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INTERPOLATING SEQUENCES FOR BOUNDED ANALYTIC FUNCTIONS

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Abstract. We prove that any sequence in the open ball of a complex Banach space E, even in that of E^{**}, whose norms are an interpolating sequence for H^∞, is interpolating for the space of all bounded analytic functions on B_E. The construction made yields that the interpolating functions depend linearly on the interpolated values.

1. Introduction and preliminaries

Let A be a space of functions defined on X. A sequence $\{x_n\}$ in X is called interpolating for A if for any sequence $(a_n) \in \ell_\infty$, there exists $f \in A$ such that $f(x_n) = a_n$ for all $n \in \mathbb{N}$. It is a classical result in function theory that a sequence (z_n) in the open unit disc $\mathbf{D} \subset \mathbb{C}$ is interpolating for H^∞, if, and only if, Carleson's condition holds, i.e.:

$$\text{(1)} \qquad \text{There is } \delta > 0 \text{ such that} \qquad \prod_{k \neq j} \left| \frac{z_k - z_j}{1 - \overline{z_k} z_j} \right| \geq \delta \qquad \forall j \in \mathbb{N},$$

Some years later B. Berndtsson [B] showed that a sequence (x_n) in the open unit ball B_n of $\mathbb{C}^n$ is interpolating for $A = H^\infty(B_n)$ if the following condition holds:

$$\text{There is } \delta > 0 \text{ such that} \qquad \prod_{k \neq j} \rho_A(x_j, x_k) \geq \delta > 0 \qquad \forall j \in \mathbb{N},$$

where ρ_A denotes the pseudohyperbolic distance in B_n. Shortly after that, B. Berndtsson, S. Chang and K. Lin [BCL] proved that such a type of condition is also sufficient to guarantee that if (x_n) lies in the polidisc, $\mathbf{D}^n$, then it is interpolating for $A = H^\infty(\mathbf{D}^n)$.

Very recently, T. Gamelin, M. Lindström and the first author [GGL1] have extended the above type of results to the ball of a complex Hilbert space. However, when turning to complex Banach spaces E there were, to the best of our knowledge, no known general sufficient conditions for a sequence to be interpolating. The aim of this paper is to provide a sufficient condition for a sequence (x_n) in the open

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unit ball, B_E, of E to be interpolating for $H^\infty(B_E)$. The resulting condition is:

$$\text{(2)} \qquad \text{There is } \delta > 0 \text{ such that} \qquad \prod_{k\neq j}\left|\frac{\|x_j\|-\|x_k\|}{1-\|x_j\|\|x_k\|}\right| \geq \delta \qquad \forall j \in \mathbb{N}.$$

Actually, we construct a sequence $(F_j) \subset H^\infty(B_E)$ such that $F_j(x_n) = \delta_{j,n}$ and $\sum_j |F_j(x)| \leq M$ for all $x \in B_E$, where M is a constant depending only on δ. Therefore the mapping $T : \ell_\infty \to H^\infty(B_E)$ defined by $T((\alpha_n)) = \sum_n \alpha_n F_n$ is a well defined operator such that $\|T\| \leq M$ and $T((\alpha_n))(x_k) = \alpha_k \ \ \forall k \in \mathbb{N}$. When such a linear interpolation operator exists, we say that $\{x_n\}$ is a *linear* interpolating sequence. Its existence has shown to be very useful in the study of composition operators; see [GLR] or [GGL].

A function $f : B_E \to \mathbb{C}$ is said to be analytic if it is Fréchet differentiable. Denote by $H^\infty(B_E)$ the space $\{f : B_E \to \mathbb{C} : f$ is analytic and bounded $\}$. It is a uniform Banach algebra when endowed with the sup-norm $\|f\| = \sup\{|f(x)| : x \in B_E\}$ and it is, obviously, the analogue of the space H^∞. Davie and Gamelin [DG] proved that each $f \in H^\infty(B_E)$ extends, by means of the Aron-Berner extension, to an element $\widetilde{f} \in H^\infty(B_{E^{**}})$ and that the extension mapping is a multiplicative linear isometry. Therefore, the question whether a sequence in $B_{E^{**}}$ is interpolating for $H^\infty(B_E)$ is meaningful. Our techniques allow us to assert that (2) is also a sufficient condition whenever the sequence (x_n) lies in $B_{E^{**}}$.

Recall that the pseudohyperbolic distance between two given points $z, w \in \mathbf{D}$ is $\rho(z,w) = \frac{|z-w|}{|1-\bar{z}w|}$.

