PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 133, Number 7, Pages 2165–2172
S 0002-9939(05)07774-9
Article electronically published on February 18, 2005

HOMOGENEITY OF POWERS OF SPACES AND THE CHARACTER

A. V. ARHANGEL'SKII

(Communicated by Alan Dow)

Abstract. A space is said to be power-homogeneous if some power of it is homogeneous. We prove that if a Hausdorff space X of point-countable type is power-homogeneous, then, for every infinite cardinal τ, the set of points at which X has a base of cardinality not greater than τ, is closed in X. Every power-homogeneous linearly ordered topological space also has this property. Further, if a linearly ordered space X of point-countable type is power-homogeneous, then X is first countable.

All spaces considered are assumed to be T_1-spaces. A space X is said to be power-homogeneous if some non-zero power of X is homogeneous. This notion was introduced and studied by E. van Douwen in [5]. Further results on power homogeneity were obtained in [6], [4], [7], [8], [9], [10], [11] and [3]. Which spaces are power-homogeneous is interesting to know in connection with the following, still unsolved, problems: is an arbitrary compact Hausdorff space a continuous image of a homogeneous compact Hausdorff space? Is there a homogeneous compact Hausdorff space with the Souslin number greater than 2^ω (E. van Douwen)? For a survey of the topic, see [7].

Let τ be an infinite cardinal. A point $x \in X$ (a set $A \subset X$) will be called *a G_τ-point* in X (a G_τ-subset of X) if there exists a family γ of open sets in X such that $|\gamma| \leq \tau$ and $\{x\} = \bigcap \gamma$ (and $A = \bigcap \gamma$). The *character* of a space X at a point x does not exceed τ (we write $\chi(x, X) \leq \tau$ in this case) if there exists a base $\mathcal{B}_x$ at x such that $|\mathcal{B}_x| \leq \tau$.

Our main results on power-homogeneity are Theorems 10, 16, and Corollary 29. They show, in particular, that under a very general restriction on a Hausdorff space X, a necessary condition for power-homogeneity of X is that the character in X does not increase under the closure operator. This is a very effective (though, not a universal) tool in proving that certain (in fact, very many of them) spaces are not power-homogeneous. Partial results in this direction were obtained in [3].

In what follows, τ is an infinite cardinal number. Suppose that $\mathcal{F} = \{X_a : a \in A\}$ is a family of spaces, and $X = \prod_{a \in A} X_a$ is the product of these spaces. *A τ-cube* in X is any subset B of X that can be represented as the product $B = \prod_{a \in A} B_a$, where B_a is a non-empty subset of X_a, for each $a \in A$, and the cardinality of $A_B = \{a \in A : B_a \neq X_a\}$ is not greater than τ. We put $X_K = \prod_{a \in K} X_a$, for every

Received by the editors August 25, 2003 and, in revised form, April 2, 2004.
2000 *Mathematics Subject Classification.* Primary 54A25, 54B10.
Key words and phrases. Power-homogeneous, point-countable type, τ-twister, τ-diagonalizable space, linearly ordered space, G_τ-tightness, first countable space.

non-empty subset K of A, and denote by p_K the natural projection mapping of X onto X_K. The *G_τ-tightness* of a space X at a point $z \in X$ is said not to exceed τ (notation: $t_\tau(z, X) \le \tau$) if, for every family γ of G_τ-subsets of X such that $z \in \overline{\bigcup \gamma}$, there is a subfamily η of γ such that $|\eta| \le \tau$ and $z \in \overline{\bigcup \eta}$.

Theorem 1. *Suppose that $\{X_a : a \in A\}$ is a family of topological spaces, τ an infinite cardinal number, and z_a a point in X_a, for each $a \in A$, such that $\chi(z_a, X_a) \le \tau$, and let $X = \prod_{a \in A} X_a$ be the topological product. Then the G_τ-tightness of X at the point $z = (z_a : a \in A)$ is not greater than τ.*

Proof. It is enough to show that, for any family γ of τ-cubes in X such that the point $z = (z_a : a \in A)$ is in the closure of the set $U = \bigcup \gamma$, there exists a subfamily η of γ such that $x \in \overline{\bigcup \eta}$ and $|\eta| \le \tau$.

