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ZEROS OF VARYING LAGUERRE–KRALL ORTHOGONAL POLYNOMIALS

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Abstract. In this paper we introduce a sequence of varying orthogonal polynomials related to a Laguerre weight where this absolutely continuous measure is perturbed by a sequence of nonnegative masses located at the origin. The main objective is to obtain asymptotic relations between the zeros of these polynomials and the zeros of the Bessel functions of the first kind (or linear combinations of them). This is done through Mehler–Heine type formulas. With these relations we can easily compute asymptotically the zeros of these polynomials. We show some numerical experiments.

1. Introduction

We call Krall polynomials those polynomials which are orthogonal with respect to a measure μ that we can describe as $\mu = \mu_{ac} + \mu_d$, where μ_{ac} is the absolutely continuous part and $\mu_d = \sum_{i=1}^{m} M_i \delta_{\xi_i}$ is the discrete part with $M_i > 0$ and δ_a the Dirac delta function at the point a. In other words, we have an absolutely continuous measure perturbed by a finite sum of Dirac delta functions. A first approach to these orthogonal polynomials is due to H.L. Krall in [9], where he tackles the problem of extending Bochner's characterization of classical orthogonal polynomials, and A.M. Krall, who in [8] obtains a few new families of nonclassical orthogonal polynomials. However, since the last quarter of the 20th century, more general situations have been considered and the literature on this topic has increased considerably. Nevai's text [10] is a nice reference when the measure μ has compact support. On the other hand, the approach considered in Gautschi's text [4] can be applied to obtain algebraic properties of these polynomials and to compute them (see also [5]).

We obtain a special case of the Krall polynomials considering the absolutely continuous part as the classical Laguerre weight function on $[0, \infty)$, i.e., $\mu_{ac} = \frac{1}{\Gamma(\alpha+1)} x^\alpha e^{-x}$, $\alpha > -1$, and the discrete part formed by a single mass located at the origin. Thus, the corresponding inner product can be written as

$$(p, q) = \frac{1}{\Gamma(\alpha+1)} \int_0^\infty p(x)q(x)x^\alpha e^{-x} dx + Mp(0)q(0), \quad M > 0, \quad \alpha > -1. \tag{1.1}$$

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The orthogonal polynomials with respect to the inner product (1.1) are the so-called Laguerre–Krall polynomials, although sometimes they are also called Laguerre–Koekoek polynomials because they were studied extensively by R. Koekoek in his doctoral dissertation [6].

The Mehler–Heine type asymptotic formula for these polynomials was obtained in [3] together with other asymptotic results which have been extended very recently to Sobolev type orthogonal polynomials [2]. Denoting $L_n^{(\alpha,M)}(x) = (-1)^n/n!\,x^n + \ldots$, it is proved in [3] that

$$\lim_{n\to\infty} \frac{L_n^{(\alpha,M)}(x/n)}{n^\alpha} = -x^{-\alpha/2} J_{\alpha+2}(2\sqrt{x}),$$

uniformly on compact subsets of $\mathbb{C}$, where J_α is the Bessel function of the first kind given by

$$J_\alpha(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,\Gamma(n+\alpha+1)} \left(\frac{x}{2}\right)^{2n+\alpha}.$$

This local asymptotic behaviour around the origin allows us to describe the asymptotics of the smallest zeros of the polynomials $L_n^{(\alpha,M)}(x)$. In fact, denoting by $s_{n,i}$ the zeros of $L_n^{(\alpha,M)}(x)$ ordered as $0 < s_{n,1} < s_{n,2} < \ldots < s_{n,n}$ and by $j_{\alpha,i}$ the positive zeros of J_α ordered as $0 < j_{\alpha,1} < j_{\alpha,2} < \ldots$, and applying Hurwitz's Theorem in a straightforward way, we can deduce that (see [3])

$$\lim_{n\to\infty} n s_{n,1} = 0, \quad \lim_{n\to\infty} n s_{n,i} = \frac{j^2_{\alpha+2,i-1}}{4}, \quad i \geq 2.$$