2. The result

Theorem 1. *Let $\{x_n\}$ be a sequence in $B_{E^{**}}$ satisfying the following condition:*

$$\text{(2) } \textit{There is } \delta > 0 \textit{ such that} \qquad \prod_{k\neq j}\left|\frac{\|x_j\|-\|x_k\|}{1-\|x_j\|\|x_k\|}\right| \geq \delta \qquad \forall j \in \mathbb{N}.$$

Then there are functions $\{F_j\} \subset H^\infty(B_E)$ and a constant M, depending only on δ, such that $\widetilde{F}_j(x_n) = \delta_{j,n}$ and $\sum_j |\widetilde{F}_j(x)| \leq M$ for all $x \in B_{E^{**}}$.

The construction of the Beurling functions $\{F_j\}$ is based on the one given in [BCL] for the case of $H^\infty(\mathbf{D}^2)$. A number of lemmas is required, and we proceed to give all of them. The first one is just a set of well–known calculations.

Lemma 2. *The following holds:*

$$\rho(a,c) \geq \rho(a,b) \textit{ for real numbers } 0 \leq a \leq b \leq c < 1. \tag{3}$$

$$\rho(|z|,|w|) \leq \rho(z,w) \textit{ for } z,w \in \mathbf{D}. \tag{4}$$

$$1 - \left|\frac{z-w}{1-\overline{z}w}\right|^2 = \frac{(1-|z|^2)(1-|w|^2)}{|1-\overline{z}w|^2} \textit{ for } z,w \in \mathbf{D}. \tag{5}$$

$$1 - x \leq -\log x \textit{ for } 0 < x \leq 1. \tag{6}$$

$$\Re e\left[\frac{1+\alpha z}{1-\alpha z}\right] = \frac{1-\alpha^2|z|^2}{|1-\alpha z|^2} \textit{ for } 0 \leq \alpha \leq 1, z \in \mathbf{D}. \tag{7}$$

Lemma 3. *Let $\{x_n\}$ be a sequence in $B_{E^{**}}$ such that $\{\|x_n\|\}$ does not accumulate at 0. Then for $k,j \in \mathbb{N}$ and $\alpha_{k,j} > 0$, there exists $T_{k,j} \in \overline{B}_{E^*}$ such that*

$$\rho(T_{k,j}(x_k), T_{k,j}(x_j)) \geq \rho(\|x_k\|, \|x_j\|) - \alpha_{k,j}. \tag{8}$$

Moreover, there is $m > 0$, which depends only on the sequence, such that

$$m \leq |T_{k,j}(x_k)| \text{ for } k, j \in \mathbb{N} \text{ such that } \|x_k\| > \|x_j\|. \tag{9}$$

If, further, $\lim \|x_k\| = 1$, *then for any* $\alpha_k > 0$, *there is* $L_k \in \overline{B}_{E^*}$ *such that*

$$\rho(L_k(x_k), L_k(x_j)) \geq \rho(\|x_k\|, \|x_j\|) - \alpha_k \quad \text{if } \|x_k\| \geq \|x_j\|, \qquad \text{and} \tag{10}$$

$$L_k(x_k) \geq \|x_j\| \quad \text{if} \quad \|x_k\| > \|x_j\|. \tag{11}$$

Proof. Since 0 is not an accumulation point, there is $m > 0$ so that

$$m < \inf_{n \in \mathbb{N}} \{\|x_n\| : \|x_n\| \neq 0\}.$$

For every $k \in \mathbb{N}$, there is a sequence $\{T_n^k\}_n$ in $\overline{B}_{E^*}$ such that $T_n^k(x_k) \geq 0$ and

$$\lim_{n \to \infty} T_n^k(x_k) = \|x_k\|. \tag{12}$$

If $x_k \neq 0$, we may assume as well that $T_n^k(x_k) \geq m$ for any $n \in \mathbb{N}$.

