

SHARP TRACE REGULARITY FOR AN ANISOTROPIC ELASTICITY SYSTEM

IGOR KUKAVICA, ANNA L. MAZZUCATO, AND AMJAD TUFFAHA

(Communicated by James E. Colliander)

ABSTRACT. We establish a sharp regularity result for the normal trace of the solution to the anisotropic linear elasticity system with Dirichlet boundary condition on a Lipschitz domain. Using this result we obtain a new existence result for a fluid-structure interaction model in the case when the structure is an anisotropic elastic body.

1. INTRODUCTION

In this paper, we establish an optimal trace regularity theorem, also known as the hidden regularity theorem [23], for the anisotropic linear elasticity equation on a bounded domain Ω with Lipschitz boundary. In its simplest form it provides a space-time L^2 estimate for the trace of the normal derivative for the solution. Over the years, such sharp trace regularity theorems have proven to be crucial for obtaining well-posedness, controllability and boundary stabilization results for a variety of hyperbolic and partially hyperbolic systems, [8, 25–28].

For the wave equation, the analog of the result proved in this work for the anisotropic elasticity equation (see [23] and [9] for the case of Lipschitz domains) was an essential ingredient in deriving a well-posedness theorem for a fluid-structure interaction system given initial data in low regularity Sobolev spaces [18–21]. In [18, 19], it was proven that the fluid-structure problem possesses a local solution when the initial datum (u_0, w_0, w_1) belongs to $H^1 \times H^{3/2+r} \times H^{1/2+r}$ where $r > 0$ (here u_0 is the initial fluid velocity, and (w_0, w_1) are the Cauchy data for the wave equation). In these works, the wave equation $w_{tt} - \Delta w = 0$ is used as a scalar approximation to the isotropic elasticity system in the acoustic approximation, while the fluid is modeled by the Navier-Stokes equations. The extension of the above-mentioned results to the isotropic elasticity system

$$w_{tt} - \operatorname{div}(\mu(\nabla w + \nabla w^T) + \lambda \operatorname{div} w I) = 0,$$

Received by the editors October 21, 2011.

2010 *Mathematics Subject Classification*. Primary 35B65, 35Q74.

Key words and phrases. Anisotropic elasticity, Legendre-Hadamard condition, hidden regularity, Lipschitz domain, fluid-structure interaction.

The first author is the corresponding author and was supported in part by the U.S. National Science Foundation grant DMS-1009769.

The second author was supported in part by the U.S. National Science Foundation grants DMS-1009713 and DMS-1009714.

The third author was supported in part by the Petroleum Institute Research Grant Ref. Number 11014.

where $\mu(x), \lambda(x) > 0$ are the Lamé coefficients, is straightforward and follows the method in [19]. It is a more difficult problem whether the same can be accomplished for the (inhomogeneous) general anisotropic elasticity system

$$w_{tt} - \operatorname{div}(c\nabla w) = 0,$$

where $c = (c_{ijkl}(x))$, for $i, j, k, l = 1, \dots, 3$, is the elasticity tensor, a rank-four tensor field assumed to satisfy the standard symmetry assumption $c_{ijkl} = c_{jikl} = c_{klij}$ (the so-called case of hyperelastic materials), and the Legendre-Hadamard condition

$$(1.1) \quad c_{ijkl}(x)\eta_i\xi_j\eta_k\xi_l \geq \delta_0|\eta|^2|\xi|^2, \quad \eta, \xi \in \mathbb{R}^n, \quad x \in \bar{\Omega}$$

with $\delta_0 > 0$. We note that the Legendre-Hadamard condition is also called the strong ellipticity condition because it amounts to the strong ellipticity of the symbol of the operator $\nabla \cdot c\nabla$. Here and throughout the paper, we use the convention of summation over repeated indices. We refer the reader to [30] for all relevant notions of elasticity.

The main result of this paper provides a positive answer to this question and establishes the hidden regularity theorem for the fully inhomogeneous and anisotropic elasticity system. An important ingredient in the proof of a hidden regularity theorem is the so-called Rellich-Nečas identity. In order to establish such an identity, it is in fact sufficient that c has the symmetries

$$c_{ijkl} = c_{klij}.$$

The Rellich-Nečas identity for elliptic systems was obtained by Payne and Weinberger in [34], but their proof uses in an essential way a stronger inequality, namely:

$$(1.2) \quad c_{ijkl}(x)E_i^j E_k^l \geq \delta_0|E|^2,$$

for all matrices E and $x \in \bar{\Omega}$, which is the so-called Legendre condition. Clearly, the Legendre condition implies the Legendre-Hadamard condition (simply take $E_i^j = \eta_i\xi_j$); on the other hand, it is well-known that the converse implication does not hold. (See for instance [15, p. 9].) In fact, for the elasticity system, the Legendre condition is never satisfied, because the tensor c is always symmetric in the indices k and l . Consequently, c has a kernel containing all antisymmetric matrices. By Korn's second inequality, a slightly weaker condition, sometimes referred to as the strong convexity condition, replaces the Legendre condition, namely