We observe that there is an important difference between the asymptotic behaviour of the zeros of the Laguerre–Krall polynomials and of the zeros of the Laguerre polynomials. Indeed, if we denote by $x_{n,i}$ the zeros of the classical Laguerre polynomials $L_n^{(\alpha)}(x)$ ordered as $0 < x_{n,1} < x_{n,2} < \ldots < x_{n,n}$, it is well known that as a direct consequence of Theorem 8.1.3 in [11, p. 193], we have

$$\lim_{n\to\infty} n x_{n,i} = \frac{j^2_{\alpha,i}}{4}, \; i \geq 1. \tag{1.2}$$

Therefore, adding a mass at the point 0 produces an acceleration in the convergence to 0 of the first zero of Laguerre–Krall polynomials, and the other scaled zeros converge to the positive zeros of a Bessel function whose order has increased by two units.

One may ask what happens if one considers a sequence of masses $\{M_n\}_n$ at the origin instead of a fixed mass M as in (1.1). Will the asymptotic behaviour of the zeros change? If it will, why?

The main goal of this paper is to give an answer to the previous questions and to be a first approach to the local asymptotic behaviour around the origin of the orthogonal polynomials with respect to inner products involving sequences of varying masses located at points of the real line. Moreover, we are interested in computing numerically the zeros of these varying orthogonal polynomials.

Therefore, the paper is structured as follows. In Section 2, we introduce the results about varying Laguerre–Krall polynomials that we use in Section 3 to prove our main result about the local asymptotics of these polynomials around the origin and its consequences on the corresponding zeros. We deduce similar results for the varying generalized Hermite polynomials as an immediate consequence of the results for the Laguerre case. Finally, we compute the zeros of the varying Laguerre–Krall orthogonal polynomials and illustrate the results obtained previously.

2. Varying Laguerre–Krall orthogonal polynomials

Let $\{M_n\}_n$ be a sequence of nonnegative numbers such that

$$\lim_{n\to\infty} M_n n^{\beta} = M > 0, \quad \beta \in \mathbb{R}. \tag{2.1}$$

Then, we consider the varying inner products

$$(p,q)_n = \frac{1}{\Gamma(\alpha+1)} \int_0^{\infty} p(x)q(x)x^{\alpha}e^{-x}dx + M_n p(0)q(0), \quad \alpha > -1. \tag{2.2}$$

We denote by $\{L_n^{(\alpha,M_n)}\}_n$ the sequence of orthogonal polynomials with respect to the inner products (2.2), and we call them varying Laguerre–Krall orthogonal polynomials. We normalize these polynomials by taking as leading coefficient $(-1)^n/n!$. More precisely the polynomial $L_n^{(\alpha,M_n)}$ of degree n satisfies the orthogonality relations $(L_n^{(\alpha,M_n)}, x^j)_n = 0$, for $j = 0, \ldots, n-1$, but $(L_n^{(\alpha,M_n)}, x^j)_{n-1}$ may be different from zero.

Notice that for each M_n, $n \in \mathbb{N}$, we get a sequence of monic orthogonal polynomials with respect to the inner product (2.2). Thus, a square scheme appears, that is, $\{L_k^{(\alpha,M_n)}\}_k$. We focus our attention on the diagonal entries of such a scheme, i.e. on the sequence $\{L_0^{(\alpha,M_0)}, L_1^{(\alpha,M_1)}, L_2^{(\alpha,M_2)}, \ldots\}$ denoted as $\{L_n^{(\alpha,M_n)}\}_n$. Thus, the zeros of $L_n^{(\alpha,M_n)}$ are real, simple, and located in the open interval $(0,+\infty)$.

In [7, Remark 10] the authors give a relation between the Laguerre–Krall orthogonal polynomials and the classical Laguerre ones for the case $M_n = M$, for all n. In a very straightforward way we can extend that result for any sequence $\{M_n\}_n$ satisfying (2.1).

