

## STABILITY OF SOLITARY WAVES FOR THE OSTROVSKY EQUATION

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ABSTRACT. Considered herein is the Ostrovsky equation which is widely used to describe the effect of rotation on the surface and internal solitary waves in shallow water or the capillary waves in a plasma. It is shown that the solitary-wave solutions are orbitally stable for certain wave speeds.

### 1. INTRODUCTION

The equation

$$(1.1) \quad \partial_x \{ \partial_t u + \partial_x^3 u + \partial_x(u^2) \} + \gamma u = 0, \quad x \in \mathbb{R},$$

was derived by Ostrovsky [14] as a model for the propagation of small-amplitude surface and internal waves in a rotating fluid, where  $u(t, x)$  can be considered as the fluid velocity in the  $x$ -direction and the parameter  $\gamma = \pm f^2/c_0$  measures the effect of rotation with the wave speed  $c_0$  and the local Coriolis parameter  $f$  (see also [1, 5, 6] for other physical backgrounds). Setting  $\gamma = 0$  in (1.1) and integrating with respect to  $x$  in  $\mathbb{R}$  and assuming the solution  $u(t, x)$  and all the derivatives are vanishing at infinity, one obtains the well-known Korteweg-de Vries (KdV) equation

$$(1.2) \quad \partial_t u + \partial_x^3 u + \partial_x(u^2) = 0, \quad x \in \mathbb{R}.$$

Although the structure of (1.1) is similar to that of (1.2), unlike the KdV equation (1.2), the Ostrovsky equation (1.1) is evidently nonintegrable by the method of the inverse scattering transform [6, 15]. Here, we recall conserved quantities of (1.1):

$$(1.3) \quad V(u) = \frac{1}{2} \int_{\mathbb{R}} |u(x)|^2 dx \quad (\text{momentum}),$$

$$(1.4) \quad E(u) = \int_{\mathbb{R}} \left\{ \frac{1}{2} |\partial_x u(x)|^2 + \frac{\gamma}{2} |D_x^{-1} u(x)|^2 - \frac{1}{3} u(x)^3 \right\} dx \quad (\text{energy}),$$

where for  $k \in \mathbb{N}$  the operator  $D_x^{-k}$  is defined by

$$(\widehat{D_x^{-k} f})(\xi) = (i\xi)^{-k} \hat{f}(\xi)$$

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for  $f \in L^2(\mathbb{R})$  such that  $\xi^{-k} \hat{f}(\xi) \in L^2(\mathbb{R})$ . Note that one can write (1.1) as the Hamiltonian formulation

$$\frac{du}{dt} = \partial_x E'(u).$$

For  $s \geq 0$ , we define the space  $X_s$  by

$$X_s = \{f \in H^s(\mathbb{R}) : D_x^{-1} f \in L^2(\mathbb{R})\}$$

with inner product

$$(f, g)_{X_s} = (f, g)_{H^s} + (D_x^{-1} f, D_x^{-1} g)_{L^2}.$$

Actually, a natural space to look for solutions to the Ostrovsky equation (1.1) is the energy space  $X_1$  suggested by the conservation laws of momentum (1.3) and energy (1.4). In what follows, we write  $X$  for  $X_1$ .

Assume  $u \in X_1$  is the solution of (1.1). Then  $v = D_x^{-1} u \in L_2$  such that  $v_x = u$  at least in the sense of distribution. In essence,  $u$  can be regarded as the fluid velocity in the  $x$ -direction, which in the weakly nonlinear long wave approximation is equivalent to the vertical wave displacement at leading order, while  $v$  is the fluid velocity in the transverse direction.

Regarding the initial-value problem for (1.1), it was first proved by Varlamov and Liu [16] that (1.1) is locally well posed in  $X_s$  for  $s > 3/2$ . This local well posedness result is recently improved by Linares and Milanés [11] to the case  $s > 3/4$ . Consequently, if  $\gamma > 0$ , (1.1) is globally well posed in the energy space  $X$  due to the conservation laws of momentum (1.3) and energy (1.4). Furthermore, use of bilinear estimates in a Bourgain’s space enables us to establish some finer existence results (see Gui and Liu [9]).