$$(1.3) \quad c_{ijkl}(x)E_i^j E_k^l \geq \delta_0|E|^2,$$

for all symmetric matrices E and $x \in \bar{\Omega}$. It can be shown that such a convexity condition implies the coercivity of the Dirichlet form associated to the operator $\nabla \cdot c\nabla$. (We refer again to [30] for a discussion.) A simple calculation gives that the strong convexity condition implies the Legendre-Hadamard condition (cf. for instance [30, Proposition 3.10, page 241]), but again the converse is not true. Conditions on the anisotropic elasticity tensor c that give strong ellipticity, but not strong convexity, are discussed for example in [32] (cf. also [33] and the references therein). The Legendre-Hadamard condition is sufficient to establish Gårding inequality, for example by means of the Fourier transform [15].

The novelty of our work consists in treating a system of differential operators that does not satisfy the Legendre or strong convexity condition, but rather the weaker Legendre-Hadamard condition, on a domain with rough boundary of Lipschitz class.

The paper is structured as follows. In the beginning of Section 2, we introduce the necessary notation and state the main theorem. The rest of the section contains the proof of the theorem. In Section 3 we then provide an application on the fluid-structure interaction model coupling the incompressible Navier-Stokes equation and the anisotropic elasticity system.

2. TRACE REGULARITY FOR THE ELASTICITY SYSTEM

We begin by stating and proving a sharp regularity theorem for the boundary trace of the normal derivative of the solution to the anisotropic linear elasticity system. Even though the most interesting case is three-dimensional, it is convenient to state our result for an arbitrary dimension $n \geq 2$.

Let $\Omega \subseteq \mathbb{R}^n$ be a bounded Lipschitz domain with boundary Γ , and let u be the solution to the linear elasticity systems of equations

$$(2.1) \quad u_{tt} - \operatorname{div}(c \cdot \nabla u) = f \quad \text{in } \Omega \times (0, T)$$

with Dirichlet boundary condition

$$(2.2) \quad u = g \quad \text{on } \Gamma \times (0, T)$$

and with initial conditions

$$(2.3) \quad u(\cdot, 0) = u_0 \quad \text{in } \Omega,$$

$$(2.4) \quad u_t(\cdot, 0) = u_1 \quad \text{in } \Omega.$$

Above, u is a vector field and $c = (c_{ijkl})_{i,j,k,l=1}^n$ is a rank-four tensor field, respectively, on $\bar{\Omega}$.

We may rewrite equation (2.1) in coordinates as

$$(2.5) \quad \partial_{tt}u_j - \partial_i(c_{ijkl}\partial_k u_l) = f_j \quad \text{in } \Omega \times (0, T), \quad j = 1, \dots, n.$$

We assume that c satisfies the following additional conditions:

a) the symmetry relations

$$(2.6) \quad c_{ijkl} = c_{klij}, \quad i, j, k, l = 1, \dots, n,$$

and

$$(2.7) \quad c_{ijkl} = c_{ijlk}, \quad i, j, k, l = 1, \dots, n,$$

b) the Legendre-Hadamard condition

$$(2.8) \quad c_{ijkl}(x)\eta_i\xi_j\eta_k\xi_l \geq \delta_0|\eta|^2|\xi|^2, \quad \forall \eta, \xi \in \mathbb{R}^n, \forall x \in \bar{\Omega},$$

where $\delta_0 > 0$ is a constant. The theory of elasticity guarantees that the tensor c has the symmetry (2.7), as it gives the relation between the stress and the strain, which is the symmetric part of the gradient. The symmetry (2.6) is an additional condition, which is satisfied in practice by most materials. We also assume that $c \in C^1(\bar{\Omega} \times [0, T])$.

Our main result asserts the regularity of the trace $\partial u / \partial \nu$ on the boundary Γ of Ω when the initial data is in the finite energy space and with given appropriate Dirichlet data.

Theorem 2.1. *Let $\Omega \subseteq \mathbb{R}^n$ be a Lipschitz domain. Let $u_0 \in H^1(\Omega)$, $u_1 \in L^2(\Omega)$, $g \in H^1(\Gamma \times [0, T])$, and $f \in L^2([0, T], L^2(\Omega))$. Let u be the solution of the Cauchy problem (2.1)–(2.4). Then*

$$(2.9) \quad \left\| \frac{\partial u}{\partial \nu} \right\|_{L^2([0, T]; L^2(\Gamma))}^2 \leq C_T (\|g\|_{H^1(\Gamma \times [0, T])}^2 + \|f\|_{L^2([0, T]; L^2(\Omega))}^2 + \|u_0\|_{H^1(\Omega)}^2 + \|u_1\|_{L^2(\Omega)}^2),$$

where $\partial u / \partial \nu$ is the outer normal derivative of u .

