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DOMINATED POLYNOMIALS ON INFINITE DIMENSIONAL SPACES

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Abstract. The aim of this paper is to prove a stronger version of a conjecture posed earlier on the existence of nondominated scalar-valued m-homogeneous polynomials, $m \geq 3$, on arbitrary infinite dimensional Banach spaces.

Introduction

The theory of absolutely summing multilinear mappings and homogeneous polynomials between Banach spaces, which was first outlined by Pietsch [43], studies multilinear and polynomial generalizations of the very successful theory of absolutely summing linear operators. The theory has been developed by several authors, and among the advances obtained thus far we mention: Pietsch-type domination/factorization theorems ([11], Geiss [26], Pérez-García [38]), different types of absolutely summing multilinear mappings and polynomials (Achour and Mezrag [1], Carando and Dimant [15], Çaliskan and Pellegrino [14], Dimant [23], Pellegrino and Souza [36], Pérez-García [38]), Grothendieck-type theorems (Bombal et al. [4], Pérez-García and Villanueva [41]), coincidence/inclusion/composition theorems (Alencar and Matos [2], Botelho et al. [8], Pérez-García [39, 37], Popa [44]), connections with the geometry of Banach spaces ([5], Floret and Matos [25], Meléndez and Tonge [33], [35], Pérez-García [40]), interplay with other multi-ideals and polynomial ideals (Botelho et al. [7], [12], Cilia and Gutiérrez [17], Jarchow et al. [27], Matos [29]), estimates for absolutely summing norms (Aron et al. [3], [13], Choi et al. [16], Defant and Sevilla-Peris [21], Zalduendo [45]), extensions of the theory to more general nonlinear mappings (Junek, Matos and Pellegrino [28], Matos [30, 31], Matos and Pellegrino [32]).

Let us sketch some of the applications this theory has produced. The following results were obtained with the help of the theory of absolutely summing multilinear mappings: (i) for every $n \in \mathbb{N}$, a tensor norm of order n constructed by Pérez-García and Villanueva [41] is shown in Defant and Pérez-García [20] to preserve unconditionality for $\mathcal{L}_p$-spaces; (ii) Defant et al. [19] provides optimal estimates for the width of Bohr's strip for Dirichlet series in infinite dimensional Banach spaces;

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(iii) applications to quantum information theory are obtained by Pérez-García et al. [42], where it is proved that, contrary to the bipartite case, tripartite Bell inequalities can be unboundedly violated; (iv) in Jarchow et al. [27], the existence of Hahn-Banach-type extension theorems for multilinear forms is strongly connected to the structural properties of the underlying spaces.

One of the central notions of this theory is that of a dominated homogeneous polynomial. A continuous m-homogeneous polynomial P from the Banach space X to the Banach space Y is *r-dominated* if $(P(x_j))_{j=1}^{\infty}$ is $\frac{r}{m}$-summable in Y whenever $(x_j)_{j=1}^{\infty}$ is weakly r-summable in X.

The following conjecture was posed in [9]:

Conjecture 1. *There is no infinite dimensional Banach space X such that for every $m \in \mathbb{N}$ and every $r \geq 1$, any continuous scalar-valued m-homogeneous polynomial on X is r-dominated.*

It is known that the conjecture holds true for Banach spaces with unconditional basis (see [9, Theorem 3.2]). The status of the problem was changed by the proof of a stronger result for multilinear forms: in Jarchow et al. [27, Lemma 5.4] (see also [10, Proposition 3.2]), it is shown that for every $m \geq 3$ and every $r \geq 1$, on every infinite dimensional Banach space, there exists a continuous m-linear form that fails to be r-dominated. Although not solving Conjecture 1 (to prove Conjecture 1 we need a *symmetric* non-r-dominated m-linear form), this result indicates that a result stronger than Conjecture 1 should be pursued for polynomials:

Conjecture 2. *For every infinite dimensional Banach space X, every $m \geq 3$ and every $r \geq 1$, there is a continuous scalar-valued m-homogeneous polynomial on X that fails to be r-dominated.*

In Section 2 we solve Conjecture 2 (hence Conjecture 1) in the positive.