Focus of the development in the present paper is the stability of solitary-wave solutions of (1.1). Localized, traveling-wave solutions of nonlinear dispersive-wave equations are known in many circumstances to play a central role in the long-time evolution of an initial disturbance. By a solitary wave, we mean a traveling-wave solution of (1.1) with the form  $u(t, x) = \varphi_c(x - ct)$ , where  $c \in \mathbb{R}$  is a given parameter and  $\varphi_c$  is a *ground state* of the stationary problem

$$(1.5) \quad -\partial_x^2 \varphi + c\varphi - \gamma D_x^{-2} \varphi - \varphi^2 = 0, \quad x \in \mathbb{R}.$$

To define the ground state, we introduce some notation.

$$\begin{aligned} L_c(u) &= E(u) + cV(u) \\ &= \int_{\mathbb{R}} \left\{ \frac{1}{2} |\partial_x u(x)|^2 + \frac{c}{2} |u(x)|^2 + \frac{\gamma}{2} |D_x^{-1} u(x)|^2 - \frac{1}{3} u(x)^3 \right\} dx, \\ \mathcal{S}_c &= \{v \in X \setminus \{0\} : L'_c(v) = 0\}, \\ \mathcal{G}_c &= \{w \in \mathcal{S}_c : L_c(w) \leq L_c(v), \forall v \in \mathcal{S}_c\}. \end{aligned}$$

Note that (1.5) is regarded as the Euler-Lagrange equation of the action  $L_c$  defined on  $X$ ,  $\mathcal{S}_c$  is the set of all nontrivial solutions for (1.5), and  $\mathcal{G}_c$  is the set of all ground states for (1.5).

It was proved by Liu and Varlamov [13] that if  $\gamma > 0$  and  $c > -2\sqrt{\gamma}$ , then  $\mathcal{G}_c$  is not empty. As is well known, for  $c > 0$ , KdV equation (1.2) has a solitary-wave solution  $\psi_c(x - ct)$ , where

$$(1.6) \quad \psi_c(x) = \frac{3c}{2} \operatorname{sech}^2 \left( \frac{\sqrt{c}}{2} x \right)$$

is a unique positive solution in  $H^1(\mathbb{R})$ , up to translations, of

$$(1.7) \quad -\partial_x^2 \psi + c\psi - \psi^2 = 0, \quad x \in \mathbb{R}.$$

It is shown in [5, 6, 13] that (1.5) does not admit any nontrivial solutions in the energy space  $X$  provided  $\gamma < 0$  and some positive  $c$  with  $c < \sqrt{140|\gamma|}$ . Hence, the question of how an initial perturbation in the form of a KdV soliton will be destroyed will be worth investigating. On the other hand, we observed that (1.7) does not have any nontrivial solitary waves in the energy space  $H^1(\mathbb{R})$  when  $c \leq 0$ . However, unlike (1.2), (1.1) does have solitary-wave solutions even for some negative  $c$  satisfying  $c > -2\sqrt{\gamma}$  when  $\gamma > 0$  (see [13]). This notable property of the equation makes a search of its stability of solitary waves highly desirable.

Our goal here with regard to (1.1) is to establish the orbital stability with respect to arbitrary perturbations of the solitary wave  $\varphi_c(x - ct)$  for large speed  $c$ .

**Definition.** For  $\varphi_c \in \mathcal{G}_c$  and  $\delta > 0$ , we put

$$U_\delta(\varphi_c) = \{v \in X : \inf_{y \in \mathbb{R}} \|v - \varphi_c(\cdot + y)\|_X < \delta\}.$$

We say that a solitary-wave solution  $\varphi_c(x - ct)$  of (1.1) is stable in  $X$  if for any  $\epsilon > 0$  there exists  $\delta > 0$  such that for any  $u_0 \in U_\delta(\varphi_c)$ , the solution  $u(t)$  of (1.1) with  $u(0) = u_0$  satisfies  $u(t) \in U_\epsilon(\varphi_c)$  for any  $t \geq 0$ . Otherwise,  $\varphi_c(x - ct)$  is said to be unstable in  $X$ .

The principal result of the present paper is the following.

