

**CORRIGENDUM TO “EXISTENCE OF SOLUTIONS  
 FOR SEMILINEAR ELLIPTIC PROBLEMS  
 WITHOUT (PS) CONDITION”**

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ABSTRACT. In this corrigendum we give a correct proof for equation (3.17), as well as the assertion on line 15, page 1364, of “Existence of solutions for semilinear elliptic problems without (PS) condition”, *Proc. Amer. Math. Soc.* 132(5) (2004), 1355-1366.

In the paper [1], the use of Hospital’s rule in (3.17), page 1363, line 15, and page 1364, line 15, is incorrect. In order to avoid the use of Hospital’s rule, let  $\sigma = -\lim_{t \rightarrow \infty} \frac{t\bar{h}'(t)}{\bar{h}(t)}$ , we need to assume further that

$$(0.1) \quad \lim_{t \rightarrow \infty} \frac{th'_t(x, t) - \sigma h(x, t)}{\bar{h}(t)} = 0$$

uniformly with respect to  $x$  and that for any  $a > 0$  there exist  $T_a > 0$  and  $C_a > 0$  such that for every  $t \geq T_a$ ,

$$(0.2) \quad \bar{h}(at) \leq C_a \bar{h}(t).$$

An example which satisfies (0.1), (0.2) and  $(f_1) - (f_4)$  in [1] is given for  $t \geq 0$  by

$$(0.3) \quad f(t) = (\ln(1+t))^\alpha (\gamma + \sin((\ln \ln t)^\beta)) t^p + g(x, t),$$

with the lower term  $g(x, t)$  as the example in [2], where  $\alpha < 1$  and  $\beta > 0$ . Observe that in this example, we have  $h(x, t) = (\ln(1+t))^\alpha (\gamma + \sin((\ln \ln t)^\beta))$ ,  $\bar{h}(t) = (\gamma + 1)(\ln(1+t))^\alpha$  and  $\sigma = 0$ . Moreover, such a function  $f$  does not satisfy the Ambrosetti-Rabinowitz condition.

Now, we use (0.1) and (0.2) to show that

$$(0.4) \quad \lim_{n \rightarrow \infty} \lambda_n M_n \frac{f'_t(x_n + \phi^{-\frac{1}{2}}(M_n)x, M_n \tilde{u}_n)}{\alpha(M_n)} = (p - \sigma)Q(x)u^{p-1},$$

where Hospital’s rule was used. We have as on page 1363 of [1] that

$$\left| \frac{1}{2} \phi^{-\frac{3}{2}}(M_n) \phi'(M_n) \alpha'^{-1}(M_n) \nabla_x f(x_n + \phi^{-\frac{1}{2}}(M_n)x, M_n \tilde{u}_n) \right| \rightarrow 0$$

and  $\frac{\alpha(M_n)}{M_n \alpha'(M_n)} \rightarrow \frac{1}{p-\sigma}$  as  $n \rightarrow \infty$ . Hence, to prove (0.4), it suffices to prove that

$$(0.5) \quad \lim_{n \rightarrow \infty} T_n(x) = 0$$

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for all  $x$  such that  $u(x) > 0$ , where

$$T_n(x) = \frac{(p - \sigma)f(x_n + \phi^{-\frac{1}{2}}(M_n)x, M_n\tilde{u}_n) - M_n\tilde{u}_nf'_t(x_n + \phi^{-\frac{1}{2}}(M_n)x, M_n\tilde{u}_n)}{\alpha(M_n)}.$$

Here we deal only with the case

$$(0.6) \quad \lim_{t \rightarrow \infty} \bar{h}(t)t^{p-q} = \infty.$$

By (0.1), for  $\varepsilon > 0$  there exists  $t_\varepsilon > 0$  such that for  $t \geq t_\varepsilon$  and all  $x \in \Omega$ ,

$$(0.7) \quad \left| \frac{th'_t(x, t) - \sigma h(x, t)}{\bar{h}(t)} \right| < \varepsilon,$$

which implies

$$\begin{aligned} |T_n(x)| &\leq \frac{|\sigma h(x_n + \phi^{-\frac{1}{2}}(M_n)x, M_n\tilde{u}_n) - M_n\tilde{u}_nh'_t(x_n + \phi^{-\frac{1}{2}}(M_n)x, M_n\tilde{u}_n)|}{\bar{h}(M_n)} \\ &\quad + \frac{M_n^q}{M_n^p \bar{h}(M_n)}. \end{aligned}$$

Since  $M_n\tilde{u}_n(x) \rightarrow \infty$  as  $n \rightarrow \infty$ , by (0.6) and (0.7) we may find  $N_\varepsilon > 0$  such that for  $n \geq N_\varepsilon$ ,  $|T_n(x)| < \varepsilon$ . Equation (0.5) follows.

Next, we prove the formula on page 1364, line 15, that is,

$$(0.8) \quad (M_n\alpha(M_n))^{-1}\lambda_n F(x_n + \phi^{-\frac{1}{2}}(M_n)x, M_n\tilde{u}_n) \rightarrow \frac{1}{p+1-\sigma}Q(x)|u(x)|^{p+1}.$$

By  $(f_3)$ , there exists  $\varepsilon > 0$  such that  $\sigma + \varepsilon < p$  and  $-t\bar{h}'(t) \leq (\sigma + \varepsilon)\bar{h}(t)$ . Integrating by part, we obtain

$$\int_0^t \bar{h}(s)s^p ds \leq C\bar{h}(t)t^{p+1}$$

for  $t$  large enough. This inequality, together with (0.1), yields

$$(0.9) \quad \lim_{t \rightarrow +\infty} \frac{tf(x, t) - (p - \sigma + 1)F(x, t)}{t^{p+1}\bar{h}(t)} = 0$$

uniformly with respect to  $x$ . Therefore, we may deduce as above that

$$\begin{aligned} &\lim_{n \rightarrow \infty} \frac{(p - \sigma + 1)F(x_n + \phi^{-\frac{1}{2}}(M_n)x, M_n\tilde{u}_n) - M_n\tilde{u}_nf(x_n + \phi^{-\frac{1}{2}}(M_n)x, M_n\tilde{u}_n)}{M_n^{p+1}\bar{h}(M_n\tilde{u}_n)} \\ &= 0, \end{aligned}$$

which implies (0.8).

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## REFERENCES

- [1] Jianfu Yang, Existence of solutions for semilinear elliptic problems without  $(PS)$  condition, *Proc. Amer. Math. Soc.* 132(5) (2004), 1355-1366. MR2053340 (2005b:35048)
- [2] D.G. de Figueiredo and Jianfu Yang, On a semilinear elliptic problem without  $(PS)$  condition, *J. Differential Equations* 187 (2003), 412-428. MR1949448 (2004c:35074)

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