1. Notation

Throughout this paper, n and m are positive integers, and X and Y will stand for Banach spaces over $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$. The Banach space of all continuous m-homogeneous polynomials $P\colon X \longrightarrow Y$, with the sup norm, is denoted by $\mathcal{P}(^m X; Y)$ ($\mathcal{L}(X;Y)$ if $m = 1$). When $m = 1$ and $Y = \mathbb{K}$, we write X^* to denote the topological dual of X. The closed unit ball of X is represented by B_X. For details on the theory of polynomials between Banach spaces, we refer to [24, 34].

Given $r \in [0, \infty)$, let $\ell_r(X)$ be the Banach (r-Banach if $0 < r < 1$) space of all absolutely r-summable sequences $(x_j)_{j=1}^{\infty}$ in X with the norm $\|(x_j)_{j=1}^{\infty}\|_r = (\sum_{j=1}^{\infty} \|x_j\|^r)^{1/r}$. We denote by $\ell_r^w(X)$ the Banach (r-Banach if $0 < r < 1$) space of all weakly r-summable sequences $(x_j)_{j=1}^{\infty}$ in X with the norm $\|(x_j)_{j=1}^{\infty}\|_{w,r} = \sup_{\varphi \in B_{X^*}} \|(\varphi(x_j))_{j=1}^{\infty}\|_r$.

Let $p, q > 0$. An m-homogeneous polynomial $P \in \mathcal{P}(^m X; Y)$ is absolutely $(p;q)$-summing if $(P(x_j))_{j=1}^{\infty} \in \ell_p(Y)$ whenever $(x_j)_{j=1}^{\infty} \in \ell_q^w(X)$. It is well known that P is absolutely $(p;q)$-summing if and only if there is a constant $C \geq 0$ such that

$$\left(\sum_{j=1}^{n} \|P(x_j)\|^p\right)^{\frac{1}{p}} \leq C \left(\|(x_j)_{j=1}^{n}\|_{w,q}\right)^m$$

for every $x_1, \dots, x_n \in X$ and n a positive integer. The infimum of such a C is denoted by $\|P\|_{as(p;q)}$. The space of all absolutely $(p;q)$-summing m-homogeneous polynomials from X to Y is denoted by $\mathcal{P}_{as(p;q)}(^mX;Y)$, and $\|\cdot\|_{as(p;q)}$ is a complete norm (p-norm if $p<1$) on $\mathcal{P}_{as(p;q)}(^mX;Y)$.

An m-homogeneous polynomial $P \in \mathcal{P}(^mX;Y)$ is said to be r-*dominated* if it is absolutely $(\frac{r}{m};r)$-summing. In this case we write $\mathcal{P}_{d,r}(^mX;Y)$ and $\|\cdot\|_{d,r}$ instead of $\mathcal{P}_{as(\frac{r}{m};r)}(^mX;Y)$ and $\|\cdot\|_{as(\frac{r}{m};r)}$. As usual we write $\mathcal{P}_{d,r}(^mX)$ and $\mathcal{P}(^mX)$ when $Y=\mathbb{K}$. The definition (and notation) of r-dominated multilinear mappings is analogous (for the notation just replace $\mathcal{P}$ by $\mathcal{L}$). The symbol $\mathcal{L}^s$ means that only symmetric multilinear mappings are considered. For details we refer to [9, 27].

2. The proof of Conjecture 2

It is well-known that a homogeneous polynomial is p-dominated if and only if its associated symmetric multilinear map is also p-dominated (see [33, Theorem 6]). However, the multilinear and polynomial parts of our problem are not so tightly connected, as is made clear by the following example of a non-p-dominated multilinear form whose symmetrization is p-dominated: let

$$T:\ell_2\times\ell_2\to\mathbb{K}\ ,\ T(x,y)=\sum_{j=1}^{\infty}x_jy_{j+1}-\sum_{j=1}^{\infty}x_{j+1}y_j.$$

Note that $T(e_j,e_{j+1})=1$ for every j. So, T fails to be p-dominated (regardless of the $p\geq 1$). On the other hand, the symmetrization of T is zero. So, even knowing that the multilinear counterpart of the conjecture is true, it is in principle not clear that the same holds for polynomials.

To prove Conjecture 2 we need the following well-known results:

Lemma 2.1. (a) [6, Propositions 41(b) and 46(a)] *If* $\mathcal{P}_{d,r}(^nX;Y)=\mathcal{P}(^nX;Y)$, *then* $\mathcal{P}_{d,r}(^mX;Y)=\mathcal{P}(^mX;Y)$ *for every* $m\leq n$.

