PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 145, Number 5, May 2017, Pages 2081–2087
http://dx.doi.org/10.1090/proc/13364
Article electronically published on October 26, 2016

AUTOMATIC CONTINUITY FOR LINEAR SURJECTIVE MAPS COMPRESSING THE LOCAL SPECTRUM AT FIXED VECTORS

CONSTANTIN COSTARA

(Communicated by Thomas Schlumprecht)

Abstract. Let X and Y be complex Banach spaces and denote by $\mathcal{L}(X)$ and $\mathcal{L}(Y)$ the algebras of all bounded linear operators on X, respectively Y. Let also $x_0 \in X$ and $y_0 \in Y$ be nonzero vectors. We prove that if $\varphi : \mathcal{L}(X) \to \mathcal{L}(Y)$ is a linear surjective map such that for each $T \in \mathcal{L}(X)$ we have that the local spectrum of $\varphi(T)$ at y_0 is a subset of the local spectrum of T at x_0, then φ is automatically continuous. We also give a new proof for the automatic continuity of linear surjective maps decreasing the local spectral radius at some fixed nonzero vector. As a corollary, we obtain that the characterizations of J. Bračič and V. Müller for linear surjective maps on $\mathcal{L}(X)$ preserving the local spectrum/local spectral radius at some fixed vector can be obtained with no continuity assumptions on them.

1. Introduction and background from local spectral theory

Let X be a complex Banach space and denote by $\mathcal{L}(X)$ the Banach algebra of all bounded linear operators on X. We denote by X' the dual of X, by I_X the identity operator on X and respectively by $I_{X'}$ the identity on X'. For $T \in \mathcal{L}(X)$, by $T^* \in \mathcal{L}(X')$ we shall denote its adjoint operator. Also, for $x \in X$ and $f \in X'$ we denote by $x \otimes f \in \mathcal{L}(X)$ the rank one operator sending $y \in X$ into $f(y)\, x \in X$.

Fix now a nonzero vector $x_0 \in X$. For $T \in \mathcal{L}(X)$, we shall denote by $\sigma(T)$, $\sigma_p(T)$ and $r(T)$ respectively the classical spectrum, the point spectrum and the spectral radius of T. Also, we shall denote by $\sigma_T(x_0)$ the local spectrum of T at x_0 and by $r_T(x_0)$ its local spectral radius at the same vector.

Let us recall that $\sigma_T(x_0) = \mathbb{C} \backslash \rho_T(x_0)$, where $\rho_T(x_0)$ is the local resolvent set of T at x_0, that is, the union of all open subsets $U \subseteq \mathbb{C}$ for which there exists $u : U \to X$ analytic such that $(T - \mu I_X)\, u(\mu) = x_0$ for all $\mu \in U$. We have that $\sigma_T(x_0) \subseteq \sigma(T)$, but unlike the classical spectrum, the local spectrum might be empty! If T has the so-called single-valued extension property (SVEP, for short), then $\sigma_T(x_0)$ is nonempty. (An operator $T \in \mathcal{L}(X)$ is said to have SVEP if given any open subset $U \subseteq \mathbb{C}$, the only analytic function $u : U \to X$ satisfying $(T - \mu I_X)\, u(\mu) = 0$ for every $\mu \in U$ is the trivial one, namely u identically zero.) Any $T \in \mathcal{L}(X)$ for which $\sigma_p(T)$ has no interior points has SVEP, but there are examples of operators which do not have this property (see, e.g., [1, pg. 71]).

Let us also recall that $r_T(x_0) = \limsup_{n \to \infty} \|T^n(x_0)\|^{1/n}$. Then the X-valued series $u(\mu) := -\sum_{n=1}^{\infty} \mu^{-n} T^{n-1}(x_0)$ converges locally uniformly for $|\mu| > r_T(x_0)$,

Received by the editors February 24, 2016 and, in revised form, June 28, 2016.
2010 *Mathematics Subject Classification.* Primary 46H40; Secondary 47A11.
This work was supported by a grant of the Romanian National Authority for Scientific Research, CNCS - UEFISCDI, project number PN-II-RU-TE-2012-3-0042.

so it defines an analytic function on this set. Clearly, we have $(T-\mu I_X)\,u(\mu)=x_0$ for all $|\mu|>r_T(x_0)$, and therefore $|\mu|>r_T(x_0)$ implies $\mu\in\rho_T(x_0)$. Thus, the elements $\lambda\in\sigma_T(x_0)$ (if any) all satisfy the inequality $|\lambda|\le r_T(x_0)$. Therefore

$$\Gamma_T(x_0):=\max\{|\lambda|:\lambda\in\sigma_T(x_0)\}\le r_T(x_0).$$

(If $\sigma_T(x_0)=\emptyset$, by convention we have $\Gamma_T(x_0)=-\infty$.) Unlike the corresponding property for the classical spectrum, in the general case we do not have equality: the two quantities $\Gamma_T(x_0)$ and $r_T(x_0)$ are equal for example when T has SVEP (see, e.g., [10, Prop. 3.3.13]).

We shall finish the preliminaries on local spectral theory with the following simple but very useful observation. We have

$$x_0\notin(T-\lambda I_X)(X)\Rightarrow\lambda\in\sigma_T(x_0),$$

and therefore

$$x_0\notin(T-\lambda I_X)(X)\Rightarrow|\lambda|\le\Gamma_T(x_0). \tag{1.1}$$

For more background information on general local spectral theory, we refer the reader to the excellent monographs [1] and [10].

