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BORSUK-ULAM TYPE THEOREMS FOR COMPACT LIE GROUP ACTIONS

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Abstract. Borsuk-Ulam type theorems for arbitrary compact Lie group actions are proven. The transfer plays a major role in this approach.

We present Borsuk-Ulam type theorems for arbitrary compact Lie group actions. The essence of our approach is a generalization of the ideal-valued index of Fadell-Husseini [FH88] using transfer [Boa66], [BG75], [Dol76], [KP72], [Rou71]. Once an appropriate concept (Definition 0.3) is established in terms of transfer, then the proofs of our main results (Theorem 0.5, Theorem 0.6, and Corollary 0.9) are straightforward and simple.

We start with some standard definitions of the transfer and Gysin homomorphism.

Definition 0.1. (i) (cf. [BG75], [LMS86]) When G is a compact Lie group, any compact G-manifold F admits a G-embedding $F \subset W$ in a finite-dimensional real G-representation W equipped with an invariant G-metric, by a theorem of Mostow [Mos57]. Then the Pontryagin-Thom construction gives us a map $t : S^W \to F^\nu$, where ν is the G-equivariant normal bundle of F. However, we prefer to rewrite this in the equivariant stable homotopy category [LMS86] as $t : S^0 \to F^{-\tau(F)}$, where $\tau(F)$ is the tangent G-equivariant bundle of G satisfying $\tau(F) \oplus \nu \cong F \times W$. Then for any honest G-equivariant vector bundle ξ on F, we have the composite

$$t(\xi) : S^0 \xrightarrow{\gamma} F^{-\tau(F)} \to F^{-\tau(F)\oplus\xi},$$

where the second map between Thom spectra is induced by the inclusion of G-equivariant virtual bundles $-\tau(F) \subset -\tau(F) \oplus \xi$.

Given a G-CW complex X, smashing $t(\xi)$ with the identity of the suspension spectrum $\Sigma^{-\infty}X_+$ yields another morphism, which is also denoted by $t(\xi)$:

$$t(\xi) : \Sigma^{-\infty}X_+ \to F^{-\tau(F)\oplus\xi} \wedge \Sigma^{-\infty}X_+.$$

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(ii) We now specialize to the case $F = G/H$ and recall (e.g. [Kaw91, Th. 3.15]) that $\tau(G/H)$, the tangent G-bundle of G/H, is nothing but the bundle

$$G \times_H \tau_{eH}(G/H) \to G/H$$

associated with the principal H-bundle $G \to G/H$, where $\tau_{eH}(G/H)$, the tangent space of G/H at eH, is regarded as the isotropy representation of H, and that an honest G-equivariant vector bundle on G/H is nothing but a finite-dimensional real H-representation. Thus, for any finite-dimensional real representation V of H, $t(V)$ may be written as

$$t_H(V) : \Sigma^\infty X_+ \to (G/H)^{-\tau(G/H)\oplus V} \wedge \Sigma^\infty X_+ \cong G_+ \wedge_H (S^{-\tau_{eH}(G/H)\oplus V} \wedge \Sigma^\infty X_+),$$

which is natural with respect to X and is denoted by $t_H(V)$ to emphasize H.

Remark 0.2. (i) When $\xi = 0$, $t(\xi)$ is the equivariant version of Boardman's umkehr map [Boa66], [BG75, §4], which yields the equivariant Gysin type homomorphism in Definition 0.3(ii). (For related constructions, consult [Kaw91, 6.1] for the equivariant Gysin homomorphism, and [ASi71, 3] for the family version of the topological index.) When $\xi = \tau(F)$, $t(\xi)$ is the equivariant version of the Becker-Gottlieb transfer [BG75].

(ii) When G is a finite group and $F = G/H$, we will consider the case $\xi = 0$, and the resulting transfer $t_H(\xi)$ is the equivariant version of the usual transfer of Roush [Rou71], Kahn-Priddy [KP72], and Becker-Gottlieb [BG75], which all agree in this case.

Now the following concept is the essence of our approach.

Definition 0.3. (i) Let $\tilde{h}_G$ be a G-equivariant reduced generalized cohomology theory. For any G-CW complex X, the pointed version $p_X : X_+ \to S^0$ of the canonical projection $X \to \{\text{a singleton}\}$ induces

$$p_X^*; \tilde{h}_G^*(S^0) \to \tilde{h}_G(X_+),$$

and we call an element $x \in \tilde{h}_G^*(S^0)$ *essential in* X if $p_X^*(x) \neq 0 \in \tilde{h}_G(X_+)$.