Let $j \in \mathbb{N}$. There is nothing to prove if $\|x_k\| = \|x_j\|$. So, suppose $\|x_k\| > \|x_j\|$. When considering the pseudohyperbolic distance between $\|x_k\|$ and $\|x_j\|$, there are two possibilities:

i) Either, there is $n_1 \in \mathbb{N}$ so that $\rho(T_{n_1}^k(x_k), |T_{n_1}^k(x_j)|) \geq \rho(\|x_k\|, \|x_j\|)$. In this case, we choose $T_{k,j} = T_{n_1}^k$ and then (8) holds for arbitrary $\alpha_{k,j} \geq 0$.

ii) Or such n_1 does not exist, that is, $\rho(|T_n^k(x_k)|, |T_n^k(x_j)|) < \rho(\|x_k\|, \|x_j\|)$ $\forall n$. Since $\|x_j\| < \|x_k\|$, there is $n_2 \in \mathbb{N}$ such that $\|x_j\| \leq T_n^k(x_k)$ for $n \geq n_2$. Thus, after using (3), we have $\rho(T_n^k(x_k), |T_n^k(x_j)|) \geq \rho(T_n^k(x_k), \|x_j\|)$. Consequently,

$$\rho(T_n^k(x_k), \|x_j\|) \leq \rho(T_n^k(x_k), |T_n^k(x_j)|) \leq \rho(\|x_k\|, \|x_j\|) \qquad \forall n \geq n_2.$$

Hence by the choice in (12), $\lim_{n\to\infty} \rho(T_n^k(x_k), |T_n^k(x_j)|) = \rho(\|x_k\|, \|x_j\|)$. Then, for $\alpha_{k,j} > 0$ there is $n_3 > n_2$ such that

$$\rho(T_n^k(x_k), T_n^k(x_j)) \geq \rho(T_n^k(x_k), |T_n^k(x_j)|) \geq \rho(\|x_k\|, \|x_j\|) - \alpha_{k,j} \text{ for } n \geq n_3.$$

So we choose $T_{k,j} = T_{n_3}^k$, and (8) holds as well in this case.

Assume now that $\lim \|x_k\| = 1$. Fix $k \in \mathbb{N}$. Observe that there are only a finite number of terms x_j in the sequence with $\|x_j\| < \|x_k\|$. For such $j \in \mathbb{N}$ we consider

$$B_j = \left\{n : \rho(T_n^k(x_k), |T_n^k(x_j)|) < \rho(\|x_k\|, \|x_j\|)\right\}.$$

In case B_j is a finite set, we may find $n_j \in \mathbb{N}$ so that

$$\rho(T_n^k(x_k), |T_n^k(x_j)|) \geq \rho(\|x_k\|, \|x_j\|) \geq \rho(\|x_k\|, \|x_j\|) - \alpha_k \ \ \forall n \geq n_j. \tag{13}$$

Whenever B_j is not finite, we argue as in ii) above to show that

$$\lim_{n \in B_j} \rho(T_n^k(x_k), |T_n^k(x_j)|) = \rho(\|x_k\|, \|x_j\|).$$

Therefore, an $n_j \in \mathbb{N}$ can be found so that

$$\rho(T_n^k(x_k), |T_n^k(x_j)|) \geq \rho(\|x_k\|, \|x_j\|) - \alpha_k \ \ \forall n \in B_j, n \geq n_j.$$

Note that this inequality also holds for $n \notin B_j$. Consequently, (13) also holds in this case.

Since $\max\{n_j\}$ is finite, $\lim_{n\to\infty} T_n^k(x_k) = \|x_k\|$ and $\max\{\|x_j\| : \|x_j\| < \|x_k\|\} < \|x_k\|$, we may take n_0 big enough so that $T_{n_0}^k(x_k) > \|x_j\|$ if $\|x_j\| < \|x_k\|$. By setting $L_k = T_{n_0}^k$ and using (4), we are done. $\square$

Lemma 4. *Let $\{x_n\} \subset B_{E^{**}}$ and $\delta > 0$ satisfying condition* (2)*. Then there are functionals $\{T_{k,j}\} \subset \overline{B}_{E^*}$ satisfying* (9) *such that*

$$\prod_{k\neq j} \rho(T_{k,j}(x_k), T_{k,j}(x_j)) \geq \frac{\delta}{2} \qquad \text{and} \tag{14}$$

$$\sum_{k=1}^{\infty}(1-|T_{k,j}(x_k)|^2) \leq (1+2\log\frac{2}{\delta})\frac{1+\|x_j\|}{1-\|x_j\|} \qquad \forall j \in \mathbb{N}. \tag{15}$$

Proof. Choose a sequence (β_k) such that $\prod(1-\beta_k) \geq \frac{1}{2}$ and then put $\alpha_{k,j} = \beta_k \rho(\|x_k\|, \|x_j\|)$. Now, apply Lemma 3 to find the functionals $\{T_{k,j}\}$ such that

$$\rho(T_{k,j}(x_k), T_{k,j}(x_j)) \geq \rho(\|x_k\|, \|x_j\|) - \alpha_{k,j} = (1-\beta_k)\rho(\|x_k\|, \|x_j\|).$$

