5] had also established that each quasi-proximity class with at least two members contains at least 2^c quasi-uniformities. Of course, it follows from that result that if a quasi-proximity class without transitive members contains at least two members, then it contains at least 2^c *nontransitive* members. However all the stated results leave open the following two natural questions (compare Problem 2 of [10]):

Problem 1. *If a quasi-proximity class of a topological space contains at least two quasi-uniformities, does it contain a nontransitive quasi-uniformity?*

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Problem 2. *If a quasi-proximity class of a topological space contains a nontransitive quasi-uniformity, does it contain at least* 2^c *nontransitive quasi-uniformities*?

While these questions remain open for an arbitrary quasi-proximity class of a topological space, in this note we shall prove the following positive result for the Pervin quasi-proximity class: If a topological space admits at least two quasi-uniformities, then its Pervin quasi-proximity class contains at least 2^c nontransitive quasi-uniformities.

Let us note that our result answers Problem 4 of [10] and the problem formulated in Remark 1 of [4]. Our method of proof partially relies on ideas contained in [6] and [8]. For basic facts about quasi-uniformities we refer the reader to [1]. As usual, for a binary relation R on a set X, R^∞ will denote the relation $\bigcup_{n\in\mathbf{N}} R^n$. Furthermore for a topological space X, $\mathcal{P}_X$ will denote the Pervin quasi-uniformity of X. Recall also that a quasi-uniform space $(X,\mathcal{U})$ is called *hereditarily precompact* provided that for each $V \in \mathcal{U}$ and each subset A of X, $\{V(x) : x \in A\}$ possesses a finite subcollection covering A. Of course, for an arbitrary topological space, the (totally bounded) Pervin quasi-uniformity is hereditarily precompact.

A nonempty topological space is said to be *irreducible* provided that each pair of nonempty open sets intersects.

2. Preliminary results

It is easily seen that the supremum (quasi-uniformity) of two nontransitive quasi-uniformities can be transitive. It is also known (see e.g. [3]) that the supremum of a nontransitive quasi-uniformity and a totally bounded transitive quasi-uniformity can be transitive.

However there are nontransitive quasi-uniform spaces that cannot be made transitive by taking the supremum with a totally bounded quasi-uniformity, as we are now going to show. In the next section the following construction will be our main tool to prove the result stated in the abstract.

Example 1. Let F be the set of all finite sequences $(x_i)_{i\in n+1}$ where $n \in \omega$ over the alphabet ω. We shall find it convenient to also consider the set Y of all sequences $(y_i)_{i\in\omega}$ that are eventually 0 over ω, where we shall assume that Y is equipped with its lexicographic ordering $\leq$. Observe that Y is countable. For each $x \in F$, let $[x]$ denote all sequences in Y with initial segment x. For two sequences x and y, where $x \in F$ and y belongs to F or Y, denote by $(x : y)$ the sequence obtained from x and y under the operation of concatenation. Of course, concatenation is assumed to be associative. In particular for each $x \in F$,$x^!$ will denote the sequence $(x : \overline{0})$ where $\overline{0}$ denotes the constant zero sequence in Y.

In the following, for $x, y \in Y$ with $x \neq y$, let $l(x, y) \in \omega$ denote the (cardinal) number of coordinates of the (longest) common initial segment of x and y. Define an (extended) distance function $d : Y \times Y \to [0,\infty]$ as follows: Let $x, y \in Y$. Set $d(x, x) = 0$ whenever $x \in Y$; $d(x, y) = \sum_{k=x_i}^{y_i-1} \frac{1}{k+1}$ if $x < y$ and the i^{th}-coordinate is the first coordinate where x and y differ; $d(x, y) = \infty$ otherwise.