Following [FH88, Def. 2.1], we set the *index of* X by

$$\mathrm{Index}_{\tilde{h}_G^*}(X) = \mathrm{Ker}(p_X^*; \tilde{h}_G^*(S^0) \to \tilde{h}_G(X_+)).$$

(ii) Let $\mathcal{F}$ be a family of conjugacy classes of proper closed subgroups of G, and

$$\tilde{\mathcal{F}} = \{((H), V_H) \mid (H) \in \mathcal{F}, V_H : \text{a finite-dimensional real representation of } H\}.$$

For a G-CW complex X, we say that an element $x \in \tilde{h}_G^*(S^0)$ *is transferred from* $\tilde{\mathcal{F}}$ *in* X if $p_X^*(x) \in \tilde{h}_G^*(X_+)$ is contained in the image of

$$\begin{aligned}\bigoplus_{((H),V_H)\in\tilde{\mathcal{F}}} t_H(V_H)^* : \bigoplus_{((H),V_H)\in\tilde{\mathcal{F}}} & \tilde{h}_H^*(S^{-\tau_{eH}(G/H)\oplus V_H} \wedge \Sigma^\infty X_+) \\ \cong \bigoplus_{((H),V_H)\in\tilde{\mathcal{F}}} & \tilde{h}_G^*(G_+ \wedge_H (S^{-\tau_{eH}(G/H)\oplus V_H} \wedge \Sigma^\infty X_+)) \\ \to & \tilde{h}_G^*(X_+).\end{aligned}$$

We set the *transfer index of* X *with respect to* $\tilde{\mathcal{F}}$ by

$$\mathrm{Index}_{\tilde{h}_G^*}(X; \tilde{\mathcal{F}}) = (p_X^*)^{-1}(\mathrm{Im}(\bigoplus_{((H),V_H)\in\tilde{\mathcal{F}}} t_H(V_H)^*)) \subseteqq \tilde{h}_G^*(S^0),$$

which generalizes $\mathrm{Index}_{\tilde{h}_G^*}(X)$ in (i), for $\mathrm{Index}_{\tilde{h}_G^*}(X;\emptyset) = \mathrm{Index}_{\tilde{h}_G^*}(X)$.

(iii) We call an element $x \in \tilde{h}_G^*(S^0)$ *$\tilde{\mathcal{F}}$-essential in X* if x is not contained in the submodule generated by $\mathrm{Index}_{\tilde{h}_G^*}(X)$ and $\mathrm{Index}_{\tilde{h}_G^*}(\{\text{a singleton}\};\tilde{\mathcal{F}})$.

When $\tilde{h}_G^*$ is a multiplicative theory, $\mathrm{Im}(\bigoplus_{((H),V_H)\in\tilde{\mathcal{F}}} t_H(V_H)^*)$ becomes an (not necessarily proper; cf. Remark 0.4 (ii)) ideal of $\tilde{h}_G^*(X_+)$ by the projection formula [BG75, §5], and so $\mathrm{Index}_{\tilde{h}_G^*}(X;\tilde{\mathcal{F}})$ is an (not necessarily proper) ideal of $\tilde{h}_G^*$.

Remark 0.4. (i) Examples of $\tilde{h}_G^*$ include the $RO(G)$-graded equivariant generalized cohomology theories [LMS86] and the Borel cohomology type theory $\tilde{h}^*(EG_+ \wedge_G -)$ associated with a non-equivariant reduced generalized cohomology theory $\tilde{h}^*$. We denote the corresponding unreduced theories by h_G^* and h^* so that $\tilde{h}_G^*(X_+) = h_G^*(X)$ and $\tilde{h}^*(Y_+) = h^*(Y)$, respectively.

(ii) When $G = U(n)$, $\tilde{\mathcal{F}} = \{(T^n, 0)\}$, and $h_G^* = K_G^*$ (the equivariant K-theory), the equivariant Gysin homomorphism $t_{T^n}(0)^*$ yields, via the equivariant Thom isomorphism, the split surjection

$$K_{T^n}^*(X) \to K_{U(n)}^*(X)$$

for any finite G-CW complex X. This fact played a major role in [ASe69].

We have now arrived at the main results of this paper.

Theorem 0.5. (i) *Given G-CW complexes X and Y, suppose there exists an element $x \in \tilde{h}_G^*(S^0)$ such that for some $\tilde{\mathcal{F}}$ as in Definition 0.3*(ii)*,*

(1) *x is not transferred from $\tilde{\mathcal{F}}$ in X,*
(2) *x is transferred from $\tilde{\mathcal{F}}$ in Y.*

Then there is no G-equivariant map from X to Y.