Before giving the proof, we recall the definition of a Lipschitz domain. A bounded domain $\Omega \subseteq \mathbb{R}^n$ is Lipschitz if, for every point $p \in \Gamma$, there exists a neighborhood $U(p)$ of p and a direction $\gamma(p) \in \mathbb{R}^n$ such that in the direction $\gamma(p)$ the domain $\Omega \cap U(p)$ is below a Lipschitz graph. This means that if we rotate coordinates so that $\gamma(p) = (0, \dots, 0, 1)$, there exists a Lipschitz function $\phi: \mathbb{R}^{n-1} \rightarrow \mathbb{R}$ such that

$$U(p) \cap \Omega = \{(x_1, \dots, x_{n-1}, x_n) \in U(p) : x_n < \phi(x_1, \dots, x_{n-1})\}$$

and

$$U(p) \cap \Gamma = \{(x_1, \dots, x_{n-1}, x_n) \in U(p) : x_n = \phi(x_1, \dots, x_{n-1})\}.$$

The outer normal ν and the normal derivative $\partial / \partial \nu = \nu \cdot \nabla$ are then well-defined a.e. on Γ .

Proof. The existence, uniqueness of solutions and their interior regularity, $(u, u_t) \in L^\infty([0, T]; H^1(\Omega) \times L^2(\Omega))$ with

$$(2.10) \quad \|u\|_{L^\infty([0, T]; H^1(\Omega))}, \|u_t\|_{L^\infty([0, T]; L^2(\Omega))} \leq C_T (\|u_0\|_{H^1(\Omega)} + \|u_1\|_{L^2(\Omega)} + \|g\|_{H^1(\Gamma \times [0, T])} + \|f\|_{L^2([0, T]; L^2(\Omega))}),$$

are standard results. For a proof, cf. [30, Section 6.3]; see also [14, 29, 36].

The key for obtaining the sharp trace regularity for a solution of system (2.5) is to test (2.5) with $h \cdot \nabla u$ for a suitably chosen h . For smooth domains, we can simply set h to be an extension of the normal into the interior of Ω . However, for Lipschitz domains, we shall take h to be a smooth field that is not parallel to the tangent plane, which exists for almost every point on the boundary. For the time being, we allow h to depend on t . Differentiating by parts gives

$$(2.11) \quad \begin{aligned} \partial_{tt} u_j h_m \partial_m u_j &= \partial_t (\partial_t u_j h_m \partial_m u_j) - \frac{1}{2} \partial_m (\partial_t u_j h_m \partial_t u_j) \\ &+ \frac{1}{2} \partial_t u_j \partial_m h_m \partial_t u_j - \partial_t u_j \partial_t h_m \partial_m u_j \end{aligned}$$

and similarly

$$(2.12) \quad \begin{aligned} \partial_i (c_{ijkl} \partial_k u_l) h_m \partial_m u_j &= \partial_i (c_{ijkl} \partial_k u_l h_m \partial_m u_j) - c_{ijkl} \partial_k u_l h_m \partial_{mi} u_j \\ &- c_{ijkl} \partial_k u_l \partial_i h_m \partial_m u_j. \end{aligned}$$

In order to rewrite the second term on the right, first observe that

$$(2.13) \quad \begin{aligned} c_{ijkl} \partial_k u_l h_m \partial_{mi} u_j &= \partial_m (c_{ijkl} \partial_k u_l h_m \partial_i u_j) - \partial_m c_{ijkl} \partial_k u_l h_m \partial_i u_j \\ &- c_{ijkl} \partial_{mk} u_l h_m \partial_i u_j - c_{ijkl} \partial_k u_l \partial_m h_m \partial_i u_j. \end{aligned}$$

By the symmetry relation (2.6), the third term on the right is identical to the term on the left, and we get

$$(2.14) \quad \begin{aligned} c_{ijkl} \partial_k u_l h_m \partial_m u_j &= \frac{1}{2} \partial_m (c_{ijkl} \partial_k u_l h_m \partial_i u_j) - \frac{1}{2} \partial_m c_{ijkl} \partial_k u_l h_m \partial_i u_j \\ &\quad - \frac{1}{2} c_{ijkl} \partial_k u_l \partial_m h_m \partial_i u_j. \end{aligned}$$

Replacing (2.14) for the second term on the right side of (2.12), we obtain

$$(2.15) \quad \begin{aligned} \partial_i (c_{ijkl} \partial_k u_l) h_m \partial_m u_j &= \partial_i (c_{ijkl} \partial_k u_l h_m \partial_m u_j) - \frac{1}{2} \partial_m (c_{ijkl} \partial_k u_l h_m \partial_i u_j) \\ &\quad + \frac{1}{2} \partial_m c_{ijkl} \partial_k u_l h_m \partial_i u_j + \frac{1}{2} c_{ijkl} \partial_k u_l \partial_m h_m \partial_i u_j \\ &\quad - c_{ijkl} \partial_k u_l \partial_i h_m \partial_m u_j. \end{aligned}$$