Let us first verify that (Y, d) is an (extended) quasi-pseudometric space: Clearly $d(x, x) = 0$ whenever $x \in Y$. Obviously, in order to verify the triangle inequality $d(x, z) \leq d(x, y)+d(y, z)$, it suffices to consider the case where $x, y, z \in Y$, $x \neq y$ and $y \neq z$. Hence $l(x, y)$ and $l(y, z)$ are well defined. Note that if $l(x, y) < l(y, z)$, then $d(x, y) = d(x, z)$. Similarly, if $l(x, y) > l(y, z)$, then $d(x, z) = d(y, z)$. Therefore in these two cases the triangle inequality is trivially satisfied. So suppose that $t := l(x, y) = l(y, z)$. If either $d(x, y)$ or $d(y, z)$ equals infinity, then the triangle inequality is also clearly satisfied. So it remains to consider the case that $x_t < y_t < z_t$. But then $l(x, z) = t$ and one readily verifies that $d(x, z) \leq d(x, y) + d(y, z)$ in this case. We have shown that d is an (extended) quasi-pseudometric on Y.

For each real $\epsilon > 0$, set $B_\epsilon = \{(x, y) \in Y \times Y : d(x, y) < \epsilon\}$. In the following, $\mathcal{U}_d = \text{fil}\{B_\epsilon : \epsilon > 0\}$ will denote the quasi-pseudometric quasi-uniformity induced by d on Y. Note that each $x \in Y$ has a smallest $\tau(d)$-neighborhood, namely $B_{\frac{1}{m(x)}}(x) = \{x\}$ where $m(x) = \max\{x_i : i \in \omega\} + 1$. For each $x \in Y$, set $L_x = \{y \in Y : y \leq x\}$.

Let us define a subset R of ω as follows: Set $r_0 = 1$. If we have already defined r_n, then let $r_{n+1} > r_n + 1$ be chosen such that $\sum_{k=r_n+1}^{r_{n+1}-1} \frac{1}{k+1} \geq 1$. Set $R = \{r_n : n \in \omega\}$. For a subset A of R let $\eta_A = \{Y\} \cup \{L_{(x:n)^!} : x = (x_i)_{i \in m+1} \in F, m \in \omega, x_i = n + 1\,(i \in m + 1), n \in R \setminus A\}$.

Since the topology of (Y, d) is discrete, it is clear that η_A is an interior-preserving open cover of Y. Set $U_A(x) = \bigcap\{C \in \eta_A : x \in C\}$ $(x \in Y)$. Let us note that for $x, y \in Y$ with $x < y$, $y \notin U_A(x)$ if and only if there is $z \in Y$ such that $x \leq z < y$ and $L_z \in \eta_A$.

Obviously, U_A is a transitive neighbornet of Y. Note that $U_A \cap U_B = U_{A\cap B}$ $(A, B \subseteq R)$. Set $\mathcal{U}_\sigma = \text{fil}\{U_A : A \in \sigma\}$ where σ is a free filter on R.

For an arbitrary free filter σ on R consider now the filter $\mathcal{V}_\sigma$ on $Y \times Y$ that is generated by $\mathcal{U}_\sigma \cup \mathcal{U}_d \cup \mathcal{V}$, where $\mathcal{V}$ denotes an arbitrary hereditarily precompact quasi-uniformity on Y. We want to show next that the constructed quasi-uniformity $\mathcal{V}_\sigma$ is not transitive:

In order to reach a contradiction suppose the contrary. Then B_1 contains a transitive entourage $T \in \mathcal{V}_\sigma$. Consequently there are $A \in \sigma$, $V \in \mathcal{V}$ and a real $\rho > 0$ such that $(U_A \cap V \cap B_\rho)^\infty \subseteq B_1$.

By induction over $n \in \omega$ we shall construct, for each $n \in \omega$, $p_n \in \omega$ and three sequences $x_n, y_n, a_n \in F$ such that $[x_{n+1}] \cap V({y_n}^!) = \emptyset$ where $x_{n+1} = (x_n : p_n + 1)$, $y_n = (x_n : p_n : a_n)$ and each sequence x_n is strictly increasing.