Let A_0 be any non-empty subset of A such that $|A_0| \le \tau$. Assume that a subset A_n is already defined and satisfies the condition $|A_n| \le \tau$. Put $K = A_n$ and $z_K = (z_a : a \in K)$. Obviously, $p_K(z) = z_K$. Since $\chi(z_K, X_K) \le \tau$, there exists a subfamily γ_n of γ such that $|\gamma_n| \le \tau$ and z_K is in the closure of $\bigcup\{p_K(V) : V \in \gamma_n\}$ in X_K. Put $A_{n+1} = A_n \cup \bigcup\{A_B : B \in \gamma_n\}$. The inductive step is complete.

Put $M = \bigcup\{A_n : n \in \omega\}$ and $\eta = \bigcup\{\gamma_n : n \in \omega\}$. Clearly, η is a subfamily of γ such that $|\eta| \le \tau$. Let H be the closure of $\bigcup \eta$. Let us show that $z \in H$. Every standard open neighbourhood O_1 of z in X has a common point with H. Indeed, $O_1 = p_S^{-1} p_S(O_1)$, for some finite $S \subset A$. Put $F = S \cap M$ and $O = p_F^{-1} p_F(O_1)$. Then, clearly, $O_1 \subset O$ and $O = p_F^{-1} p_F(O)$. The conditions $O \cap H \ne \emptyset$ and $O_1 \cap H \ne \emptyset$ are equivalent. To see this, assume that $O \cap H \ne \emptyset$, and fix $y \in O \cap H$. There exists $y' \in O_1$ such that $p_M(y') = p_M(y)$. Since $p_M^{-1} p_M(H) = H$ and $y \in H$, we have $y' \in H$. Therefore, $y' \in O_1 \cap H$ and $O_1 \cap H \ne \emptyset$. Since the sequence $\{A_n : n \in \omega\}$ is increasing, there exists $n \in \omega$ such that $F \subset A_n$. Then, by the choice of γ_n, $p_F(z)$ is in the closure of the set $\bigcup\{p_F(V) : V \in \gamma_n\}$ in the space X_F. Therefore, there exists a point $y \in \bigcup \eta$ such that $p_F(y) \in p_F(O)$. Since $O = p_F^{-1} p_F(O)$, it follows that y is in $O \cap \bigcup \eta$. Hence, $z \in H$. $\square$

The next statement is proved in a standard way.

Proposition 2. *If $f : X \to Y$ is a pseudo-open continuous mapping of X onto Y, and the G_τ-tightness of X does not exceed τ at every point of X, then the G_τ-tightness of Y does not exceed τ at every point of Y.*

A *τ-twister* at a point e of a space X is a binary operation on X, written as a product operation xy on x, y in X, satisfying the following conditions:

a) $ex = xe = x$, for each $x \in X$;

b) for every $y \in X$ and every G_τ-subset V in X containing y, there is a G_τ-subset P of X such that $e \in P$ and $xy \in V$, for each $x \in P$ (that is, $Py \subset V$) (this is the G_τ-continuity of the operation at e on the right); and

c) if $e \in \overline{B}$, for some $B \subset X$, then, for every $x \in X$, $x \in \overline{xB}$ (this is the continuity of the operation at e on the left).

When in the above definition we replace condition b) with the following condition: b′) for every $y \in X$ and every open neighbourhood V of y, there is an open neighbourhood W of e such that $Wy \subset V$, then we obtain a definition of *a twister* on X.

Proposition 3. *If Z is a retract of X and $e \in Z$, and there is a τ-twister (a twister) at e on X, then there is a τ-twister (a twister) on Z at e.*

Proof. Fix a retraction r of X onto Z and a τ-twister on X at e, and define an operation ϕ on Z by the rule: $\phi(z,h) = r(zh)$. Clearly, ϕ is a τ-twister (a twister) on Z at e. □

If a space X has a τ-twister (a twister) at a point $e \in X$, we say that X is *τ-diagonalizable (respectively, diagonalizable) at e*. A space is called *τ-diagonalizable* (*diagonalizable*) if it is τ-diagonalizable (respectively, diagonalizable) at every point. Diagonalizability was first introduced and applied in [2], [3].