By (2.5), the left sides of (2.11) and (2.15) differ by $f_j h_m \partial_m u_j$, and we get

$$\begin{aligned} &\partial_i (c_{ijkl} \partial_k u_l h_m \partial_m u_j) - \frac{1}{2} \partial_m (c_{ijkl} \partial_k u_l h_m \partial_i u_j) \\ &= c_{ijkl} \partial_k u_l \partial_i h_m \partial_m u_j - \frac{1}{2} c_{ijkl} \partial_k u_l \partial_m h_m \partial_i u_j + \partial_t (\partial_t u_j h_m \partial_m u_j) \\ &\quad - \frac{1}{2} \partial_m (\partial_t u_j h_m \partial_t u_j) + \frac{1}{2} \partial_t u_j \partial_m h_m \partial_t u_j - \partial_t u_j \partial_t h_m \partial_m u_j \\ &\quad - f_j h_m \partial_m u_j - \frac{1}{2} \partial_m c_{ijkl} \partial_k u_l h_m \partial_i u_j. \end{aligned}$$

Integrating this identity over $\Omega \times [0, T]$, we get

$$(2.16) \quad \begin{aligned} &\int_0^T \int_{\partial\Omega} c_{ijkl} \partial_k u_l h_m \partial_m u_j \nu_i \, d\sigma \, dt - \frac{1}{2} \int_0^T \int_{\partial\Omega} c_{ijkl} \partial_k u_l h_m \partial_i u_j \nu_m \, d\sigma \, dt \\ &= \int_0^T \int_{\Omega} c_{ijkl} \partial_k u_l \partial_i h_m \partial_m u_j \, dx \, dt - \frac{1}{2} \int_0^T \int_{\Omega} c_{ijkl} \partial_k u_l \partial_m h_m \partial_i u_j \, dx \, dt \\ &\quad + \int_{\Omega} \partial_t u_j h_m \partial_m u_j \, dx \Big|_{t=0}^{t=T} - \frac{1}{2} \int_0^T \int_{\partial\Omega} \partial_t u_j h_m \partial_t u_j \nu_m \, d\sigma \, dt \\ &\quad + \frac{1}{2} \int_0^T \int_{\Omega} \partial_t u_j \partial_m h_m \partial_t u_j \, dx \, dt - \int_0^T \int_{\Omega} \partial_t u_j \partial_t h_m \partial_m u_j \, dx \, dt \\ &\quad - \int_0^T \int_{\Omega} f_j h_m \partial_m u_j \, dx \, dt - \frac{1}{2} \int_0^T \int_{\Omega} \partial_m c_{ijkl} \partial_k u_l h_m \partial_i u_j \, dx \, dt, \end{aligned}$$

where $d\sigma$ denotes the surface measure on $\partial\Omega$ and $\nu = (\nu_1, \dots, \nu_n)$ stands for the outward unit normal. In order to complete the proof, we rewrite the left side of the identity (2.16) in terms of the normal and tangential vectors and then choose a suitable h .

Fix an arbitrary $x_0 \in \partial\Omega$ where the outward normal ν exists, and let $\tau^1, \dots, \tau^{n-1}$ be tangential vectors to the boundary such that $\tau^1, \dots, \tau^{n-1}, \nu$ form an orthonormal system. Then, expanding the vector $(\partial_1 u_j, \dots, \partial_m u_j)$ in the basis $\tau^1, \dots, \tau^{n-1}, \nu$, we have

$$(2.17) \quad \partial_k u_j = \frac{\partial u_j}{\partial \nu} \nu_k + \sum_{r=1}^{n-1} \frac{\partial g_j}{\partial \tau^r} \tau_k^r, \quad j, k = 1, \dots, n.$$

Using this identity, we obtain at x_0 ,

$$\begin{aligned}
 & c_{ijkl}\partial_k u_l h_m \partial_m u_j \nu_i - \frac{1}{2} c_{ijkl}\partial_k u_l h_m \partial_i u_j \nu_m \\
 &= c_{ijkl}\partial_k u_l h_m \frac{\partial u_j}{\partial \nu} \nu_m \nu_i + \sum_{r=1}^{n-1} c_{ijkl}\partial_k u_l h_m \frac{\partial g_j}{\partial \tau^r} \tau_m^r \nu_i \\
 (2.18) \quad & - \frac{1}{2} c_{ijkl}\partial_k u_l h_m \frac{\partial u_j}{\partial \nu} \nu_i \nu_m - \frac{1}{2} \sum_{r=1}^{n-1} c_{ijkl}\partial_k u_l h_m \frac{\partial g_j}{\partial \tau^r} \tau_i^r \nu_m.
 \end{aligned}$$