Set $x_0 = (0)$. Suppose now that for some $n \in \omega$, x_n and p_k, x_k, y_k, a_k (whenever $k \in \omega$ with $k < n$), are all defined according to our conditions. First choose $l \in R$, say $l = r_j$, such that l is strictly larger than all the coordinates of x_n and $\frac{1}{l+2} < \rho$. Furthermore let $m = r_{j+1}$.

Set $s_{l+1} = (0)$. Inductively for each $k \geq l + 1$ as long as possible find $s_{k+1} \in F$ such that $((x_n : k : s_k)^!, (x_n : k + 1 : s_{k+1})^!) \in V$. Suppose first that this is possible until $k + 1 = m$. Since for each such k, both $(x_n : k)$ and $(x_n : k + 1)$ are strictly increasing, there is no $L_z \in \eta_A$ such that z has either of these sequences as its initial segment; it follows that $((x_n : k : s_k)^!, (x_n : k + 1 : s_{k+1})^!) \in U_A \cap B_\rho$. We conclude that $((x_n : l + 1 : s_{l+1})^!, (x_n : m : s_m)^!) \in (U_A \cap V \cap B_\rho)^\infty \subseteq B_1$ but $\sum_{k=l+1}^{m-1} \frac{1}{k+1} \geq 1$ by definition of the set R — a contradiction to the definition of d.

Thus there is $p_n \in \omega$ with $m > p_n \geq l + 1$ such that for each $h \in F$, $((x_n : p_n : s_{p_n})^!, (x_n : p_n + 1 : h)^!) \notin V$. We conclude that $V((x_n : p_n : s_{p_n})^!) \cap [(x_n : p_n + 1)] = \emptyset$. Now set $a_n = s_{p_n}$, $x_{n+1} = (x_n : p_n + 1)$ and $y_n = (x_n : p_n : a_n)$. Clearly the sequence x_{n+1} is strictly increasing. This concludes the induction over n.

Observe next that for each $n \in \omega$, $[x_{n+1}] \subseteq [x_n]$ by the definition of x_{n+1}. Note finally that for each $n, k \in \omega$ with $k \geq n$, ${y_{k+1}}^! = (x_{k+1} : p_{k+1}, a_{k+1})^! \in [x_{k+1}] \subseteq [x_{n+1}]$; it follows that ${y_{k+1}}^! \notin V({y_n}^!)$. We have reached another contradiction, since $\mathcal{V}$ is hereditarily precompact, but $\{y_n : n \in \omega\}$ obviously is not precompact in $(Y, \mathcal{V})$.

We deduce that the quasi-uniformity $\mathcal{V}_\sigma$ is not transitive.

Finally we want to verify that distinct free filters on R yield distinct quasi-uniformities. So let σ and σ' be two distinct free filters on R. Assume for instance that there is $A \in (\sigma \setminus \sigma')$. In order to reach a contradiction, we suppose indirectly that $U_A \in \mathcal{V}_{\sigma'}$. Then there are $B \in \sigma'$, $V \in \mathcal{V}$ and $\rho > 0$ such that $U_B \cap V \cap B_\rho \subseteq U_A$. Choose $s \in B \setminus A$ such that $\frac{1}{s+1} < \rho$. We can find such an s, since $B \setminus A$ is infinite, and assume that $s = r_j \in R$. Let $t = r_j + 1$.

For each $n \in \omega$ define $x_n = (z_i)_{i \in n+1}$ where $z_i = t$ whenever $i \in n+1$. Let $y_n = (x_n : s)$ whenever $n \in \omega$. Since $\{y_n{}^! : n \in \omega\}$ is hereditarily precompact in $(Y, \mathcal{V})$, there are $k, n \in \omega$ with $k > n$ such that $y_k{}^! \in V(y_n{}^!)$, that is, $((x_n : s)^!, (x_k : s)^!) \in V$. Clearly also $((x_n : s)^!, (x_k : s)^!) \in B_\rho$. Consider $z \in Y$ such that $(x_n : s)^! \leq z < (x_k : s)^!$. If $z = (q+1, q+1, \ldots, q+1, q)^!$ for $q \in R$, then necessarily by the aforementioned interval condition, $z = y_p{}^!$ for some $p \in \omega$ such that $n \leq p < k$; but since $s \in B$, such $L_{y_p{}^!} \notin \eta_B$. We conclude that $((x_n : s)^!, (x_k : s)^!) \in U_B$. Thus $((x_n : s)^!, (x_k : s)^!) \in U_A$ by our assumption. However this is impossible, because $L_{(x_n:s)^!}$ belongs to η_A. Therefore we conclude that $\mathcal{V}_\sigma \nsubseteq \mathcal{V}_{\sigma'}$. Hence we have shown that the constructed quasi-uniformities $\mathcal{V}_\sigma$, where σ is a free filter on R, are pairwise distinct.