(ii) *Given G-CW complexes X and Y, suppose there exists an element $x \in \tilde{h}_G^*(S^0)$ such that for some $\tilde{\mathcal{F}}$ as in Definition 0.3* (ii), (iii)*,*

(1) *x is $\tilde{\mathcal{F}}$-essential in X,*
(2) *x is transferred from $\tilde{\mathcal{F}}$ in Y.*

Suppose furthermore that all the members of the corresponding family $\mathcal{F}$ are subconjugate to some proper closed subgroup K of G. Then any G-equivariant map $f : X \to Y$, when regarded as a K-equivariant map by restriction, is not K-equivariantly null-homotopic even in the K-equivariant stable homotopy category.

Proof. (i) follows immediately from the naturality of $\bigoplus_{((H),V_H)\in\tilde{\mathcal{F}}} t_H(V_H)^*$ with respect to X.

For (ii), since $x \in \tilde{h}_G^*(S^0)$ is transferred from $\tilde{\mathcal{F}}$ in Y, there is some $y \in \bigoplus_{((H),V_H)\in\tilde{\mathcal{F}}} \tilde{h}_H^*(S^{-\tau_{eH}(G/H)\oplus V_H} \wedge \Sigma^\infty Y_+)$ such that

$$(\bigoplus_{((H),V_H)\in\tilde{\mathcal{F}}} t_H(V_H)^*)(y) = p_Y^*(x) \in \tilde{h}_G^*(Y_+).$$

Without loss of generality, we may assume that $Y^K \neq \emptyset$ and choose a representative H of (H) to be a closed subgroup of K for any $(H) \in \mathcal{F}$. Thus, consider a fixed $y_0 \in Y^K (\subseteq Y^H,\ (H) \in \mathcal{F})$ as the basepoint, and we have

$$\bigoplus_{((H),V_H)\in\tilde{\mathcal{F}}} \tilde{h}_H^*(S^{-\tau_{eH}(G/H)\oplus V_H} \wedge \Sigma^\infty Y_+) = \mathrm{Im}\, q_Y^* \oplus \mathrm{Im}\, p_Y^*,$$

where

$$q_Y^* : \tilde{h}_H^*(S^{-\tau_{eH}(G/H)\oplus V_H} \wedge \Sigma^\infty Y) \to \tilde{h}_H^*(S^{-\tau_{eH}(G/H)\oplus V_H} \wedge \Sigma^\infty Y_+),$$
$$p_Y^* : \tilde{h}_H^*(S^{-\tau_{eH}(G/H)\oplus V_H} \wedge \Sigma^\infty S^0) \to \tilde{h}_H^*(S^{-\tau_{eH}(G/H)\oplus V_H} \wedge \Sigma^\infty Y_+)$$

are respectively induced by the K-equivariant pointed maps

$$q_Y : Y_+ \to Y,$$
$$p_Y : Y_+ \to S^0,$$

where q_Y sends the extra basepoint of Y_+ to $y_0 \in Y$. In particular, for some $y' \in \tilde{h}_H^*(S^{-\tau_{eH}(G/H)\oplus V_H} \wedge \Sigma^\infty Y)$ and $y'' \in \tilde{h}_H^*(S^{-\tau_{eH}(G/H)\oplus V_H} \wedge \Sigma^\infty S^0)$,

$$y = q_Y^*(y') + p_Y^*(y'') \in \tilde{h}_H^*(S^{-\tau_{eH}(G/H)\oplus V_H} \wedge \Sigma^\infty Y_+).$$

Now, contrary to the claim, suppose that

$$q_Y \circ f_+ : X_+ \to Y$$

is K-equivariantly stably null-homotopic. Then,

$$\begin{aligned}
p_X^*(x) = f_+^* \circ p_Y^*(x) &= f_+^*((\bigoplus_{((H),V_H)\in\tilde{\mathcal{F}}} t_H(V_H)^*)(y)) \\
&= (\bigoplus_{((H),V_H)\in\tilde{\mathcal{F}}} t_H(V_H)^*)(f_+^*(y)) \\
&= (\bigoplus_{((H),V_H)\in\tilde{\mathcal{F}}} t_H(V_H)^*)(f_+^* q_Y^*(y') + f_+^* p_Y^*(y'')) \\
&= (\bigoplus_{((H),V_H)\in\tilde{\mathcal{F}}} t_H(V_H)^*)((q_Y \circ f_+)^*(y') + (p_Y \circ f_+)^*(y'')) \\
&= (\bigoplus_{((H),V_H)\in\tilde{\mathcal{F}}} t_H(V_H)^*)(0 + (p_X)^*(y'')) \\
&= p_X^*((\bigoplus_{((H),V_H)\in\tilde{\mathcal{F}}} t_H(V_H)^*)(y'')),
\end{aligned}$$