**Theorem 1.1** (Stability). *Let  $\gamma > 0$ ,  $c > -2\sqrt{\gamma}$  and  $\varphi_c \in \mathcal{G}_c$ . Then, there exists  $c_* > 0$  such that if  $c > c_*$ , then the solitary-wave solution  $\varphi_c(x - ct)$  of (1.1) is stable in  $X$ .*

Stability of the set of ground states  $\mathcal{G}_c$  was proved in [13] under the assumption of convexity of action  $d(c) = L_c(\varphi_c)$  with  $\varphi_c \in \mathcal{G}_c$ . It can be done by showing the solitary wave  $\varphi_c(x - ct)$  is a local constrained minimizer of a Hamiltonian functional with this condition of  $d(c)$ . However, being different from the case of the KdV equation, the scaling and dilation technique does not give the description of the action  $d(c)$  explicitly. On the other hand, it is noted that the spectra of a linearized operator which is a fourth-order differential operator at the solitary wave  $\varphi_c$  is not easy to deal with, so that it seems difficult to apply the abstract theory in Grillakis, Shatah and Strauss [7] directly. To prove Theorem 1.1, we basically follow the proof in Fukuizumi and Ohta [4] to establish the following sufficient condition for stability of the solitary wave  $\varphi_c(x - ct)$ . We emphasize that we do not need to study the spectra of a linearized operator, for example, the simpleness of zero and negative eigenvalues.

**Proposition 1.2.** *Let  $\gamma > 0$ ,  $c > -2\sqrt{\gamma}$  and  $\varphi_c \in \mathcal{G}_c$ . Assume that there exists  $\delta > 0$  such that  $\langle L_c''(\varphi_c)v, v \rangle \geq \delta \|v\|_X^2$  for any  $v \in X$  satisfying  $(v, \varphi_c)_{L^2} = (v, \partial_x \varphi_c)_{L^2} = 0$ . Then the solitary-wave solution  $\varphi_c(x - ct)$  of (1.1) is stable in  $X$ .*

For the sake of completeness, we give the proof of Proposition 1.2 in Section 2. In Section 3, we prove Theorem 1.1 using Proposition 1.2.

2. PROOF OF PROPOSITION 1.2

In this section, we give the proof of Proposition 1.2. Based on Theorem 3.4 in [7] and Lemma 2.1 in [4] (see also Lemma 2.1 in [3]), we first prove the following lemma.

**Lemma 2.1.** *Let  $\gamma > 0$ ,  $c > -2\sqrt{\gamma}$  and  $\varphi_c \in \mathcal{G}_c$ . Assume that there exists  $\delta > 0$  such that  $\langle L_c''(\varphi_c)v, v \rangle \geq \delta \|v\|_X^2$  for any  $v \in X$  satisfying  $(v, \varphi_c)_{L^2} = (v, \partial_x \varphi_c)_{L^2} = 0$ . Then there exist  $C > 0$  and  $\varepsilon > 0$  such that*

$$E(u) - E(\varphi_c) \geq C \inf_{y \in \mathbb{R}} \|u - \varphi_c(\cdot + y)\|_X^2$$

for any  $u \in U_\varepsilon(\varphi_c)$  satisfying  $V(u) = V(\varphi_c)$ .

*Proof.* Let  $u \in U_\varepsilon(\varphi_c)$  with  $V(u) = V(\varphi_c)$ . By the implicit function theorem, if  $\varepsilon > 0$  is small, there exists  $y(u) \in \mathbb{R}$  such that

$$(2.1) \quad \|u - \varphi_c(\cdot + y(u))\|_X^2 = \min_{y \in \mathbb{R}} \|u - \varphi_c(\cdot + y)\|_X^2.$$

Let  $v := u(\cdot - y(u)) - \varphi_c$ . Then, the Taylor expansion gives

$$L_c(u) = L_c(u(\cdot - y(u))) = L_c(\varphi_c) + \langle L_c'(\varphi_c), v \rangle + \frac{1}{2} \langle L_c''(\varphi_c)v, v \rangle + o(\|v\|_X^2).$$

Since  $L_c'(\varphi_c) = 0$  and  $V(\varphi_c) = V(u)$ , we have

$$(2.2) \quad E(u) - E(\varphi_c) = \frac{1}{2} \langle L_c''(\varphi_c)v, v \rangle + o(\|v\|_X^2).$$