Proposition 2.1. *Let $L_n^{(\alpha,M_n)}(x) = \frac{(-1)^n}{n!}x^n + \ldots$ be the orthogonal polynomials with respect to the inner product* (2.2). *Then,*

$$L_n^{(\alpha,M_n)}(x) = a_n\, L_n^{(\alpha)}(x) + b_n\, x\, L_{n-1}^{(\alpha+2)}(x)\,, \quad n \geq 1, \tag{2.3}$$

where

$$a_n = \frac{1}{1 + M_n \frac{\Gamma(n+\alpha+1)}{\Gamma(n)\Gamma(\alpha+2)}}, \quad b_n = \frac{-M_n \frac{\Gamma(n+\alpha+1)}{\Gamma(n+1)\Gamma(\alpha+2)}}{1 + M_n \frac{\Gamma(n+\alpha+1)}{\Gamma(n)\Gamma(\alpha+2)}}. \tag{2.4}$$

If one does not want to repeat for the varying case the proof of this proposition given in [7], then it is easy to check that the polynomial on the right-hand side of (2.3) is orthogonal to all lower-degree polynomials with respect to the inner product (2.2).

We can obtain the asymptotic behaviour of the coefficients a_n and b_n when $n \to \infty$.

Lemma 2.2. *We have*

$$\lim_{n\to\infty} a_n = \begin{cases} 0, & \text{if } \beta < \alpha+1, \\ \dfrac{\Gamma(\alpha+2)}{\Gamma(\alpha+2)+M}, & \text{if } \beta = \alpha+1, \\ 1, & \text{if } \beta > \alpha+1; \end{cases}$$

$$\lim_{n\to\infty} nb_n = \begin{cases} -1, & \text{if} \quad \beta < \alpha+1, \\ \dfrac{-M}{\Gamma(\alpha+2)+M}, & \text{if} \quad \beta = \alpha+1, \\ 0, & \text{if} \quad \beta > \alpha+1. \end{cases}$$

Proof. It is enough to use in (2.4) the relation (see, e.g., [1])

$$\lim_{n\to\infty} \frac{n^{b-a}\Gamma(n+a)}{\Gamma(n+b)} = 1$$

and the assumption (2.1). □

3. Local asymptotic formula and its consequences on the zeros

We obtain the local asymptotics around the origin through the Mehler–Heine type formula. Thus, we get for varying Laguerre–Krall orthogonal polynomials the following Mehler–Heine type formula.

Theorem 3.1. *Let* $g_i(x) = x^{-\alpha/2} J_{\alpha+2i}(2\sqrt{x})$. *Then, for* $\alpha > -1$, *we have*

$$\lim_{n\to\infty} \frac{L_n^{(\alpha,M_n)}(x/n)}{n^\alpha} = \begin{cases} -g_1(x), & \text{if} \quad \beta < \alpha+1, \\ \dfrac{\Gamma(\alpha+2)\, g_0(x) - M\, g_1(x)}{\Gamma(\alpha+2)+M}, & \text{if} \quad \beta = \alpha+1, \\ g_0(x), & \text{if} \quad \beta > \alpha+1, \end{cases}$$

uniformly on compact subsets of $\mathbb{C}$.

Proof. We use the well-known Mehler–Heine formula for the classical Laguerre polynomials (see [11, p. 193]). Indeed, this result states, for $j \in \mathbb{Z}$ fixed,

$$\lim_{n\to\infty} \frac{L_n^{(\alpha)}(x/(n+j))}{n^\alpha} = x^{-\alpha/2} J_\alpha(2\sqrt{x}), \tag{3.1}$$

uniformly on compact subsets of $\mathbb{C}$. Then, scaling the variable x in (2.3) we obtain

$$\frac{L_n^{(\alpha,M_n)}(x/n)}{n^\alpha} = a_n \frac{L_n^{(\alpha)}(x/n)}{n^\alpha} + nb_n \left(\frac{n-1}{n}\right)^{\alpha+2} \frac{x L_{n-1}^{(\alpha+2)}(x/n)}{(n-1)^{\alpha+2}}.$$

Now, applying Lemma 2.2 and (3.1) we get the result. □

Remark 3.2. It is important to highlight the differences with respect to the particular case $M_n = M$, for all n, considered in the previous literature. As we have commented in the introduction, in this *constant case* we have

$$\lim_{n\to\infty} \frac{L_n^{(\alpha,M)}(x/n)}{n^\alpha} = -x^{-\alpha/2} J_{\alpha+2}(2\sqrt{x}) = -g_1(x).$$

Now, we have three possible cases which are related to the *size* of the sequence of masses $\{M_n\}_n$.