Proposition 4. *Suppose that $\{X_a : a \in A\}$ is a family of topological spaces, τ an infinite cardinal number, and e_a a point in X_a, for each $a \in A$. Let $X = \prod_{a\in A} X_a$ be the topological product and $e = (e_a : a \in A)$. Suppose further that X_a is τ-diagonalizable (diagonalizable) at e_a, for each $a \in A$. Then the product space X is τ-diagonalizable (diagonalizable) at e.*

Proof. Fix a τ-twister on X_a at e_a, and define the product operation on X coordinatewise. This is a τ-twister on X. □

Theorem 5. *Suppose that X is a Hausdorff space and τ an infinite cardinal such that X is τ-diagonalizable and the G_τ-tightness of X does not exceed τ. Then the set A of all G_τ-points in X is closed.*

Proof. Take any $e \in \overline{A}$, and fix a τ-twister at e. Since the G_τ-tightness of X at e does not exceed τ, and each point in A is a G_τ-point in X, there exists a subset B of A such that $|B| \le \tau$ and $e \in \overline{B}$. For every $b \in B$ we can find a G_τ-subset P_b in X such that $e \in P_b$ and $P_b b = \{b\}$. Put $P^* = \bigcap\{P_b : b \in B\}$. Then P^* is a G_τ-set in X, $e \in P^*$, and $xb = b$, for every $b \in B$ and every $x \in P^*$.

Claim: $P^* = \{e\}$. Assume the contrary, and fix $c \in P^*$ such that $c \ne e$. There is an open neighbourhood W of c such that e is not in the closure of W. We have $ce = c \in W$. By the continuity assumption, there is an open neighbourhood V of e such that $cV \subset W$. We can also assume that $V \cap W = \emptyset$. Put $B_1 = B \cap V$. Then $B_1 \ne \emptyset$ and $cB_1 = B_1 \subset V$. On the other hand, $cB_1 \subset cV \subset W$. It follows that $cB_1 \subset W \cap V$, a contradiction with $W \cap V = \emptyset$. Hence, $P^* = \{e\}$. Since P^* is a G_τ-set in X, it follows that e is a G_τ-point in X, that is, $e \in A$ and A is closed. □

Proposition 6. *If e is a G_τ-point in a space X, then there exists a τ-twister on X at e.*

Proof. Put $ey = y$, for every $y \in X$, and $xy = x$ for every x and y in X such that $x \ne e$. This operation is a τ-twister on X. □

Theorem 7. *Suppose that X is a power-homogeneous Hausdorff space and μ an infinite cardinal such that the character of X at least at one point is not greater than μ. Then, for any cardinal τ such that $\mu \le \tau$, the set of all G_τ-points in X is closed.*

Proof. Fix an infinite cardinal number λ such that the space X^λ is homogeneous. It follows from Theorem 1 that the G_τ-tightness of X^λ at least at one point does not exceed τ. Hence, the G_τ-tightness of X^λ is not greater than τ at all points, since X^λ is homogeneous. The natural projection of X^λ onto X is open and continuous. Therefore, by Proposition 2, the G_τ-tightness of X also does not exceed τ.

The set of G_τ-points in X is not empty. From Proposition 6 it follows that X is τ-diagonalizable at some point. By Proposition 4, X^λ is τ-diagonalizable at some

point. Since X^λ is homogeneous, the space X^λ is τ-diagonalizable. Since X is a retract of X^λ, it follows from Proposition 3 that X is τ-diagonalizable. It remains to refer to Theorem 5. □

Corollary 8. *If X is a power-homogeneous Hausdorff space with a dense set of isolated points, then every point in X is a G_δ.*

Corollary 9. *If X is a power-homogeneous compact Hausdorff space with a dense set of G_δ-points, then X is first countable and $|X| \leq 2^\omega$.*

A space X is of *point-countable type* if every point of X is contained in a compact subspace $F \subset X$ such that F has a countable base of neighbourhoods in X. The class of spaces of point-countable type contains all locally compact Hausdorff spaces, all Čech-complete spaces, and all p-spaces [1].

Theorem 10. *For every Hausdorff power-homogeneous space of point-countable type and every infinite cardinal number τ, the set C_τ of all G_τ-points in X is closed.*

Proof. If C_τ is empty, we have nothing to prove. Assume that C_τ is non-empty, and fix a G_τ-point a in X. Then, since X is a space of point-countable type, the character of X at a does not exceed τ [1]. Now it follows from Theorem 7 that the set C_τ is closed in X. □

We will call a space X *character-closed* if, for every infinite cardinal number τ, the set of all points of X at which X has a base of cardinality not greater than τ is closed in X.