Hence (14) follows. From here we get

$$-\sum_{k\neq j} \log\left|\frac{T_{k,j}(x_j) - T_{k,j}(x_k)}{1-\overline{T_{k,j}(x_k)}T_{k,j}(x_j)}\right|^2 \leq -2\log\frac{\delta}{2}.$$

Further, by (5) and then bearing in mind (6), we obtain

$$\sum_{k\neq j} \frac{(1-|T_{k,j}(x_k)|^2)(1-|T_{k,j}(x_j)|^2|)}{|1-\overline{T_{k,j}(x_k)}T_{k,j}(x_j)|^2} \leq -2\log\frac{\delta}{2}.$$

Finally,

$$\begin{aligned}\sum_{k=1}^{\infty}(1-|T_{k,j}(x_k)|^2) &= \sum_{k=1}^{\infty}\frac{(1-|T_{k,j}(x_k)|^2)(1-|T_{k,j}(x_j)|^2)}{|1-\overline{T_{k,j}(x_k)}T_{k,j}(x_j)|^2}\frac{|1-\overline{T_{k,j}(x_k)}T_{k,j}(x_j)|^2}{1-|T_{k,j}(x_j)|^2} \\ &\leq (1-2\log\frac{\delta}{2})\frac{(1+\|x_j\|)^2}{1-\|x_j\|^2} = (1+2\log\frac{2}{\delta})\frac{1+\|x_j\|}{1-\|x_j\|}. \qquad \square\end{aligned}$$

Lemma 5. *Let $\{x_n\} \subset B_{E^{**}}$ and $\delta > 0$ satisfying condition* (2)*. Then there are functionals $\{L_k\} \subset \overline{B}_{E^*}$ such that*

$$\prod_{\{k:\|x_k\|>\|x_j\|\}} \rho(L_k(x_k), L_k(x_j)) \geq \frac{\delta}{2} \qquad \text{and} \tag{16}$$

$$\sum_{\{k:\|x_k\|>\|x_j\|\}} (1-|L_k(x_k)|^2) \leq (1+2\log\frac{2}{\delta})\frac{1+\|x_j\|}{1-\|x_j\|} \qquad \forall j \in \mathbb{N}. \tag{17}$$

Proof. The proof follows the same pattern as Lemma 4. Simply choose $\alpha_k = \min_{\{j:\|x_j\|<\|x_k\|\}}\{\beta_k\, \rho(\|x_k\|, \|x_j\|)\}$ and pick L_k from Lemma 3. $\square$

Lemma 6 ([BCL])**.** *Let $\{S_j\}$ be a sequence of subsets of $\mathbb{N}$ such that if $k \notin S_j$, then $j \in S_k$. Let $\{c_j\}$ be a sequence of nonnegative numbers. Then*

$$\sum_{j=1}^{\infty} c_j\ e^{-\sum_{k\in S_j} c_k} \leq 2e.$$

Now we proceed to prove our main result.

Proof of Theorem 1. Let $\{T_{k,j}\}$ be the set of functionals furnished by Lemma 4. Define

$$g_{k,j}(x) = a_{k,j}\frac{T_{k,j}(x_k) - T_{k,j}(x)}{1-\overline{T_{k,j}(x_k)}T_{k,j}(x)} \qquad \forall k, j \in \mathbb{N}, \quad k \neq j,$$

where $a_{k,j}$ is given by

$$a_{k,j} = \begin{cases} \frac{\overline{T_{k,j}(x_k)}}{|T_{k,j}(x_k)|} & \text{if } T_{k,j}(x_k) \neq 0, \\ -1 & \text{if } T_{k,j}(x_k) = 0. \end{cases}$$

For each $j \in \mathbb{N}$ we define $B_j(x) = \prod_{k \neq j} g_{k,j}(x)$. First we check that this infinite product converges uniformly on $rB_{E^{**}}$ for fixed $0 < r < 1$. We have

$$\begin{aligned} 1 - g_{k,j}(x) &= \frac{1}{|T_{k,j}(x_k)|}\left[1 - \frac{|T_{k,j}(x_k)|^2 - \overline{T_{k,j}(x_k)}T_{k,j}(x)}{1 - \overline{T_{k,j}(x_k)}T_{k,j}(x)}\right] + 1 - \frac{1}{|T_{k,j}(x_k)|} \\ &= \frac{1 - |T_{k,j}(x_k)|}{|T_{k,j}(x_k)|}\left[\frac{1 + |T_{k,j}(x_k)|}{1 - \overline{T_{k,j}(x_k)}T_{k,j}(x)} - 1\right]. \end{aligned}$$