- When the size of the sequences of masses $\{M_n\}_n$ is small enough ($\beta > \alpha+1$), then the Mehler–Heine asymptotic formula for varying Laguerre–Krall orthogonal polynomials is the same as the one for Laguerre orthogonal polynomials. Thus, the discrete part of the measure does not influence the asymptotics.

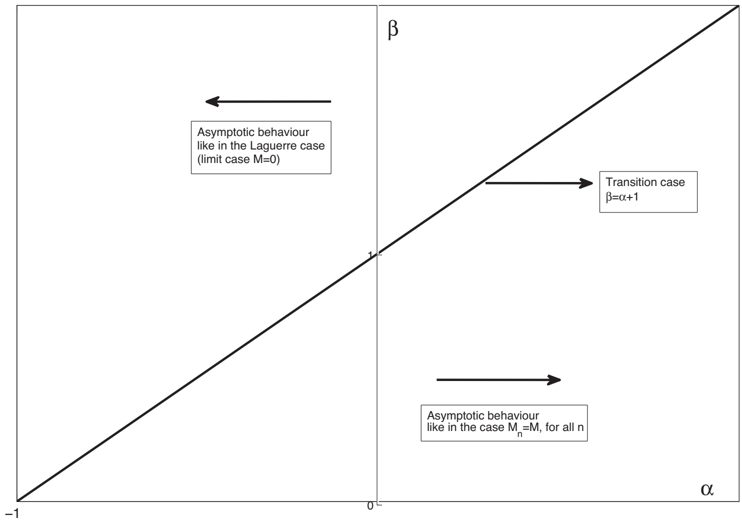

FIGURE 1. Different types of Mehler–Heine type asymptotics for varying Laguerre–Krall orthogonal polynomials.

- When the size of the sequences of masses $\{M_n\}_n$ is large enough ($\beta < \alpha + 1$), then the Mehler–Heine asymptotic formula for varying Laguerre–Krall orthogonal polynomials is the same as the one obtained for the *constant case*. For example, if $\alpha = -0.5$, the Mehler–Heine type formula is the same for the sequences with *n–th* term $M_n = \frac{1}{\sqrt[4]{n+1}}$, or $M_n = 5$, or $M_n = 3n^{1000}$.
- The third case corresponds to $\beta = \alpha + 1$. We call it the *transition case* because we have a smooth change between the two previous cases. If we take appropriate limits in the expression $\frac{\Gamma(\alpha+2)\, g_0(x) - M\, g_1(x)}{\Gamma(\alpha+2)+M}$, we obtain $g_0(x)$ when $M \to 0$, and $-g_1(x)$ when $M \to +\infty$, that is, the results obtained in the other two cases. We illustrate this remark with Figure 1.

In conclusion, the asymptotic behaviour of the sequence of masses $\{M_n\}_n$ has its influence on this type of asymptotics. On the other hand, we claim that Mehler–Heine asymptotics is the type of asymptotics that deserves to be studied because it describes in a precise way the behaviour of the polynomials around the point where we have introduced the perturbation.

Now we can apply the Hurwitz Theorem (see, e.g., [11]) and get the following corollary.