which implies that

$$x = (\bigoplus_{((H),V_H)\in\tilde{\mathcal{F}}} t_H(V_H)^*)(y'') + z$$

for some $z \in \operatorname{Ker} p_X^* = \operatorname{Ind}_{\tilde{h}_G^*}(X)$. But, this means that x is not $\tilde{\mathcal{F}}$-essential, which is a contradiction. This completes the proof. $\square$

Theorem 0.5 may be restated in terms of the index of Definition 0.3:

Theorem 0.6. (i) (*cf.* [FH88, 2.2(a)]) *Given G-CW complexes X and Y, suppose that* $\operatorname{Index}_{\tilde{h}_G^*}(Y;\tilde{\mathcal{F}})$ *is not contained in* $\operatorname{Index}_{\tilde{h}_G^*}(X;\tilde{\mathcal{F}})$ *for some* $\tilde{\mathcal{F}}$.

Then there is no G-equivariant map from X to Y.

(ii) *Given G-CW complexes X and Y, suppose that* $\operatorname{Index}_{\tilde{h}_G^*}(Y;\tilde{\mathcal{F}})$ *is not contained in the submodule generated by* $\operatorname{Index}_{\tilde{h}_G^*}(X)$ *and* $\operatorname{Index}_{\tilde{h}_G^*}(\{\text{a singleton}\};\tilde{\mathcal{F}})$ *for some $\tilde{\mathcal{F}}$ such that all the members of the corresponding family $\mathcal{F}$ are sub-conjugate to some proper closed subgroup K of G. Then any G-equivariant map $f : X \to Y$, when regarded as a K-equivariant map by restriction, is not K-equivariantly null-homotopic even in the K-equivariant stable homotopy category.* $\square$

Example 0.7. (i) Let an elementary abelian 2-group $G = (\mathbb{Z}/2)^r$ act freely on $S^{n_1-1} \times S^{n_2-1} \times \cdots \times S^{n_r-1}$ via the diagonal antipodal action, and consider the Borel cohomology $\tilde{H}^*_{(\mathbb{Z}/2)^r}(-) = \tilde{H}^*(E(\mathbb{Z}/2)^r_+ \wedge_{(\mathbb{Z}/2)^r} -; \mathbb{Z}/2)$. Then,

$$\begin{aligned}\tilde{H}^*_{(\mathbb{Z}/2)^r}(S^0) &= \tilde{H}^*(E(\mathbb{Z}/2)^r_+ \wedge_{(\mathbb{Z}/2)^r} S^0; \mathbb{Z}/2) = H^*(B(\mathbb{Z}/2)^r; \mathbb{Z}/2) \\ &= \mathbb{Z}/2[x_1, x_2, \cdots, x_r]\end{aligned}$$

and

$$\mathrm{Index}_{\tilde{H}^*_{(\mathbb{Z}/2)^r}}(S^{n_1-1} \times S^{n_2-1} \times \cdots \times S^{n_r-1}) = (x_1^{n_1}, x_2^{n_2}, \cdots, x_r^{n_r}),$$
$$\mathrm{Index}_{\tilde{H}^*_{(\mathbb{Z}/2)^r}}(\{\text{a singleton}\}; \{(\{e\}, 0)\}) = 0,$$
$$\mathrm{Index}_{\tilde{H}^*_{(\mathbb{Z}/2)^r}}(S^{n_1-1} \times S^{n_2-1} \times \cdots \times S^{n_r-1}; \{(\{e\}, 0)\}) = (x_1^{n_1}, x_2^{n_2}, \cdots, x_r^{n_r}, x_1^{n_1-1}x_2^{n_2-1}\cdots x_r^{n_r-1}).$$

Thus, if there is a $(\mathbb{Z}/2)^r$-equivariant map

$$S^{n_1-1} \times S^{n_2-1} \times \cdots \times S^{n_r-1} \to S^{m_1-1} \times S^{m_2-1} \times \cdots \times S^{m_r-1}, \tag{0.1}$$

where the target is also given by the diagonal antipodal action, then

$$n_1 \le m_1,\ n_2 \le m_2, \cdots, n_r \le m_r.$$

Furthermore, if all of these inequalities happen to be equalities, then any such $(\mathbb{Z}/2)^r$-equivariant map (0.1) is non-equivariantly essential.