The sum of the first and the third terms on the right equals

$$\frac{1}{2} c_{ijkl}\partial_k u_l h_m \frac{\partial u_j}{\partial \nu} \nu_m \nu_i,$$

and with the help of (2.17) may be rewritten as

$$\begin{aligned}
 (2.19) \quad & \frac{1}{2} c_{ijkl}\partial_k u_l h_m \frac{\partial u_j}{\partial \nu} \nu_m \nu_i = \frac{1}{2} c_{ijkl} \frac{\partial u_l}{\partial \nu} \nu_k h_m \frac{\partial u_j}{\partial \nu} \nu_m \nu_i + \frac{1}{2} \sum_{r=1}^{n-1} c_{ijkl} \frac{\partial g_l}{\partial \tau^r} \tau_k^r h_m \frac{\partial u_j}{\partial \nu} \nu_m \nu_i.
 \end{aligned}$$

For the first term on the right, we use the Legendre-Hadamard coercivity condition (2.8) and get

$$c_{ijkl}\nu_i \frac{\partial u_j}{\partial \nu} \nu_k \frac{\partial u_l}{\partial \nu} \geq \delta_0 \left| \frac{\partial u}{\partial \nu} \right|^2,$$

and thus, if

$$h(x_0) \cdot \nu(x_0) \geq 0,$$

then

$$(2.20) \quad \frac{1}{2} c_{ijkl}\partial_k u_l h_m \frac{\partial u_j}{\partial \nu} \nu_m \nu_i \geq h_m \nu_m \delta_0 \left| \frac{\partial u}{\partial \nu} \right|^2 - C|c||D'g||h| \left| \frac{\partial u}{\partial \nu} \right|,$$

where $D'g$ denotes the tangential gradient of g . On the second and fourth terms on the right side of (2.18), we use

$$|\nabla u| \leq \left| \frac{\partial u}{\partial \nu} \right| + C|D'g|,$$

which follows from (2.17). We thus get from (2.18)

$$\begin{aligned}
 & c_{ijkl}\partial_k u_l h_m \partial_m u_j \nu_i - \frac{1}{2} c_{ijkl}\partial_k u_l h_m \partial_i u_j \nu_m \\
 & \geq h_m \nu_m \delta_0 \left| \frac{\partial u}{\partial \nu} \right|^2 - C|c||D'g||h| \left| \frac{\partial u}{\partial \nu} \right| - C|c||D'g|^2|h|
 \end{aligned}$$

at all the points on the boundary for which $h \cdot \nu \geq 0$. Substituting this inequality in (2.16), we get

$$\begin{aligned}
 & \delta_0 \int_0^T \int_{\partial\Omega} h \cdot \nu \left| \frac{\partial u}{\partial \nu} \right|^2 d\sigma(x) dt \\
 & \leq C \int_0^T \int_{\partial\Omega} |c| |D'g| |h| \left(\left| \frac{\partial u}{\partial \nu} \right| + |D'g| \right) d\sigma(x) dt + C \int_0^T \int_{\partial\Omega} |\partial_t g|^2 |h| d\sigma(x) dt \\
 & \quad + C \int_0^T \int_{\Omega} |c| |\nabla u|^2 |\nabla h| dx dt + C \int_{\Omega} |\partial_t u| |h| |\nabla u| dx \Big|_T + C \int_{\Omega} |\partial_t u| |h| |\nabla u| dx \Big|_0 \\
 & \quad + C \int_0^T \int_{\Omega} |\partial_t u|^2 |\nabla \cdot h| dx dt + C \int_0^T \int_{\Omega} |\partial_t u| |\partial_t h| |\nabla u| dx dt \\
 (2.21) \quad & \quad + C \int_0^T \int_{\Omega} |f| |h| |\nabla u| dx dt + C \int_0^T \int_{\Omega} |\nabla c| |\nabla u|^2 |h| dx dt
 \end{aligned}$$

provided $h \cdot \nu \geq 0$ on $\partial\Omega$. Now, we choose a time-independent smooth function $h: \Omega \rightarrow \mathbb{R}^n$ such that $h \cdot \nu \geq \delta$, where $\delta > 0$ depends on the Lipschitz character of Ω . (In order to do this, we use a partition of unity. Without loss of generality, we can assume that each patch of the partition has support contained in a neighborhood $U(p) \cap \Gamma$ for some $p \in \Gamma$. Then, on each patch we set h to be the constant $\gamma(p)$, where $U(p)$ and $\gamma(p)$ are as in the definition of a Lipschitz domain given above.) Then the left side of (2.21) is greater than or equal to

$$\delta_0 \delta \int_0^T \int_{\partial\Omega} \left| \frac{\partial u}{\partial \nu} \right|^2 d\sigma(x) dt,$$

and we get, using the Cauchy-Schwarz inequality,

$$\begin{aligned}
 & \int_0^T \int_{\Omega} \left| \frac{\partial u}{\partial \nu} \right|^2 d\sigma(x) dt \\
 & \leq C \int_0^T \int_{\partial\Omega} (|D'g|^2 + |g_t|^2) d\sigma(x) dt + C \int_0^T \int_{\Omega} (|\nabla u|^2 + |\partial_t u|^2 + |f|^2) dx dt \\
 (2.22) \quad & \quad + C \int_{\Omega} (|\partial_t u|^2 + |\nabla u|^2) dx \Big|_T + C \int_{\Omega} (|\partial_t u|^2 + |\nabla u|^2) dx \Big|_0.
 \end{aligned}$$