Corollary 11. *If X is a Hausdorff space of point-countable type, and X contains a G_δ-subspace Y such that the set of all G_δ-points is dense in Y and Y is not first countable, then X is not power-homogeneous.*

Example 12. Let X be a Hausdorff space of point-countable type with a G_δ-subspace Y which is one of the following: 1) $\omega_1 + 1$ or any larger ordinal space; 2) $\beta\omega$ or the Stone-Čech-compactification of a non-compact metrizable space; 3) the Alexandroff one-point compactification of an uncountable discrete space; 4) any uncountable scattered compactum. Then it follows from Corollary 11 that X is not power-homogeneous. This generalizes and explains some results of E. van Douwen and V. Malykhin in [5] and [8].

The next statement is proved by practically the same argument as Theorem 7.

Theorem 13. *Suppose that $\{X_a : a \in A\}$ is a family of Hausdorff spaces, μ an infinite cardinal number, and e_a a point in X_a, for each $a \in A$, such that the character of X_a at e_a is not greater than μ. Suppose further that the product space $X = \prod_{a \in A} X_a$ is homogeneous. Then, for each cardinal τ such that $\mu \leq \tau$, and for each $a \in A$, the set of all G_τ-points in X_a is closed.*

Corollary 14. *Suppose that X is a Hausdorff space of point-countable type first countable at a dense set of points and that Y is a Hausdorff space first countable at some point such that the space $X \times Y$ is power-homogeneous. Then X is first countable.*

Problem 15. *Is there a compact Hausdorff space Y such that $(\omega_1 + 1) \times Y$ is homogeneous?*

A large class of Hausdorff spaces for which the pseudocharacter and character coincide is the class of generalized ordered spaces (that is, subspaces of linearly ordered spaces). Repeating the proof of Theorem 10, we obtain the following result:

Theorem 16. *Every power-homogeneous generalized linearly ordered space is character-closed.*

A point x of a space X will be called *a chain-point* if there exists a family γ of open subsets of X satisfying the following conditions:

(a) $\bigcap \gamma = \bigcap\{\overline{V} : V \in \gamma\} = \{x\}$; and

(b) γ is a chain, that is, for any $V, U \in \gamma$, either $V \subset U$ or $U \subset V$.

Any such family γ will be called *a strong chain* at x.

Proposition 17. *Any space X is diagonalizable at any chain-point.*

Proof. Let e be a chain-point in X and γ a strong chain at e. Take any $x, y \in X$. Put $xy = y$ if there exists $V \in \gamma$ such that $x \in V$ and $y \notin \overline{V}$. Otherwise, put $xy = x$. In particular, it follows that $ey = y$, for each $y \in X$, and $xe = x$, for each $x \in X$. It cannot occur that, for some $V, U \in \gamma$ and $x, y \in X$, $x \in V$, $y \notin V$, $y \in U$, and $x \notin U$, since γ is a chain. Therefore, the definition of multiplication is correct.

It is easy to check that the binary operation so defined is a twister on X at e. □

Lemma 18. *Suppose that X is a space diagonalizable at a point $e \in X$. Suppose further that A is a subset of X such that $e \in \overline{A}$, and U is an open neighbourhood of A. Then there exists a family γ of open neighbourhoods of e in X such that $|\gamma| \leq |A|$ and $(\bigcap \gamma) \subset \overline{U}$.*

Proof. Fix a twister at e on X. Take any $x \in A$. We have $xe = x \in U$. By continuity of multiplication on the left, we can find an open neighbourhood V_x of e such that $xV_x \subset U$. Consider the family $\gamma = \{V_x : x \in A\}$. Clearly, $|\gamma| \leq |A|$. Put $P = \bigcap \gamma$. Then $AP \subset U$.

Let us show that $P \subset \overline{U}$. Assume the contrary. Then there exists $y \in P$ and an open neighbourhood W of y such that $W \cap U = \emptyset$. Since $ey = y$ and the multiplication is continuous on the right, there is an open neighbourhood $O(e)$ of e such that $O(e)y \subset W$. Since e is in the closure of A, $A \cap O(e) \neq \emptyset$. Fix $a \in A \cap O(e)$. Then $ay \in W$. Since W and U are disjoint, it follows that $ay \notin U$. On the other hand, $ay \in AP \subset U$, a contradiction. □

Theorem 19. *Suppose that $X, <$ is a linearly ordered topological space, and $e \in X$. Then the following conditions are pairwise equivalent:*

(i) X is diagonalizable at e;

(ii) Either the left cofinality of X at e and the right cofinality of X at e coincide, or the point e is isolated from the left or from the right.