(ii) Let the r-dimensional torus T^r act freely on $S^{2n_1-1} \times S^{2n_2-1} \times \cdots \times S^{2n_r-1}$ via the diagonal standard action, and consider the Borel cohomology $\tilde{H}^*_{T^r}(-) = \tilde{H}^*(ET^r_+ \wedge_{T^r} -; \mathbb{Z})$. Then a similar argument such as (i) implies that any T^r-equivariant self map of $S^{2n_1-1} \times S^{2n_2-1} \times \cdots \times S^{2n_r-1}$ is a T^r-equivariant self-homotopy equivalence.

We shall end this paper with an application that uses the Borel cohomology. For this purpose, we prepare a lemma.

Lemma 0.8. *Let $\tilde{h}^*$ be a non-equivariant reduced generalized cohomology theory, and X a G-CW complex. Suppose there is an integer n such that the canonical pointed projection $p_X : X_+ \to S^0$ induces an isomorphism*

$$p_X^* : \tilde{h}^d(S^0) \to \tilde{h}^d(X_+)$$

for any $d \le n$. Then, for any

$$\tilde{\mathcal{F}} = \{((H), V_H) \mid (H) \in \mathcal{F}, V_H : \textit{a finite-dimensional real representation of } H\}$$

with $V_H = \tau(G/H)$ for any $(H) \in \mathcal{F}$ (thus the Becker-Gottlieb situation as in Remark 0.2(i)*), the canonical injection*

$$p_X^* : \mathrm{Index}_{\tilde{h}^*_G}(\{\text{a singleton}\}; \tilde{\mathcal{F}}) \to \mathrm{Index}_{\tilde{h}^*_G}(X; \tilde{\mathcal{F}})$$

is an isomorphism in dimensions $\le n$.

Proof. Considering the Atiyah-Hirzebruch spectral sequence

$$E_2^{s,t} = H^s(BH; h^t(X)) \Longrightarrow h^{s+t}(EH \times_H X),$$

we see by the assumption that p_X induces isomorphisms

$$p_X^* : \tilde{h}^d(EH_+ \wedge_H S^0) \to \tilde{h}^d(EH_+ \wedge_H X_+)$$

for any $d \leq n$. Now the claim follows immediately by applying the naturality of the map $\bigoplus_{((H),V_H)\in\tilde{\mathcal{F}}} t_H(V_H)^*$ with respect to $p_X : X_+ \to S^0$. $\square$

From Lemma 0.8 and Theorem 0.5, we immediately get

Corollary 0.9. *Let X be a G-CW complex such that there is a non-equivariant reduced generalized cohomology theory $\tilde{h}^*$ and an integer n so that the canonical pointed projection $p_X : X_+ \to S^0$ induces an isomorphism*

$$p_X^* : \tilde{h}^d(S^0) \to \tilde{h}^d(X_+)$$

for any $d \leq n$. Then, for any G-CW complex Y, the following hold.

(i) *Suppose there exists an element $x \in \tilde{h}^n_G(S^0)$ such that for some $\tilde{\mathcal{F}}$ with $V_H = \tau(G/H)$ for any $(H) \in \mathcal{F}$ (thus the Becker-Gottlieb situation as in Remark 0.2*(i)*)*

(1) *x is not transferred from $\tilde{\mathcal{F}}$ in* {a singleton},
(2) *x is transferred from $\tilde{\mathcal{F}}$ in Y.*

Then there is no G-equivariant map from X to Y.

(ii) *Suppose there exists an element $x \in \tilde{h}^n_G(S^0)$ such that for some $\tilde{\mathcal{F}}$ with $V_H = \tau(G/H)$ for any $(H) \in \mathcal{F}$ (thus the Becker-Gottlieb situation as in Remark 0.2*(i)*)*

(1) *x is $\tilde{\mathcal{F}}$-essential in* {a singleton},
(2) *x is transferred from $\tilde{\mathcal{F}}$ in Y.*

Suppose furthermore that all the members of the corresponding family $\mathcal{F}$ are subconjugate to some proper closed subgroup K of G. Then any G-equivariant map $f : X \to Y$, when regarded as a K-equivariant map by restriction, is not K-equivariantly null-homotopic even in the K-equivariant stable homotopy category. $\square$

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