Corollary 3.3. *Let $s_{n,1} < s_{n,2} < \cdots < s_{n,n-1} < s_{n,n}$ be the zeros of the Laguerre–Krall orthogonal polynomials $L_n^{(\alpha,M_n)}$ and let $j_{\alpha,i}$ denote the i–th positive zero of the Bessel function of the first kind J_α. Then*

(i) *If $\beta < \alpha + 1$, then*

$$\lim_{n\to\infty} n s_{n,1} = 0 \quad \text{and} \quad \lim_{n\to\infty} n s_{n,i} = \frac{j^2_{\alpha+2,i-1}}{4}, \quad i \geq 2.$$

(ii) *If $\beta = \alpha + 1$, then*

$$\lim_{n\to\infty} n s_{n,i} = h_{\alpha,i}, \quad i \geq 1,$$

where $h_{\alpha,i}$ denotes the i–th positive zero of the function

$$h_\alpha(x) := \Gamma(\alpha+2)g_0(x) - Mg_1(x).$$

(iii) *If $\beta > \alpha + 1$, then*

$$\lim_{n\to\infty} ns_{n,i} = \frac{j^2_{\alpha,i}}{4}, \quad i \geq 1.$$

Remark 3.4. We can observe that the presence of a sequence of masses $\{M_n\}_n$ of adequate *size* ($\beta \leq \alpha+1$) located at the origin produces a change in the asymptotic behaviour of the zeros compared to the Laguerre case (see (1.2)). Moreover, if $\beta < \alpha + 1$, the convergence of the first zero $s_{n,1}$ to 0 is accelerated.

These results can be extended to another family of orthogonal polynomials, namely to the one in which the inner product involves a generalized Hermite weight function, i.e.

$$(p,q)_n = \frac{1}{\Gamma(\mu+1/2)} \int_{-\infty}^{+\infty} p(x)q(x)|x|^{2\mu} e^{-x^2} dx + M_n p(0)q(0), \quad \mu > -1/2, \tag{3.2}$$

the sequence $\{M_n\}_n$ again satisfying (2.1).

It is clear that the orthogonal polynomials $H_n^{(\mu,M_n)}$ with respect to the inner product (3.2) are symmetric, and using a symmetrization process in the same way as the one used to establish the well-known relation between classical Laguerre and Hermite polynomials (see, for example, [11, p. 106]), we have

$$H_{2n+1}^{(\mu,M_n)}(x) = (-1)^n 2^{2n+1} n! x L_n^{(\mu+1/2)}(x^2),$$
$$H_{2n}^{(\mu,M_n)}(x) = (-1)^n 2^{2n} n! L_n^{(\mu-1/2,M_n)}(x^2).$$

Thus, for the polynomials of odd degree there is nothing new with respect to generalized Hermite polynomials. However, for the polynomials with even degree we can deduce from Theorem 3.1 the following result.

Corollary 3.5. *Let $\widetilde{g}_i(x) = \left(\frac{x}{2}\right)^{-\mu+1/2} J_{\mu-1/2+2i}(x)$. Then, for $\mu > -1/2$,*

$$\lim_{n\to\infty} \frac{(-1)^n H_{2n}^{(\mu,M_n)}(x/(2\sqrt{n}))}{4^n n! n^{\mu-1/2}} = \begin{cases} -\widetilde{g}_1(x), & \text{if } \beta < \mu+1/2, \\ \dfrac{\Gamma(\mu+3/2)\,\widetilde{g}_0(x) - M\,\widetilde{g}_1(x)}{\Gamma(\mu+3/2)+M}, & \text{if } \beta = \mu+1/2, \\ \widetilde{g}_0(x), & \text{if } \beta > \mu+1/2, \end{cases}$$

uniformly on compact subsets of $\mathbb{C}$.

Note that $\widetilde{g}_i(x) = g_i(x^2/4)$ with $\alpha = \mu - 1/2$. From this result, it is possible to deduce easily the asymptotic behaviour of the zeros of $H_{2n}^{(\mu,M_n)}$ as we did in Corollary 3.3 for varying Laguerre–Krall orthogonal polynomials.

4. Numerical experiments

We illustrate the results obtained in the previous section for varying Laguerre–Krall orthogonal polynomials with some numerical examples. In the following tables, we show the asymptotic behaviour of the first four scaled zeros according to Corollary 3.3. In these examples we have taken $\alpha = 3$ and $M_n = 1.5/n^\beta$; thus $M = 1.5$. We choose β such that all the cases in Corollary 3.3 are covered. All the computations have been made using the commands provided by the software *Mathematica*^{TM} (version 7.0). We have generated the varying Laguerre–Krall orthogonal polynomials through relation (2.3) taking into account the explicit values of the coefficients in (2.3) given in (2.4).