Moreover, $|T_{k,j}(x)| \leq \|x\| \leq r < 1$, so we get

$$|1 - g_{k,j}(x)| \leq \frac{1 - |T_{k,j}(x_k)|}{|T_{k,j}(x_k)|}\left[\frac{2}{1-r} + 1\right].$$

Since $\lim_k \|x_k\| = 1$, it follows that $\|x_k\| > \|x_j\|$, for $k = k_j, k_j + 1, \ldots$ and recalling (9),

$$|1 - g_{k,j}(x)| \leq \frac{1}{m}\left[\frac{2}{1-r} + 1\right](1 - |T_{k,j}(x_k)|) \qquad \forall k \geq k_j.$$

Now we use inequality (15) to show that the series $\sum_{k=1}^{\infty} |1 - g_{k,j}(x)|$ is uniformly convergent on $\{x : \|x\| \leq r\}$, as we wanted. In particular, $\prod_{k \neq j} g_{k,j}(x)$ converges uniformly on compact sets, so $B_j \in H^\infty(B_{E^{**}})$.

Note that $\|B_j\|_\infty \leq 1$, $B_j(x_k) = 0$ for $k \neq j$ and that, according to (14), $|B_j(x_j)| \geq \frac{\delta}{2}$.

Next we take the sequence $\{L_k\}$ found in Lemma 5 and we define

$$A_j(x) = \sum_{\{k : \|x_k\| \geq \|x_j\|\}} \left[\frac{1 + L_k(x_k)L_k(x)}{1 - L_k(x_k)L_k(x)}\right](1 - L_k(x_k)^2).$$

Since $\left|\frac{1+L_k(x_k)L_k(x)}{1-L_k(x_k)L_k(x)}\right| \leq \frac{1+\|x\|}{1-\|x\|}$, it follows from (17) that the convergence of this series is uniform on $rB_{E^{**}}$. Therefore $G_j := A_j - A_j(x_j) \in H(B_{E^{**}})$.

Let $F_j \in H^\infty(B_{E^{**}})$ be given by

$$F_j(x) = \frac{B_j(x)}{B_j(x_j)} q_j(x) \exp\left\{-\frac{1}{2(1 + 2\log\frac{2}{\delta})} G_j(x)\right\}$$

where $q_j(x) = \left[\frac{1-|L_j(x_j)|^2}{1-L_j(x_j)L_j(x)}\right]^2$.

Recall that the Aron-Berner extension is also continuous whenever both $H^\infty(B_E)$ and $H^\infty(B_{E^{**}})$ are endowed with the topology of the uniform convergence on balls of radii less than 1 (see [ACG] (10.1) p. 86). Consequently, each of the functions $g_{k,j}, B_j, q_j$ and F_j is the Aron-Berner extension of its restriction to B_E.

We claim that

$$F_j(x_k) = \delta_{j,k} \text{ for } j, k \in \mathbb{N} \text{ and that} \tag{18}$$

$$\text{there exists } M > 0 \text{ such that } \sum |F_j(x)| \leq M. \tag{19}$$

Condition (18) is trivially verified. To prove (19), we begin by recalling that $|B_j(x_j)| \geq \delta/2$. Then

$$\sum_{j=1}^{\infty} |F_j(x)| \leq \frac{2}{\delta} \sum_{j=1}^{\infty} |q_j(x)| \exp\left\{ -\frac{1}{2(1+2\log\frac{2}{\delta})} \Re e\ G_j(x) \right\}.$$

Applying (7) we have

$$\Re e\ A_j(x) = \sum_{\{k:\|x_k\|\geq\|x_j\|\}} \frac{(1-L_k(x_k)^2|L_k(x)|^2)(1-L_k(x_k)^2)}{|1-L_k(x_k)L_k(x)|^2}. \tag{20}$$