The inequality (2.9) then follows immediately. \square

3. AN APPLICATION TO THE FLUID-STRUCTURE INTERACTION PROBLEM

In this section, we present an application of Theorem 2.1 to a fluid-structure system addressed in [6, 7, 18, 19, 22]. As in [6, 18, 22], we consider the case when the boundary is stationary, which is a physical approximation for the case when the transversal motion is small while the lateral motion may be large. The fluid velocity u is modeled in a smooth domain Ω_f by the incompressible Navier-Stokes equations

$$(3.1) \quad \partial_t u - \Delta u + (u \cdot \nabla) u + \nabla p = 0 \text{ in } \Omega_f \times (0, T),$$

$$(3.2) \quad \nabla \cdot u = 0 \text{ in } \Omega_f \times (0, T),$$

while the elastic structure displacement w is modeled in a smooth domain Ω_e by the elasticity system

$$(3.3) \quad \partial_{tt}w^j - \partial_i(c_{ijkl}\partial_k w^l) = 0 \quad \text{in } \Omega_e \times (0, T), \quad j = 1, \dots, n.$$

Here, c_{ijkl} are assumed smooth, satisfying the strong ellipticity condition and all the symmetry relations

$$(3.4) \quad c_{ijkl} = c_{jikl} = c_{klij}, \quad i, j, k, l = 1, 2, 3,$$

from Section 2. We assume that $\Gamma_c = \partial\Omega_e \subseteq \partial\Omega_f$, which corresponds to the situation when the body is immersed in the fluid. Then, Γ_c represents the interface between the fluid and the elastic structure.

We also impose the standard transmission conditions across the interface $\Gamma_c = \partial\Omega_e$ between Ω_f and Ω_e ; i.e., we assume continuity of the velocities, $w^j = w_t^j$, and the tractions

$$c_{ijkl}\partial_k w^l \nu_i = \partial_i w^j \nu_i - p \nu_j$$

for $j = 1, 2, 3$, where $\nu = (\nu_1, \nu_2, \nu_3)$ is the outward unit normal with respect to Ω_e . Also, we impose the no-slip boundary conditions at the fluid boundary, $u = 0$ on $\Gamma_f = \partial\Omega_f \setminus \Gamma_c$. Finally, the functions u and w satisfy the initial condition $(u(x, 0), w(x, 0), w_t(x, 0)) = (u_0(x), w_0(x), w_1(x))$. The next statement is our main application of the hidden regularity theorem from the previous section. We denote $V = \{u \in H^1(\Omega_f) : \operatorname{div} u = 0, u|_{\Gamma_f} = 0\}$.

Theorem 3.1. *Let $r \in (0, (\sqrt{2}-1)/2)$. Assume that $u_0 \in V$, $w_0 \in H^{3/2+r}(\Omega_e)$ and $w_1 \in H^{1/2+r}(\Omega_e)$ with the compatibility condition $w_1^i = u_0^i$ on Γ_c for all $i = 1, 2, 3$. Then there exists a unique local-in-time solution*

$$u \in L^2([0, T]; H^{3/2+r}(\Omega_f)) \cap L^\infty([0, T]; V),$$

$$\partial_t^j w \in C([0, T]; H^{3/2+r-j}(\Omega_e)), \quad j = 0, 1, 2, 3,$$

with $p \in L^\infty([0, T]; H^{1/2+r}(\Omega_f))$, for a time $T > 0$ depending on the initial data.

The proof is obtained by a direct adaptation of the proof in [19]. The first difference is the hidden regularity theorem and is dealt with in Section 2. The second difference is that the coefficients in the elasticity equation are not constant. This however does not create any additional difficulties, as the presence of derivatives on the coefficients c_{ijkl} creates only lower order terms. We defer to [19] for additional details.

ACKNOWLEDGMENT

The authors thank the referee for corrections and useful suggestions.

REFERENCES

- [1] S. Agmon, A. Douglis, and L. Nirenberg, *Estimates near the boundary for solutions of elliptic partial differential equations satisfying general boundary conditions. I*, Comm. Pure Appl. Math. **12** (1959), 623–727. MR0125307 (23 #A2610)
- [2] F. D. Araruna, F. O. Matias, M. P. Matos, and S. M. S. Souza, *Hidden regularity for the Kirchhoff equation*, Commun. Pure Appl. Anal. **7** (2008), no. 5, 1049–1056, DOI 10.3934/cpaa.2008.7.1049. MR2410866 (2009g:35189)
- [3] G. Avalos and R. Triggiani, *The coupled PDE system arising in fluid/structure interaction. I. Explicit semigroup generator and its spectral properties*, Fluids and waves, Contemp. Math., vol. 440, Amer. Math. Soc., Providence, RI, 2007, pp. 15–54, DOI 10.1090/conm/440/08475. MR2359448 (2008k:35465)