TABLE 1. Case $\beta < \alpha + 1$. $\beta = 3$.

	$ns_{n,1}$	$ns_{n,2}$	$ns_{n,3}$	$ns_{n,4}$
$n = 50$	3.57082042	19.40801926	37.51351782	60.23628195
$n = 100$	2.31491605	19.39345757	37.84701150	60.96428898
$n = 150$	1.70451233	19.35996601	37.93568192	61.19599999
$n = 300$	0.94894652	19.30805230	38.00813567	61.41728772
Limit	**0**	$\frac{\mathbf{j}_{5,1}^2}{4} = 19.23473208$	$\frac{\mathbf{j}_{5,2}^2}{4} = 38.06028839$	$\frac{\mathbf{j}_{5,3}^2}{4} = 61.62386653$

TABLE 2. Case $\beta = \alpha + 1$. $\beta = 4$.

	$ns_{n,1}$	$ns_{n,2}$	$ns_{n,3}$	$ns_{n,4}$
$n = 50$	9.51286576	22.65503421	40.51315286	63.14210328
$n = 100$	9.71684451	23.10384109	41.28254627	64.29579872
$n = 150$	9.78683986	23.25927241	41.55180422	64.70495730
$n = 300$	9.85788691	23.41778486	41.82786954	65.12726400
Limit	$\mathbf{h}_{3,1} = 9.93001784$	$\mathbf{h}_{3,2} = 23.57948589$	$\mathbf{h}_{3,3} = 42.11101059$	$\mathbf{h}_{3,4} = 65.56328391$

TABLE 3. Case $\beta > \alpha + 1$. $\beta = 5$.

	$ns_{n,1}$	$ns_{n,2}$	$ns_{n,3}$	$ns_{n,4}$
$n = 50$	9.78386639	22.91731617	40.77278333	63.40054799
$n = 100$	9.97555573	23.35490050	41.53126668	64.54347135
$n = 150$	10.04149319	23.50660552	41.79689600	64.94904449
$n = 300$	10.10850253	23.66140031	42.06933817	65.36776991
Limit	$\frac{\mathbf{j}_{3,1}^2}{4} = 10.17661645$	$\frac{\mathbf{j}_{3,2}^2}{4} = 23.81939314$	$\frac{\mathbf{j}_{3,3}^2}{4} = 42.34886246$	$\frac{\mathbf{j}_{3,4}^2}{4} = 65.80021356$

Taking into account the same values considered in the above numerical experiments, except for Figure 3, where we put $M = 500$, we now illustrate Theorem 3.1 with four plots (Figures 2–5). In each one, we plot the scaled polynomials of degree 10 and 50 as well as the corresponding limit function according to Theorem 3.1. In all the figures, the dashed line, the thin continuous line, and the thick continuous line correspond respectively to the scaled varying Laguerre–Krall orthogonal polynomial of degree 10, of degree 50, and the limit function in Theorem 3.1. Figures 3 and 4 show how the transition case $\beta = \alpha + 1$ is a *continuous* transition between the cases $\beta < \alpha + 1$ and $\beta > \alpha + 1$ as we mentioned in Remark 3.2.

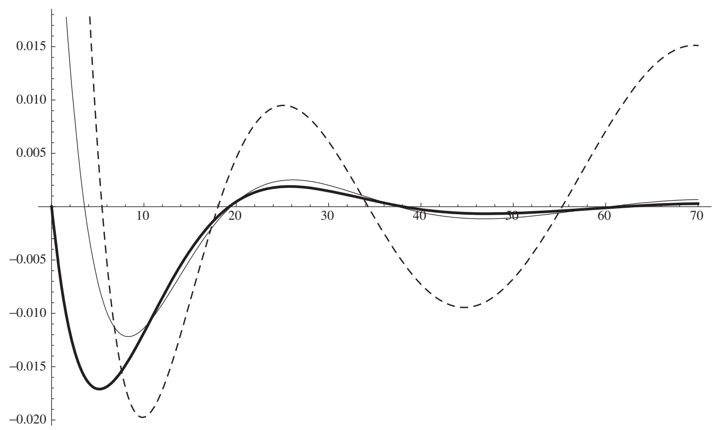

FIGURE 2. Case $\beta < \alpha + 1$.