(b) ([11, Proposition 3.4] and [22, Theorem 2.8]) $\mathcal{P}_{d,r}(^nX;Y)\subseteq\mathcal{P}_{d,q}(^nX;Y)$ *if* $r\leq q$.

The following result has its proof inspired by the proof of [27, Lemma 5.4]; its final form was kindly suggested by the referee.

Theorem 2.2. *If X is infinite dimensional and* $\mathcal{P}_{d,2}(^2X)=\mathcal{P}(^2X)$, *then* $\mathcal{P}_{d,r}(^mX)\neq\mathcal{P}(^mX)$ *for every* $m\geq 3$ *and every* $r\geq 1$.

Proof. By Lemma 2.1(b) we may assume $r\geq 2$, and by Lemma 2.1(a) we just need to prove the case $m=3$. Since X is infinite dimensional, by Dvoretzky's theorem there is a sequence $(E_n)_{n=1}^{\infty}$ of subspaces of X such that X/E_n is 2-isomorphic to ℓ_2^n for every n (details can be found in the proof of [22, Proposition 19.17(b)]). So we can consider isomorphisms $j_n\colon X/E_n\longrightarrow\ell_2^n$ such that $\|j_n\|\leq 4$ and $\|j_n^{-1}\|=\frac{1}{2}$ for every n. Letting $\pi_n\colon X\longrightarrow X/E_n$ be the corresponding quotient maps and defining $q_n:=j_n\circ\pi_n$ we have that

$$\|q_n\|\leq 4 \text{ and } B_{\ell_2^n}\subseteq q_n(B_X) \text{ for every } n.$$

By assumption we know that $\mathcal{L}^s_{d,2}(^2X)=\mathcal{L}^s(^2X)$, so by the open mapping theorem there exists $C>0$, depending only on X, such that $\|A\|_{d,2}\leq C\|A\|$ for every $A\in\mathcal{L}^s(^2X)$. Given $n\in\mathbb{N}$, defining

$$A\colon X\times X\longrightarrow\mathbb{K}\ ,\ A(x,y)=\langle q_n(x),q_n(y)\rangle,$$

we have that A is a continuous symmetric (in the complex case we consider its symmetrization) bilinear form on X and $\|A\| \leq 16$ because $\|q_n\| \leq 4$. Given $x_1, \ldots, x_k \in X$,

$$\begin{aligned} \sum_{j=1}^{k} \|q_n(x_j)\|_2^2 &= \sum_{j=1}^{k} \langle q_n(x_j), q_n(x_j)\rangle = \sum_{j=1}^{k} |A(x_j, x_j)| \\ &\leq \|A\|_{d,2} \|(x_j)_{j=1}^k\|_{w,2}^2 \leq C\|A\| \|(x_j)_{j=1}^k\|_{w,2}^2 \\ &\leq 16C \|(x_j)_{j=1}^k\|_{w,2}^2, \end{aligned}$$

showing that $\|q_n\|_{as(2,2)} \leq 4\sqrt{C}$ for every n.

For each positive integer n, consider the canonical embedding $i_n\colon \ell_2^n \longrightarrow \ell_\infty^n$. It is plain that $\|i_n\| = 1$ for every n and $\lim_{n\to\infty} \|i_n\|_{as(r;r)} = \infty$. Again, by Dvoretzky's theorem, there is an n-dimensional subspace X_n of X and an isomorphism $k_n\colon X_n \longrightarrow \ell_2^n$ such that $\|k_n\| \leq 1$ and $\|k_n^{-1}\| \leq 2$ (for every n). Since ℓ_∞^n is an injective space, there is a norm-preserving extension u_n of $i_n \circ k_n$ to the whole of X. Let us see that $\sup_n \|u_n\|_{as(r;r)} = \infty$. Given $M > 0$, since $\sup_n \|i_n\|_{as(r,r)} = \infty$, there exists n such that $\|i_n\|_{as(r,r)} > M$. Then, there exist $m \in \mathbb{N}$ and $x_1, \ldots, x_m \in \ell_2^n$ such that

$$\left(\sum_{j=1}^{m} \|i_n(x_j)\|_\infty^r\right)^{1/r} > M \|(x_j)_{j=1}^m\|_{w,r}. \tag{1}$$