(iii) e is a chain-point of X.

Proof. In each of the three cases in (ii), an obvious argument shows that e is a chain-point in X. Thus, (ii) implies (iii). If (iii) holds, then X is diagonalizable at e, by Proposition 17. Thus, (iii) implies (i). If (i) holds, and e is neither isolated from the left nor from the right, then it obviously follows from Lemma 18 that the left cofinality of X at e and the right cofinality of X at e coincide. Thus, (i) implies (ii), and all three conditions (i), (ii), and (iii) are equivalent. □

Corollary 20. *Suppose that X is a linearly ordered topological space, diagonalizable at $e \in X$. Suppose also that there exists a countable set $A \subset X \setminus \{e\}$ such that $e \in \overline{A} \setminus A$. Then X is first countable at e.*

The next lemma is proved by a standard argument.

Lemma 21. *Suppose that X is a linearly ordered space, $e \in X$, and the G_ω-tightness of X at e is countable. Then X is first countable at e.*

Theorem 22. *Suppose that $\{X_a : a \in A\}$ is a family of linearly ordered spaces, each of which is diagonalizable at least at one point and either contains a non-closed countable subset or is first countable at some point. Suppose further that the product space $X = \prod_{a \in A} X_a$ is homogeneous. Then each of the spaces X_a is first countable.*

Proof. The space $X = \prod_{a \in A} X_a$ is diagonalizable at some point, by Proposition 4. Since X is homogeneous, it follows that X is diagonalizable (at every point). Since X_a is a retract of X, the space X_a is diagonalizable. If e is a point of X_a such that $e \in \overline{A} \setminus A$, for some countable subset A of X_a, then, by Lemma 20, X_a is first countable at e. Hence, each X_a is first countable at some point. Arguing as in the proof of Theorem 7, we conclude that the G_ω-tightness of X_a is countable. From Lemma 21 it follows that each X_a is first countable. □

Corollary 23. *Suppose that X is a power-homogeneous linearly ordered topological space containing a non-closed countable subset A. Then either X is not diagonalizable at any point or X is first countable.*

Corollary 24. *Suppose that $\{X_a : a \in A\}$ is a family of σ-countably compact linearly ordered spaces, each of which has the first element. Suppose further that the product space $X = \prod_{a \in A} X_a$ is homogeneous. Then each of the spaces X_a is first countable.*

Proof. Every linearly ordered space with the first element e is diagonalizable at e (put $xy = max\{x, y\}$ [2]). Each X_a either contains a non-closed countable subset or is discrete. It remains to apply Theorem 22. □

Every non-empty linearly ordered compact space has the first element. Hence, if the product of non-empty linearly ordered compact spaces is homogeneous, then all factors are first countable. This generalizes a result of M. Bell [4].

Lemma 25. *Suppose that X is a linearly ordered topological space containing a non-empty open locally compact subspace V. Then there exists a chain-point in X (belonging to V).*

Proof. For arbitrary $a, b \in X$ we put $[a, b] = \{x \in X : a \leq x \leq b\}$. Clearly, there exist $a_0, b_0 \in X$ such that $a_0 < b_0$ and $[a_0, b_0]$ is compact. Assume that α is an ordinal and $a_\beta \in X$ and $b_\beta \in X$ have already been defined for every $\beta < \alpha$ such that the following inequalities are satisfied whenever $\beta_1 < \beta_2 < \alpha$: $a_{\beta_1} < a_{\beta_2} < b_{\beta_2} < b_{\beta_1}$. Put $\xi_\alpha = \{[a_\beta, b_\beta] : \beta < \alpha\}$, $V_\beta = \{x \in X : a_\beta < x < b_\beta\}$, $\eta_\alpha = \{V_\beta : \beta < \alpha\}$, and $P_\alpha = \bigcap \xi_\alpha = \bigcap\{[a_\beta, b_\beta] : \beta < \alpha\}$. Since each $[a_\beta, b_\beta]$ is non-empty and closed in X and $[a_0, b_0]$ is compact, it follows that P_α is a non-empty compact subset of X. If P_α is a singleton, that is, $P_\alpha = \{z\}$, for some $z \in X$, then, clearly, η_α is a base of X at z and, hence, z is a chain-point in X. If P_α contains more than one point but is finite, we put $z = min(P_\alpha)$. Then z is isolated from the right and therefore, z is a chain-point in X. If P_α is infinite, we can choose