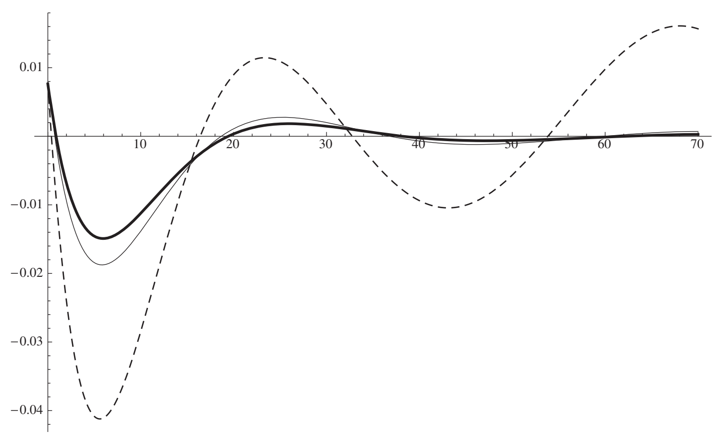

FIGURE 3. Case $\beta = \alpha + 1$. $M = 500$.

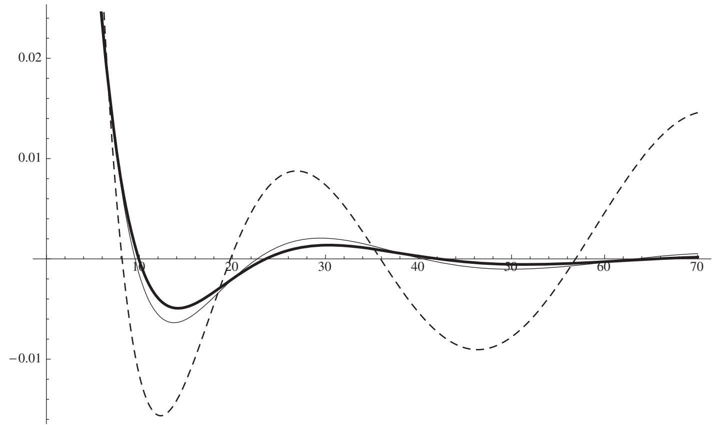

FIGURE 4. Case $\beta = \alpha + 1$. $M = 1.5$.

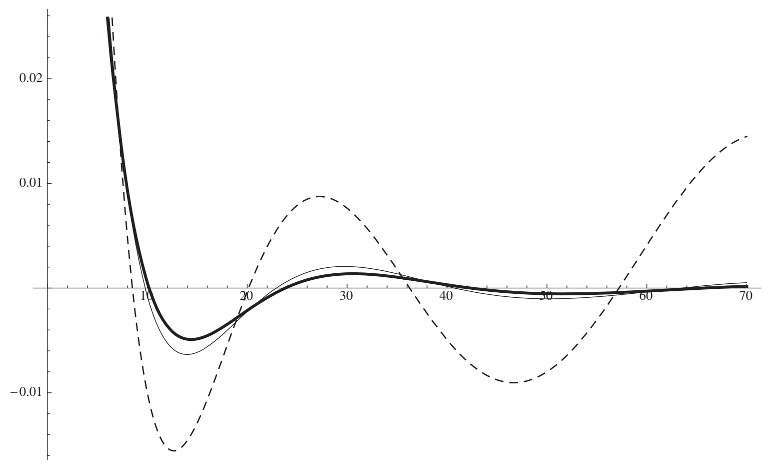

FIGURE 5. Case $\beta > \alpha + 1$.

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