Lemma 1. *Let (X, τ) be a topological space that possesses a closed subspace Z which contains a sequence $(G_n)_{n \in \omega}$ of pairwise disjoint nonempty Z-open sets. Then the Pervin quasi-proximity class of X contains at least 2^c nontransitive quasi-uniformities.*

Proof. Set $G = \bigcup_{n \in \omega} G_n$. Choose a fixed bijection $h : Y \to \{G_n : n \in \omega\}$. For each $x \in G$, there is a unique $b \in Y$ such that $x \in G_{h(b)}$. Define $p : (G, \tau|G) \to (Y, \tau(d))$ by $p(x) = b$. Since the preimage of each $\{b\}$ under p is equal to $G_{h(b)}$ and thus open in G, p is continuous. For a free filter σ on R (defined as in Example 1) let $\mathcal{S}_\sigma$ be the (compatible) quasi-uniformity $\mathcal{U}_\sigma \vee \mathcal{U}_d$ on Y from Example 1. We extend the inverse image $(p \times p)^{-1} \mathcal{S}_\sigma$ on G to a quasi-uniformity

$$\mathcal{W}_\sigma = \text{fil}\{[(p \times p)^{-1} V] \cup [(Z \setminus G) \times Z] \cup [X \times (X \setminus Z)] : V \in \mathcal{S}_\sigma\}$$

on X (see [1, Proposition 2.19]). Of course, $\tau(\mathcal{W}_\sigma) \subseteq \tau$. Put $\mathcal{Q}_\sigma = \mathcal{W}_\sigma \vee \mathcal{P}_X$. Then $\mathcal{Q}_\sigma$ is a quasi-uniformity belonging to the Pervin quasi-uniformity class of X. For each $a \in Y$ choose $x_a \in G_{h(a)}$. Set $Y' = \{x_a : a \in Y\}$. Clearly the subspace $(Y', \mathcal{Q}_\sigma | Y')$ of $(X, \mathcal{Q}_\sigma)$ is isomorphic to $(Y, \mathcal{S}_\sigma \vee \mathcal{R})$ where $\mathcal{R}$ is a totally bounded quasi-uniformity on Y. By Example 1, $\mathcal{S}_\sigma \vee \mathcal{R}$ is a nontransitive quasi-uniformity; furthermore the quasi-uniformities $\mathcal{S}_\sigma \vee \mathcal{R}$ (where σ is a free filter on R) are pairwise distinct. We deduce that we have constructed 2^c nontransitive pairwise distinct quasi-uniformities $\mathcal{Q}_\sigma$ (where σ is a free filter on R) belonging to the Pervin quasi-proximity class of X.

3. Main result

We shall now prove the result stated in the abstract.

Theorem 1. *Let X be a topological space that admits more than one quasi-uniformity. Then the Pervin quasi-proximity class of X contains at least 2^c nontransitive quasi-uniformities.*

Proof. Case 1: Suppose that X is hereditarily compact. Then the statement follows from Theorem 2.1 in [6] (use part (2) of its proof or the theorem itself together with the fact that all quasi-uniformities of a hereditarily compact space lie in the same (unique) quasi-proximity class [1, Theorem 2.36]).