We decompose  $v$  as  $v = a\varphi_c + b\partial_x \varphi_c + w$  with  $a, b \in \mathbb{R}$ ,  $w \in X$  satisfying  $(w, \varphi_c)_{L^2} = (w, \partial_x \varphi_c)_{L^2} = 0$ . Another expansion gives

$$\begin{aligned} V(\varphi_c) &= V(u) = V(u(\cdot - y(u))) = V(\varphi_c) + \langle V'(\varphi_c), v \rangle + O(\|v\|_X^2), \\ \langle V'(\varphi_c), v \rangle &= (\varphi_c, v)_{L^2} = (\varphi_c, a\varphi_c + b\partial_x \varphi_c + w)_{L^2} = a\|\varphi_c\|_2^2. \end{aligned}$$

Thus, we have  $a = O(\|v\|_X^2)$ . Moreover, since  $(v, \partial_x \varphi_c)_X = 0$  by (2.1) and since  $(\varphi_c, \partial_x \varphi_c)_X = 0$ , we have  $0 = (v, \partial_x \varphi_c)_X = b\|\partial_x \varphi_c\|_X^2 + (w, \partial_x \varphi_c)_X$ . Thus, we have  $|b|\|\partial_x \varphi_c\|_X \leq \|w\|_X$  and

$$\|v\|_X \leq |a|\|\varphi_c\|_X + |b|\|\partial_x \varphi_c\|_X + \|w\|_X \leq 2\|w\|_X + O(\|v\|_X^2).$$

Therefore, we have

$$(2.3) \quad \|w\|_X^2 \geq \frac{1}{4}\|v\|_X^2 + O(\|v\|_X^3).$$

Furthermore, since  $L_c''(\varphi_c)\partial_x \varphi_c = 0$ , we have

$$(2.4) \quad \begin{aligned} \langle L_c''(\varphi_c)w, w \rangle &= \langle L_c''(\varphi_c)v, v \rangle - 2a\langle L_c''(\varphi_c)\varphi_c, v \rangle + a^2\langle L_c''(\varphi_c)\varphi_c, \varphi_c \rangle \\ &= \langle L_c''(\varphi_c)v, v \rangle + O(\|v\|_X^3). \end{aligned}$$

Since  $w \in X$  satisfies  $(w, \varphi_c)_{L^2} = (w, \partial_x \varphi_c)_{L^2} = 0$ , by the assumption of Lemma 2.1, there exists  $\delta > 0$  such that

$$(2.5) \quad \langle L_c''(\varphi_c)w, w \rangle \geq \delta \|w\|_X^2.$$

By (2.2)–(2.5), we have

$$E(u) - E(\varphi_c) \geq \frac{\delta}{2}\|w\|_X^2 + o(\|v\|_X^2) \geq \frac{\delta}{8}\|v\|_X^2 + o(\|v\|_X^2).$$

Finally, since  $u \in U_\varepsilon(\varphi_c)$  and  $\|v\|_X = \|u - \varphi_c(\cdot + y(u))\|_X < \varepsilon$ , we can take  $\varepsilon = \varepsilon(\delta) > 0$  such that

$$E(u) - E(\varphi_c) \geq \frac{\delta}{9} \|u - \varphi_c(\cdot + y(u))\|_X^2.$$

This completes the proof.  $\square$

Proposition 1.2 is proved using Lemma 2.1 in the same way as the proof of Theorem 3.5 of [7]. We omit the detail.

### 3. PROOF OF THEOREM 1.1

Throughout this section, we assume  $\gamma > 0$ ,  $c > 0$  and  $\varphi_c \in \mathcal{G}_c$ . Following Lemma 3.1 in [4] (see also [3]), we prove Theorem 1.1 by checking the assumption on the linearized operator  $L_c''(\varphi_c)$  in Proposition 1.2 (see also Esteban and Strauss [2]). Notice that

$$\langle L_c''(\varphi_c)v, v \rangle = \int_{\mathbb{R}} \{ |\partial_x v(x)|^2 + c|v(x)|^2 + \gamma |D_x^{-1}v(x)|^2 - 2\varphi_c(x)|v(x)|^2 \} dx$$

for  $v \in X$ , and we put

$$\|v\|_{X(c)}^2 = \int_{\mathbb{R}} \{ |\partial_x v(x)|^2 + c|v(x)|^2 + \gamma |D_x^{-1}v(x)|^2 \} dx, \quad v \in X.$$

Note that  $\|\cdot\|_{X(c)}$  is an equivalent norm on  $X$  to  $\|\cdot\|_X$ .