$a_\alpha, b_\alpha \in P_\alpha$ such that $min(P_\alpha) < a_\alpha < b_\alpha < max(P_\alpha)$. This process must stop at some ordinal α. Then P_α must be finite, which provides us with a chain-point in X. □

Lemma 26. *If a linearly ordered topological space X contains a non-empty compact G_δ-subspace F, then some point of F is a chain-point in X and therefore, X is diagonalizable at some point of F.*

Proof. Fix a point y in F and a countable family $\{W_n : n \in \omega\}$ of open subsets of X such that $F = \bigcap\{W_n : n \in \omega\}$. We may assume that y is not isolated neither from the right, nor from the left (since otherwise y is a chain-point in X and there is nothing to prove). Then we can define, by induction, open intervals U_n in X such that $y \in U_n$ and $\overline{U_{n+1}} \subset U_n \subset W_n$, for each $n \in \omega$. Put $B = \bigcap\{U_n : n \in \omega\}$. Clearly, B is a closed subset of X and $y \in B \subset F$. Therefore, B is a non-empty compactum. Put $a = min(B)$ and $b = max(B)$. Then, since each U_n is an interval in X, $B = [a, b] = \{x \in X : a \leq x \leq b\}$. If the open interval V between a and b is empty, then, clearly, both a and b are chain-points in X. Otherwise this interval V is an open non-empty locally compact subspace of X, and there exists a chain-point z of X in V, by Lemma 25. Then X is diagonalizable at z, by Theorem 19. □

Theorem 27. *Suppose that $\{X_a : a \in A\}$ is a family of linearly ordered spaces each of which contains a non-empty compact G_δ-subspace. Suppose further that the product space $X = \prod_{a\in A} X_a$ is homogeneous. Then each of the spaces X_a is first countable.*

Proof. Every X_a contains a non-closed countable subset or is first countable at some point. By Lemma 26, each X_a is diagonalizable at some point. It remains to refer to Theorem 22. □

Corollary 28. *Suppose that $\{X_a : a \in A\}$ is a family of linearly ordered spaces of point-countable type. Suppose further that the product space $X = \prod_{a\in A} X_a$ is homogeneous. Then each X_a is first countable.*

Corollary 29. *If a linearly ordered space X of point-countable type is power-homogeneous, then X is first countable.*

References

[1] A.V. Arhangel'skii, *Bicompact sets and the topology of spaces*, Trans. Mosc. Math. Soc. 13 (1965), 1-62. MR0195046 (33:3251)

[2] A.V. Arhangel'skii, *An operation on topological spaces.* Applied General Topology 1:1 (2000), 13-28. MR1796929 (2001k:54040)

[3] A.V. Arhangel'skii, *Diagonal structures on topological spaces.* Topology and its Applications 125 (2002), 419-446. MR1934280 (2003j:54035)

[4] M. Bell, *Nonhomogeneity of powers of cor images.* Rocky Mountain J. Math. 22 (1992), 805-812. MR1183687 (93j:54014)

[5] Eric K. van Douwen, *Nonhomogeneity of products of preimages and π-weight.* Proc. Amer. Math. Soc. 69:1 (1978), 183-192. MR0644652 (58:30998)

[6] A. Dow and E. Pearl, *Homogeneity in powers of zero-dimensional first-countable spaces*, Proc. Amer. Math. Soc. 125 (1997), 2503-2510. MR1416083 (97j:54008)

[7] K. Kunen, *Large homogeneous compact spaces*, Open Problems in Topology (J. van Mill and G.M. Reed, eds.), North-Holland Publishing Co., Amsterdam, 1990, pp. 261-270. MR1078652

[8] V.I. Malykhin, *On homogeneous spaces*, in: Proc. of Leningrad International Topology Conference (August 1982), Nauka (Leningrad), 1983, 50-61.

[9] J. van Mill, *On the character and π-weight of homogeneous compacta*, Isr. J. Math. 133 (2003), 321-338. MR1968433 (2004b:54006)
[10] J. van Mill, *On the cardinality of power homogeneous compacta*, A preprint, 2003.
[11] R. de la Vega and K. Kunen, *A compact homogeneous S-space*, Topology Appl. 136 (2004), no. 1-3, 123-127. MR2023413

Department of Mathematics, Ohio University, Athens, Ohio 45701
E-mail address: arhangel@math.ohiou.edu