Case 2: Suppose that X is not hereditarily compact and that each closed set is the union of finitely many irreducible (closed) sets. We first show that X possesses a strictly decreasing sequence $(F_n)_{n\in\mathbf{N}}$ of irreducible closed sets none of which is hereditarily compact: Since X is not hereditarily compact, but the finite union of irreducible closed sets, X contains an irreducible closed set F that is not hereditarily compact. Set $F_1 = F$. Suppose that for some $n \in \mathbf{N}$, $(F_k)_{k\leq n}$ is constructed according to our assumption. Since F_n is not hereditarily compact, there exists a strictly increasing sequence $(G_n)_{n\in\omega}$ of F_n-open nonempty subsets of F_n. Then $F_n \setminus G_0$ is closed in X and not hereditarily compact. By our general assumption on X, $F_n \setminus G_0$ is the finite union of irreducible closed sets in X. Hence $F_n \setminus G_0$ contains an irreducible closed subset E of X that is not hereditarily compact. Set $F_{n+1} = E$. This concludes the induction.

Consider now an arbitrary open set G that hits some F_n of the constructed strictly decreasing sequence $(F_n)_{n\in\mathbf{N}}$. Then G hits $F_{p-1} \setminus F_p$ whenever $p \in \mathbf{N} \setminus \{1\}$ and $p \leq n$; otherwise $G \cap F_p \neq \emptyset$, but $G \cap (F_{p-1} \setminus F_p) = \emptyset$. Thus $F_{p-1} \setminus F_p$ and $G \cap F_p$ are nonempty open sets in F_{p-1} with an empty intersection—contradicting that F_{p-1} is irreducible. The auxiliary statement follows.

Set $H = X \setminus \bigcap_{n\in\mathbf{N}} F_n$. Furthermore let $F_0 = X$. For each $x \in H$ let n_x be the maximal $n \in \omega$ such that $x \in F_n$.

Let us work with the subset R of ω defined in Example 1. For a subset A of R let $\eta_A = \{X\} \cup \{X \setminus F_{n+1} : n \in R \setminus A\}$.

It is clear that η_A is an interior-preserving (well-monotone) open cover of X. Let $U_A(x) = \bigcap\{C \in \eta_A : x \in C\}$ $(x \in X)$. Obviously, U_A is a transitive neighbornet on X. Similarly as above note that $U_A \cap U_B = U_{A\cap B}$ $(A, B \subseteq R)$. Set

$$\mathcal{U}_\sigma = \mathrm{fil}\{U_A : A \in \sigma\}$$

where σ is a free filter on R.

Let σ be a free filter on R. Furthermore let $\mathcal{V}_\sigma$ be the filter on $X \times X$ that is generated by $\mathcal{P}_X \cup \mathcal{U}_\sigma \cup \{V_\epsilon \cup [(X \setminus H) \times X] : \epsilon > 0\}$ where $V_\epsilon = \{(x, y) \in H \times H : \sum_{k=n_x}^{n_y-1} \frac{1}{k+1} < \epsilon\}$.

Then $\mathcal{V}_\sigma$ is a quasi-uniformity on X belonging to the Pervin quasi-proximity class of X. Let us show that $\mathcal{V}_\sigma$ is not transitive. Otherwise there is a transitive entourage $T \in \mathcal{V}_\sigma$ such that $T \subseteq V_1 \cup [(X \setminus H) \times X]$. Hence there are $A \in \sigma$, $\rho > 0$ and a finite collection $\mathcal{G}$ of open sets of X such that $U_A \cap P \cap (V_\rho \cup [(X \setminus H) \times X]) \subseteq T$ where $P = \bigcap_{G\in\mathcal{G}}([G \times G] \cup [(X \setminus G) \times X])$. Set $\mathcal{G}_1 = \{G \in \mathcal{G} : \text{there is } n_G \in \mathbf{N}$ such that $G \cap F_{n_G} = \emptyset\}$ and $\mathcal{G}_2 = \mathcal{G} \setminus \mathcal{G}_1$. Moreover choose $n_0 = r_j \in R$ such that $n_0 > \max\{n_G : G \in \mathcal{G}_1\}$ and $\frac{1}{n_0+1} < \rho$.