We define  $\tilde{\varphi}_c$  by  $\varphi_c(x) = c\tilde{\varphi}_c(\sqrt{c}x)$ . Then  $\tilde{\varphi}_c$  satisfies

$$(3.1) \quad -\partial_x^2 \tilde{\varphi}_c + \tilde{\varphi}_c - \frac{\gamma}{c^2} D_x^{-2} \tilde{\varphi}_c - \tilde{\varphi}_c^2 = 0, \quad x \in \mathbb{R}.$$

Moreover, we define the norm  $\|\cdot\|_{\tilde{X}(c)}$  by

$$(3.2) \quad \|v\|_{\tilde{X}(c)}^2 = \|v\|_{H^1}^2 + \frac{\gamma}{c^2} \|D_x^{-1}v\|_{L^2}^2, \quad v \in X,$$

and we define the operator  $\tilde{H}_c$  by

$$\langle \tilde{H}_c v, v \rangle = \|v\|_{\tilde{X}(c)}^2 - 2 \int_{\mathbb{R}} \tilde{\varphi}_c(x)|v(x)|^2 dx, \quad v \in X.$$

Then, for  $v(x) = \tilde{v}(\sqrt{c}x)$ , we have

$$\begin{aligned} \|v\|_{X(c)}^2 &= c^{1/2} \|\tilde{v}\|_{\tilde{X}(c)}^2, & \langle L_c''(\varphi_c)v, v \rangle &= c^{1/2} \langle \tilde{H}_c \tilde{v}, \tilde{v} \rangle, \\ (v, \varphi_c)_{L^2} &= c^{1/2} (\tilde{v}, \tilde{\varphi}_c)_{L^2}, & (v, \partial_x \varphi_c)_{L^2} &= c (\tilde{v}, \partial_x \tilde{\varphi}_c)_{L^2}. \end{aligned}$$

Therefore, Theorem 1.1 follows from Proposition 1.2 and the following lemma.

**Lemma 3.1.** *Let  $\gamma > 0$ ,  $c > 0$  and  $\varphi_c \in \mathcal{G}_c$ . Then there exists  $c_* > 0$  with the following property: if  $c > c_*$ , then there exists  $\delta > 0$  such that  $\langle \tilde{H}_c v, v \rangle \geq \delta \|v\|_{\tilde{X}(c)}^2$  for any  $v \in X$  satisfying  $(v, \tilde{\varphi}_c)_{L^2} = (v, \partial_x \tilde{\varphi}_c)_{L^2} = 0$ .*

In the proof of Lemma 3.1, the following two lemmas play an important role.

**Lemma 3.2.** *Let  $\gamma > 0$ ,  $c > 0$  and  $\varphi_c \in \mathcal{G}_c$ . Let  $\psi_1$  be the unique positive solution of (1.7) with  $c = 1$  given by (1.6). Then, for any sequence  $\{c_j\}$  satisfying  $c_j \rightarrow \infty$ , there exists a subsequence of  $\{c_j\}$  (still denoted by the same letter) and a sequence  $\{y_j\} \subset \mathbb{R}$  such that*

$$(3.3) \quad \lim_{j \rightarrow \infty} \|\tilde{\varphi}_{c_j}(\cdot + y_j) - \psi_1\|_{H^1} = 0.$$

For the proof of Lemma 3.2, see Theorem 3.1 in [10].

Next, we define the operator  $H$  by

$$\langle Hv, v \rangle = \|v\|_{H^1}^2 - 2 \int_{\mathbb{R}} \psi_1(x)|v(x)|^2 dx, \quad v \in H^1(\mathbb{R}).$$

**Lemma 3.3.** *There exists  $\delta_1 > 0$  such that  $\langle Hw, w \rangle \geq \delta_1 \|w\|_{L^2}^2$  for any  $w \in H^1(\mathbb{R})$  satisfying  $(w, \psi_1)_{L^2} = (w, \partial_x \psi_1)_{L^2} = 0$ .*

For the proof of Lemma 3.3, see Proposition 1 in [8] and Lemma 4.2 in [3].