- [4] Muriel Boulakia, *Existence of weak solutions for the three-dimensional motion of an elastic structure in an incompressible fluid*, J. Math. Fluid Mech. **9** (2007), no. 2, 262–294, DOI 10.1007/s00021-005-0201-7. MR2329269 (2008i:74039)
- [5] M. Boulakia and S. Guerrero, *Regular solutions of a problem coupling a compressible fluid and an elastic structure*, J. Math. Pures Appl. (9) **94** (2010), no. 4, 341–365, DOI 10.1016/j.matpur.2010.04.002 (English, with English and French summaries). MR2719892 (2011j:35176)
- [6] Viorel Barbu, Zoran Grujić, Irena Lasiecka, and Amjad Tuffaha, *Existence of the energy-level weak solutions for a nonlinear fluid-structure interaction model*, Fluids and waves, Contemp. Math., vol. 440, Amer. Math. Soc., Providence, RI, 2007, pp. 55–82, DOI 10.1090/conm/440/08476. MR2359449 (2008m:35260)
- [7] Viorel Barbu, Zoran Grujić, Irena Lasiecka, and Amjad Tuffaha, *Smoothness of weak solutions to a nonlinear fluid-structure interaction model*, Indiana Univ. Math. J. **57** (2008), no. 3, 1173–1207, DOI 10.1512/iumj.2008.57.3284. MR2429089 (2009c:35352)
- [8] Francesca Bucci and Irena Lasiecka, *Optimal boundary control with critical penalization for a PDE model of fluid-solid interactions*, Calc. Var. Partial Differential Equations **37** (2010), no. 1-2, 217–235, DOI 10.1007/s00526-009-0259-9. MR2564405 (2011b:49055)
- [9] Abdellatif Chaïra, *Équation des ondes et régularité sur un ouvert lipschitzien*, C. R. Acad. Sci. Paris Sér. I Math. **316** (1993), no. 1, 33–36 (French, with English and French summaries). MR1198745 (93k:35155)
- [10] Daniel Coutand and Steve Shkoller, *Motion of an elastic solid inside an incompressible viscous fluid*, Arch. Ration. Mech. Anal. **176** (2005), no. 1, 25–102, DOI 10.1007/s00205-004-0340-7. MR2185858 (2006g:74045)
- [11] Daniel Coutand and Steve Shkoller, *The interaction between quasilinear elastodynamics and the Navier-Stokes equations*, Arch. Ration. Mech. Anal. **179** (2006), no. 3, 303–352, DOI 10.1007/s00205-005-0385-2. MR2208319 (2006j:74023)
- [12] Maria Cristina de Campos Vieira, *Hidden regularity for a coupled system of nonlinear hyperbolic equations*, An. Acad. Brasil. Ciênc. **63** (1991), no. 2, 105–113. MR1125167 (92j:35118)
- [13] G. Duvaut and J.-L. Lions, *Inequalities in mechanics and physics*, Springer-Verlag, Berlin, 1976. Translated from the French by C. W. John; Grundlehren der Mathematischen Wissenschaften, 219. MR0521262 (58 #25191)
- [14] Lawrence C. Evans, *Partial differential equations*, 2nd ed., Graduate Studies in Mathematics, vol. 19, American Mathematical Society, Providence, RI, 2010. MR2597943 (2011c:35002)
- [15] Mariano Giaquinta, *Introduction to regularity theory for nonlinear elliptic systems*, Lectures in Mathematics ETH Zürich, Birkhäuser Verlag, Basel, 1993. MR1239172 (94g:49002)
- [16] Mary Ann Horn, *Sharp trace regularity for the solutions of the equations of dynamic elasticity*, J. Math. Systems Estim. Control **8** (1998), no. 2, 11 pp. (electronic). MR1651449 (99j:73021)
- [17] Thomas J. R. Hughes and Jerrold E. Marsden, *Classical elastodynamics as a linear symmetric hyperbolic system*, J. Elasticity **8** (1978), no. 1, 97–110. MR0468469 (57 #8301)
- [18] Igor Kukavica, Amjad Tuffaha, and Mohammed Ziane, *Strong solutions to a nonlinear fluid structure interaction system*, J. Differential Equations **247** (2009), no. 5, 1452–1478, DOI 10.1016/j.jde.2009.06.005. MR2541417 (2010g:35239)
- [19] Igor Kukavica, Amjad Tuffaha, and Mohammed Ziane, *Strong solutions to a Navier-Stokes-Lamé system on a domain with a non-flat boundary*, Nonlinearity **24** (2011), no. 1, 159–176, DOI 10.1088/0951-7715/24/1/008. MR2739836 (2012b:35228)
- [20] Igor Kukavica and Amjad Tuffaha, *Well-posedness for the compressible Navier-Stokes-Lamé system with a free interface*, Nonlinearity **25** (2012), no. 11, 3111–3137, DOI 10.1088/0951-7715/25/11/3111. MR2991431
- [21] I. Kukavica and A. Tuffaha, *Solutions to a fluid-structure interaction free boundary problem*, Discrete Contin. Dyn. Syst. **32** (2012), no. 4, 1355–1389. MR2851902 (2012m:35244)
- [22] J.-L. Lions, *Quelques méthodes de résolution des problèmes aux limites non linéaires*, Dunod, 1969 (French). MR0259693 (41 #4326)
- [23] J.-L. Lions, *Hidden regularity in some nonlinear hyperbolic equations*, Mat. Apl. Comput. **6** (1987), no. 1, 7–15 (English, with Portuguese summary). MR902998 (88k:35139)
- [24] I. Lasiecka, J.-L. Lions, and R. Triggiani, *Nonhomogeneous boundary value problems for second order hyperbolic operators*, J. Math. Pures Appl. (9) **65** (1986), no. 2, 149–192. MR867669 (88c:35092)