Since $(F_n)_{n\in\mathbf{N}}$ is a strictly decreasing sequence of irreducible sets and because of the definition of $\mathcal{G}_2$, $\bigcap \mathcal{G}_2 \cap F_n \neq \emptyset$ whenever $n \in \mathbf{N}$. By the observation made above, we can choose $x_i \in \bigcap \mathcal{G}_2 \cap (F_i \setminus F_{i+1})$ whenever $n_0 + 1 \leq i \leq r_{j+1}$. Then $(x_i, x_{i+1}) \in U_A \cap P \cap V_\rho \subseteq T$ whenever $n_0 + 1 \leq i < r_{j+1}$. It follows that $(x_{n_0+1}, x_{r_{j+1}}) \in T$, because T is transitive. But $\sum_{k=n_0+1}^{r_{j+1}-1} \frac{1}{k+1} \geq 1$ by definition of R, that is, $(x_{n_0+1}, x_{r_{j+1}}) \notin V_1$ by definition of V_1—a contradiction to $T \subseteq V_1 \cup [(X \setminus H) \times X]$. We conclude that $\mathcal{V}_\sigma$ is not transitive.

Finally we want to verify that distinct free filters on R yield distinct quasi-uniformities. So let σ and σ' be two distinct free filters on R. Assume that there is some $A \in (\sigma \setminus \sigma')$. In order to reach a contradiction let us suppose indirectly that $U_A \in \mathcal{V}_{\sigma'}$. Then there are $B \in \sigma'$, $\rho > 0$ and $P \in \mathcal{P}_X$ such that

$$U_B \cap P \cap (V_\rho \cup [(X \setminus H) \times X]) \subseteq U_A$$

where $P = \bigcap_{G \in \mathcal{G}}([G \times G] \cup [(X \setminus G) \times X])$ for some finite collection $\mathcal{G}$ of open sets in X.

Set $\mathcal{G}_1 = \{G \in \mathcal{G} : \text{there is } n_G \in \mathbf{N} \text{ such that } G \cap F_{n_G} = \emptyset\}$ and $\mathcal{G}_2 = \mathcal{G} \setminus \mathcal{G}_1$. Moreover choose $f \in B \setminus A$ such that $f > \max\{n_G : G \in \mathcal{G}_1\}$ and $\frac{1}{f+1} < \rho$. We can find such an f, since $B \setminus A$ is infinite, and assume that $f = r_j \in R$. Let $s = r_j + 1$. By similar arguments as given above we can find $x \in \bigcap \mathcal{G}_2 \cap (F_{r_j} \setminus F_{r_j+1})$ and $y \in \bigcap \mathcal{G}_2 \cap (F_{r_j+1} \setminus F_{r_j+2})$. Note that $(x, y) \in U_B \cap P \cap V_\rho$, hence $(x, y) \in U_A$—a contradiction to $X \setminus F_{r_j+1} \in \eta_A$. We conclude that $\mathcal{V}_\sigma \not\subseteq \mathcal{V}_{\sigma'}$. Therefore we have constructed 2^c pairwise distinct nontransitive quasi-uniformities belonging to the Pervin quasi-proximity class of X.

Case 3: Suppose that there is a closed subset F of X that is not the union of finitely many irreducible (closed) sets. (Let us first note that then X cannot be hereditarily compact, since a hereditarily compact space is the union of finitely many irreducible sets; see e.g. [11, p. 903].) Then F contains a collection $(G_n)_{n \in \mathbf{N}}$ of pairwise disjoint nonempty F-open sets, since it follows from our assumption that the subspace F is not semi-irreducible (see [11, Theorem 3]). Our assertion is now a consequence of Lemma 1.

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Department of Mathematics and Applied Mathematics, University of Cape Town, Rondebosch 7701, South Africa

E-mail address: kunzi@maths.uct.ac.za