We are now in a position to prove Lemma 3.1.

*Proof of Lemma 3.1.* We prove by contradiction. Suppose that Lemma 3.1 is false. Then there exist sequences  $\{c_j\} \subset \mathbb{R}$  and  $\{v_j\} \subset X$  such that  $c_j \rightarrow \infty$  and

$$(3.4) \quad \limsup_{j \rightarrow \infty} \langle \tilde{H}_{c_j} v_j, v_j \rangle \leq 0, \quad \|v_j\|_{\tilde{X}(c_j)} = 1, \quad (v_j, \tilde{\varphi}_{c_j})_{L^2} = (v_j, \partial_x \tilde{\varphi}_{c_j})_{L^2} = 0.$$

By Lemma 3.2, there exists a subsequence of  $\{c_j\}$  (still denoted by the same letter) and a sequence  $\{y_j\} \subset \mathbb{R}$  such that  $\tilde{\varphi}_{c_j}(\cdot + y_j) \rightarrow \psi_1$  strongly in  $H^1(\mathbb{R})$ . By (3.2) and (3.4), we see that  $\{v_j(\cdot + y_j)\}$  is bounded in  $H^1(\mathbb{R})$ , so there exists a subsequence of  $\{v_j(\cdot + y_j)\}$  (still denoted by the same letter) and  $w \in H^1(\mathbb{R})$  such that  $v_j(\cdot + y_j) \rightarrow w$  weakly in  $H^1(\mathbb{R})$  and  $|v_j(\cdot + y_j)|^2 \rightarrow |w|^2$  weakly in  $L^2(\mathbb{R})$ . Thus, we have

$$(3.5) \quad \lim_{j \rightarrow \infty} \int_{\mathbb{R}} \tilde{\varphi}_{c_j}(x)|v_j(x)|^2 dx = \lim_{j \rightarrow \infty} \int_{\mathbb{R}} \tilde{\varphi}_{c_j}(x+y_j)|v_j(x+y_j)|^2 dx = \int_{\mathbb{R}} \psi_1(x)|w(x)|^2 dx.$$

It then follows from (3.4) and (3.5) that

$$(3.6) \quad \begin{aligned} 0 &\geq \limsup_{j \rightarrow \infty} \langle \tilde{H}_{c_j} v_j, v_j \rangle = \limsup_{j \rightarrow \infty} \left( \|v_j\|_{\tilde{X}(c_j)}^2 - 2 \int_{\mathbb{R}} \tilde{\varphi}_{c_j}(x)|v_j(x)|^2 dx \right) \\ &= 1 - 2 \int_{\mathbb{R}} \psi_1(x)|w(x)|^2 dx. \end{aligned}$$

In view of (3.4), (3.5) and  $\gamma > 0$ , we have

$$\begin{aligned} 0 &\geq \limsup_{j \rightarrow \infty} \langle \tilde{H}_{c_j} v_j, v_j \rangle \\ &\geq \liminf_{j \rightarrow \infty} \left( \|v_j(\cdot + y_j)\|_{H^1}^2 + \frac{\gamma}{c^2} \|D_x^{-1} v_j\|_{L^2}^2 - 2 \int_{\mathbb{R}} \tilde{\varphi}_{c_j}(x)|v_j(x)|^2 dx \right) \\ &\geq \|w\|_{H^1}^2 - 2 \int_{\mathbb{R}} \psi_1(x)|w(x)|^2 dx = \langle Hw, w \rangle. \end{aligned}$$

Moreover, by (3.4), it is found that

$$\begin{aligned} (w, \psi_1)_{L^2} &= \lim_{j \rightarrow \infty} (v_j(\cdot + y_j), \tilde{\varphi}_{c_j}(\cdot + y_j))_{L^2} = 0, \\ (w, \partial_x \psi_1)_{L^2} &= \lim_{j \rightarrow \infty} (v_j(\cdot + y_j), \partial_x \tilde{\varphi}_{c_j}(\cdot + y_j))_{L^2} = 0. \end{aligned}$$

It thus follows from Lemma 3.3 that  $w = 0$ . However, this contradicts (3.6). This completes the proof of Lemma 3.1. □

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