Since the restriction of u_n to X_n is $i_n \circ k_n$, for any $x \in \ell_2^n$ we have

$$\|i_n(x)\|_\infty = \|i_n \circ k_n(k_n^{-1}(x))\|_\infty = \|u_n(k_n^{-1}(x))\|_\infty.$$

From (1) it follows that

$$\begin{aligned} \left(\sum_{j=1}^{m} \|u_n(k_n^{-1}(x_j))\|_\infty^r\right)^{1/r} &> M \|(x_j)_{j=1}^m\|_{w,r} \\ &= \frac{M}{2} \sup_{\varphi \in 2B_{(\ell_2^n)^*}} \left(\sum_{j=1}^{m} |\varphi(x_j)|^r\right)^{1/r} \\ &\overset{(*)}{\geq} \frac{M}{2} \sup_{\psi \in B_{(X_n)^*}} \left(\sum_{j=1}^{m} \left|\psi(k_n^{-1}(x_j))\right|^r\right)^{1/r} \\ &= \frac{M}{2} \|(k_n^{-1}(x_j))_{j=1}^m\|_{w,r}. \end{aligned}$$

$(*)$ $\psi \in B_{(X_n)^*} \Longrightarrow \|\psi \circ k_n^{-1}\| \leq 2 \Longrightarrow \psi \circ k_n^{-1} \in 2B_{(\ell_2^n)^*}$.

Hence, $\sup_n \|u_n\|_{as(r,r)} = \infty$. Following the steps of the proof of [27, Lemma 5.4], ℓ_∞^n can be identified with the diagonal of $\ell_2^n \otimes_\varepsilon \ell_2^n$, so that u_n can be regarded as a linear operator from X to the diagonal of $\ell_2^n \otimes_\varepsilon \ell_2^n$ whose norm does not depend on n. From

$$\ell_2^n \otimes_\varepsilon \ell_2^n \overset{(**)}{=} ((\ell_2^n)^* \otimes_\pi (\ell_2^n)^*)^* = (\ell_2^n \otimes_\pi \ell_2^n)^* = \mathcal{L}(^2\ell_2^n)$$

(for $(**)$ see [18, Proposition 4.1(1)]), u_n can now be regarded as a linear operator from X to $\mathcal{L}(^2\ell_2^n)$ such that $u_n(x)$ is a symmetric bilinear form (elements of the diagonal of the tensor product are symmetric tensors, to which correspond symmetric bilinear forms) for every $x \in X$.

Consider the 3-linear form

$$T_n : X \times X \times X \longrightarrow \mathbb{K}\ ,\ T_n(x,y,z) = u_n(x)(q_n(y), q_n(z)),$$

its symmetrization $T_n^s \in \mathcal{L}^s(^3X)$,

$$T_n^s(x,y,z) = \frac{1}{3}\left[T_n(x,y,z) + T_n(y,x,z) + T_n(z,x,y)\right],$$

and the linear operator $\widetilde{T}_n^s\colon X \longrightarrow \mathcal{L}(^2X)$ associated to T_n^s,

$$\begin{aligned}\widetilde{T}_n^s(x)(y,z) &= \frac{1}{3}\left[T_n(x,y,z) + T_n(y,x,z) + T_n(z,x,y)\right]\\ &= \frac{1}{3}\left[u_n(x)(q_n(y), q_n(z)) + u_n(y)(q_n(x), q_n(z)) + u_n(z)(q_n(x), q_n(y))\right].\end{aligned}$$

Write $3\widetilde{T}_n^s = A_n + B_n + C_n$, where

$$\begin{aligned}&A_n(x)(y,z) = u_n(x)(q_n(y), q_n(z)),\\ &B_n(x)(y,z) = u_n(y)(q_n(x), q_n(z)),\ \text{and}\\ &C_n(x)(y,z) = u_n(z)(q_n(x), q_n(y)).\end{aligned}$$