- [25] Irena Lasiecka and Daniel Toundykov, *Semigroup generation and “hidden” trace regularity of a dynamic plate with non-monotone boundary feedbacks*, Commun. Math. Anal. **8** (2010), no. 1, 109–144. MR2551496 (2011a:35526)
- [26] I. Lasiecka and R. Triggiani, *Uniform stabilization of the wave equation with Dirichlet or Neumann feedback control without geometrical conditions*, Appl. Math. Optim. **25** (1992), no. 2, 189–224, DOI 10.1007/BF01182480. MR1142681 (93b:93099)
- [27] I. Lasiecka and R. Triggiani, *Sharp regularity theory for elastic and thermoelastic Kirchhoff [Kirchhoff] equations with free boundary conditions*, Rocky Mountain J. Math. **30** (2000), no. 3, 981–1024, DOI 10.1216/rmjm/1021477256. MR1797827 (2001i:35039)
- [28] Irena Lasiecka and Amjad Tuffaha, *Riccati theory and singular estimates for a Bolza control problem arising in linearized fluid-structure interaction*, Systems Control Lett. **58** (2009), no. 7, 499–509, DOI 10.1016/j.sysconle.2009.02.010. MR2536425 (2010i:93066)
- [29] J.-L. Lions and E. Magenes, *Non-homogeneous boundary value problems and applications. Vol. II*, Springer-Verlag, New York, 1972. Translated from the French by P. Kenneth; Die Grundlehren der mathematischen Wissenschaften, Band 182. MR0350178 (50 #2671)
- [30] Jerrold E. Marsden and Thomas J. R. Hughes, *Mathematical foundations of elasticity*, Dover Publications Inc., New York, 1994. Corrected reprint of the 1983 original. MR1262126 (95h:73022)
- [31] M. Milla Miranda and L. A. Medeiros, *Hidden regularity for semilinear hyperbolic partial differential equations*, Ann. Fac. Sci. Toulouse Math. (5) **9** (1988), no. 1, 103–120 (English, with French summary). MR971816 (90e:35026)
- [32] J. Merodio and R. W. Ogden, *A note on strong ellipticity for transversely isotropic linearly elastic solids*, Quart. J. Mech. Appl. Math. **56** (2003), no. 4, 589–591, DOI 10.1093/qj-mam/56.4.589. MR2026873
- [33] Anna L. Mazzucato and Lizabeth V. Rachele, *On transversely isotropic elastic media with ellipsoidal slowness surfaces*, Math. Mech. Solids **13** (2008), no. 7, 611–638, DOI 10.1177/1081286507078498. MR2442289 (2009e:74063)
- [34] L. E. Payne and H. F. Weinberger, *New bounds for solutions of second order elliptic partial differential equations*, Pacific J. Math. **8** (1958), 551–573. MR0104047 (21 #2809)
- [35] Jorge Alonso San Martín, Victor Starovoitov, and Marius Tucsnak, *Global weak solutions for the two-dimensional motion of several rigid bodies in an incompressible viscous fluid*, Arch. Ration. Mech. Anal. **161** (2002), no. 2, 113–147, DOI 10.1007/s002050100172. MR1870954 (2002j:35259)
- [36] William P. Ziemer, *Weakly differentiable functions*, Graduate Texts in Mathematics, vol. 120, Springer-Verlag, New York, 1989. Sobolev spaces and functions of bounded variation. MR1014685 (91e:46046)

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF SOUTHERN CALIFORNIA, LOS ANGELES, CALIFORNIA 90089

E-mail address: kukavica@usc.edu

DEPARTMENT OF MATHEMATICS, PENN STATE UNIVERSITY, UNIVERSITY PARK, PENNSYLVANIA 16802

E-mail address: alm24@psu.edu

DEPARTMENT OF MATHEMATICS, THE PETROLEUM INSTITUTE, ABU DHABI, UNITED ARAB EMIRATES

E-mail address: atuffaha@pi.ac.ae