If $\|x_k\| > \|x_j\|$, then $|L_k(x_j)| \leq \|x_j\| \leq L_k(x_k)$ by (11). Now, applying the inequality $1-\alpha\beta \leq 2(1-\beta)$ for $\alpha \geq \beta$, it turns out that $1-L_k(x_k)^2|L_k(x_j)|^2 \leq 2(1-|L_k(x_j)|^2)$. Thus

$$\begin{aligned} \Re e\ A_j(x_j) &\leq 2 \sum_{\{k:\|x_k\|\geq\|x_j\|\}} \frac{(1-|L_k(x_j)|^2)(1-L_k(x_k)^2)}{|1-L_k(x_k)L_k(x_j)|^2} \\ &= \sum_{\{k:\|x_k\|>\|x_j\|\}} 2\left[1-\rho(L_k(x_j),L_k(x_k))^2\right]+2 \end{aligned}$$

after bearing in mind (5). Next, using inequalities (6) and (16) we obtain

$$\Re e\ A_j(x_j) \leq -2\log \prod_{\{k:\|x_k\|>\|x_j\|\}} \rho(L_k(x_j),L_k(x_k))^2 + 2 \leq 2(1+2\log\frac{2}{\delta})$$

and then

$$\sum_{j=1}^{\infty} |F_j(x)| \leq \frac{2e}{\delta} \sum_{j=1}^{\infty} |q_j(x)| \exp\left\{ -\frac{1}{2(1+2\log\frac{2}{\delta})} \Re e\ A_j(x) \right\}.$$

Therefore, bearing in mind (20), we have that $\sum_{j=1}^{\infty} |F_j(x)|$ is bounded by

$$\frac{2e}{\delta} \sum_{j=1}^{\infty} \left| \frac{1-L_j(x_j)^2}{1-L_j(x_j)L_j(x)} \right|^2 \exp\left\{ -\frac{1}{2(1+2\log\frac{2}{\delta})} \sum_{\{k:\|x_k\|\geq\|x_j\|\}} \frac{(1-L_k(x_k)^2)^2}{|1-L_k(x_k)L_k(x)|^2} \right\}.$$

To apply Lemma 6 to the above series we put $c_j = \frac{1}{2(1+2\log\frac{2}{\delta})} \left| \frac{1-L_j(x_j)^2}{1-L_j(x_j)L_j(x)} \right|^2$. Then we obtain

$$\sum_{j=1}^{\infty} |F_j(x)| \leq \frac{4e(1+2\log\frac{2}{\delta})}{\delta} 2e = \frac{8e^2}{\delta}(1+2\log\frac{2}{\delta}) := M. \qquad \square$$

The converse of Theorem 1 is false for any Banach space E. It is enough to consider a unit vector $x \in E$ and an interpolating sequence (λ_n) in $]0,1[$ such that $\lambda_1 = 1/2$. Let $\phi \neq 0$ be a linear mapping whose norm is 1 and $\phi(x) = 1$. Since the sequence $\{-\lambda_1, \lambda_1, \ldots, \lambda_n, \ldots\}$ is interpolating for H^∞, the sequence $\{-\lambda_1 x, \lambda_1 x, \ldots, \lambda_n x, \ldots\}$, is interpolating for $H^\infty(B_E)$. However $\|-\lambda_1 x\| = \|\lambda_1 x\| = 1/2$, so condition (2) clearly fails.

Corollary 7. *Let $\{x_n\}$ be a sequence in $B_{E^{**}}$. If $\{\|x_n\|\}$ is an interpolating sequence for H^∞, then $\{x_n\}$ is a linear interpolating sequence for $H^\infty(B_E)$.*

Proof. Carleson's condition for the disc (1) states that $\{\|x_n\|\}$ is an interpolating sequence for H^∞ if, and only if, condition (2) is satisfied. Then Theorem 1 shows that $\{x_n\}$ is a linear interpolating sequence for $H^\infty(B_E)$. □

Corollary 8. *Let $\{x_n\}$ be a sequence in $B_{E^{**}}$ and $0 < c < 1$ such that*

$$\frac{1 - \|x_{k+1}\|}{1 - \|x_k\|} < c. \tag{21}$$

Then $\{x_n\}$ is a linear interpolating sequence for $H^\infty(B_E)$.

Proof. It is well-known (see [H]) that, if Hayman-Newman's condition (21) is satisfied by a sequence in $\mathbf{D}$, then it also fulfills Carleson's condition (1). So it suffices to apply the above corollary. □

Note that from Corollary 8 we derive the following interpolation result.

Theorem 9 ([ACG])**.** *Let $\{x_n\}$ be a sequence in $B_{E^{**}}$ satisfying $\lim_{n\to\infty} \|x_n\| = 1$. Then there is a linear interpolating subsequence for $H^\infty(B_E)$.*

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