Given $x_1, \ldots, x_k \in X$,

$$\begin{aligned}\sum_{j=1}^k \|B_n(x_j)\|^2 &= \sum_{j=1}^k \sup_{y,z\in B_X} |u_n(y)(q_n(x_j), q_n(z))|^2\\ &\le \sum_{j=1}^k \|u_n\|^2 \|q_n(x_j)\|^2 \|q_n\|^2\\ &\le \sup_n \|u_n\|^2 \sup_n \|q_n\|^2 \|q_n\|^2_{as(2,2)} \|(x_j)_{j=1}^k\|^2_{w,2},\end{aligned}$$

proving that B_n is 2-summing (hence r-summing because $r \ge 2$) and its 2-summing norm (hence its r-summing norm) is controlled by $16\sqrt{C}\sup_n \|u_n\|$. It is clear that the same happens to C_n, so

$$\sup_n \|B_n\|_{as(r,r)} < \infty \text{ and } \sup_n \|C_n\|_{as(r,r)} < \infty.$$

Let us prove that $\sup_n \|A_n\|_{as(r,r)} = \infty$. Given $M' > 0$, from $\sup_n \|u_n\|_{as(r,r)} = \infty$ there is an n such that $\|u_n\|_{as(r,r)} > M'$. So, there are $m \in \mathbb{N}$ and $y_1, \ldots, y_m \in X$ such that

$$\left(\sum_{j=1}^m \|u_n(y_j)\|^r\right)^{\frac{1}{r}} > M' \|(y_j)_{j=1}^m\|_{w,r}.$$

Using that $B_{\ell_2^n} \subseteq q_n(B_X)$, we have

$$\begin{aligned}\|u_n(x)\| &= \sup_{\lambda_1,\lambda_2\in B_{\ell_2^n}} |u_n(x)(\lambda_1,\lambda_2)|\\ &\le \sup_{y,z\in B_X} |u_n(x)(q_n(y), q_n(z))|\\ &= \sup_{y,z\in B_X} |A_n(x)(y,z)| = \|A_n(x)\|,\end{aligned}$$

for every $x \in X$. So,

$$\left(\sum_{j=1}^{m} \|A_n(y_j)\|^r\right)^{\frac{1}{r}} \geq \left(\sum_{j=1}^{m} \|u_n(y_j)\|^r\right)^{\frac{1}{r}} > M' \|(y_j)_{j=1}^m\|_{w,r},$$

proving that $\|A_n\|_{as(r,r)} > M'$; hence $\sup_n \|A_n\|_{as(r,r)} = \infty$. From $3\widetilde{T}_n^s = A_n + B_n + C_n$, it follows that $\sup_n \|\widetilde{T}_n^s\|_{as(r,r)} = \infty$. Hence $\sup_n \|T_n^s\|_{d,r} = \infty$ because $\|\widetilde{T}_n^s\|_{as(r,r)} \leq \|T_n^s\|_{d,r}$ (see, e.g., [5, Lemma 3.4]). But $\sup_n \|T_n^s\| < \infty$ because $\sup_n \|u_n\| < \infty$ and $\sup_n \|q_n\| < \infty$, so the open mapping theorem yields that $\mathcal{L}^s(^3X) \neq \mathcal{L}^s_{d,r}(^3X)$. Therefore $\mathcal{P}(^3X) \neq \mathcal{P}_{d,r}(^3X)$. □

To complete the proof of Conjecture 2 we need another well-known result (for complex scalars the proof appeared in [25, Corollary 3.2]; the general case can be found in [17, Proposition 13]):

Lemma 2.3. $\mathcal{P}_{d,r}(^2X) = \mathcal{P}_{d,2}(^2X)$ *for every Banach space X and $r \geq 2$.*

Theorem 2.4. *Let $m \geq 3$, $r \geq 1$ and X be an infinite dimensional Banach space. Then*

$$\mathcal{P}_{d,r}(^mX) \neq \mathcal{P}(^mX). \tag{2}$$

Moreover, if $\mathcal{P}_{d,2}(^2X) \neq \mathcal{P}(^2X)$, then (2) holds for every $m \geq 2$ and $r \geq 1$.

Proof. If $\mathcal{P}_{d,2}(^2X) = \mathcal{P}(^2X)$, Theorem 2.2 gives the result. If $\mathcal{P}_{d,2}(^2X) \neq \mathcal{P}(^2X)$, Lemma 2.3 gives that $\mathcal{P}_{d,r}(^2X) \neq \mathcal{P}(^2X)$ for every $r \geq 2$. Hence $\mathcal{P}_{d,r}(^2X) \neq \mathcal{P}(^2X)$ for every $r \geq 1$ by Lemma 2.1(b). Lemma 2.1(a) completes the result for every $m \geq 2$. □

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