2. The main result

The main result of this paper concerns an automatic continuity property for a linear local spectra preserver problem. The study of such preserver problems attracted the attention of many authors over the last years. The first ones to consider them were A. Bourhim and T. Ransford in [5]: they characterized additive maps on $\mathcal{L}(X)$ which preserve the local spectrum of each $T\in\mathcal{L}(X)$ at each vector $x\in X$. In [9], M. González and M. Mbekhta characterized linear maps on the space of matrices $\mathcal{M}_n$ that preserve the local spectrum at a nonzero fixed vector $x_0\in\mathbb{C}^n$, while A. Bourhim and V. Miller characterized in [4] linear maps on $\mathcal{M}_n$ that preserve the local spectral radius at x_0. Both results were extended in [6] by J. Bračič and V. Müller to infinite dimensional Banach spaces, by imposing a continuity assumption on the preserving map.

Theorem 2.1 ([6, Theorems 3.3 and 3.4])**.** *Let $\varphi:\mathcal{L}(X)\to\mathcal{L}(X)$ be a continuous surjective linear map.*

i) If

$$\sigma_{\varphi(T)}(x_0)=\sigma_T(x_0)\qquad(T\in\mathcal{L}(X)), \tag{2.1}$$

there exists an invertible $A\in\mathcal{L}(X)$ such that $Ax_0=x_0$ and

$$\varphi(T)=ATA^{-1}\qquad(T\in\mathcal{L}(X)).$$

ii) If

$$r_{\varphi(T)}(x_0)=r_T(x_0)\qquad(T\in\mathcal{L}(X)), \tag{2.2}$$

there exists an invertible $A\in\mathcal{L}(X)$ and a unimodular complex constant c such that $Ax_0=x_0$ and

$$\varphi(T)=cATA^{-1}\qquad(T\in\mathcal{L}(X)).$$

(For further results on linear and nonlinear local spectra preserver problems, see for instance the last section of the survey article [3] and the references therein.)

The proof of Theorem 2.1 is based on a density result given by [6, Theorem 3.5]: the set of all operators $T\in\mathcal{L}(X)$ such that the surjectivity spectrum of T

coincides with $\sigma_T(x_0)$ is residual in $\mathcal{L}(X)$, and in particular dense in $\mathcal{L}(X)$. As a corollary, we obtain that the set of all $T \in \mathcal{L}(X)$ such that $r_T(x_0) = r(T)$ is also residual [6, Corollary 2.6]. Then using the continuity hypothesis of Theorem 2.1 one can check then that if either (2.1) or (2.2) holds, then $r(\varphi(T)) = r(T)$ for all $T \in \mathcal{L}(X)$. The surjective spectral isometries of $\mathcal{L}(X)$ are of a standard form [7], and this was used in order to finish the proof of Theorem 2.1.

It is well known that every surjective spectral isometry on $\mathcal{L}(X)$ is continuous: this is a particular case of a more general automatic continuity result for surjective linear maps on semisimple Banach algebras which decrease the spectral radius [2, Theorem 5.5.2]. As we mentioned, Bračič and Müller used exactly the continuity in order to prove that a surjection preserving the local spectrum/local spectral radius is a spectral isometry, so the following question is a very natural one [6, Problem 3.5]: is it possible to omit the assumption of continuity on φ in the statement of Theorem 2.1? It is proved in [8, Theorem 1.2] that any surjective linear map on $\mathcal{L}(X)$ decreasing the local spectral radius at x_0 is continuous, and therefore giving a positive answer to half of [6, Problem 3.5]. When working with the classical spectrum/spectral radius instead of the local spectrum/local spectral radius, if one is able to obtain a result which is stated in terms of the spectral radius, then the result holds for the spectrum, too. If one can show that the converse holds, then the proof is usually much more complicated. This is due to the fact that the spectral radius of an operator $T \in \mathcal{L}(X)$ is given by the peripherical spectrum of T, which is a *nonempty* subset of $\sigma(T)$, and usually *much smaller* than $\sigma(T)$. In the case of the local spectral theory, the local spectrum might be empty while the local spectral radius is strictly positive, and this is the reason why a positive answer to the other half of [6, Problem 3.5] cannot be obtained directly from [8, Theorem 1.2].

The purpose of this paper is to prove that, as in the case of surjective linear maps decreasing the local spectral radius at a fixed nonzero vector, surjective linear maps compressing the local spectrum at a fixed nonzero vector are automatically continuous. We also give a new and completely different proof of [8, Theorem 1.2]. Both results are a particular case of the following theorem, which is the main result of this paper. With no extra effort, the result may be stated and proved for maps acting on different Banach spaces.

Theorem 2.2. *Suppose that X and Y are complex Banach spaces and let y_0 be a nonzero vector in Y. Let $\varphi : \mathcal{L}(X) \to \mathcal{L}(Y)$ be a linear surjective map such that*

$$\Gamma_{\varphi(T)}(y_0) \leq r(T) \qquad (T \in \mathcal{L}(X)). \tag{2.3}$$

Then φ is continuous.

Of course, if $x_0 \in X$ and $y_0 \in Y$ are nonzero vectors and $\varphi : \mathcal{L}(X) \to \mathcal{L}(Y)$ is a linear surjective map such that

$$r_{\varphi(T)}(y_0) \leq r_T(x_0) \qquad (T \in \mathcal{L}(X)), \tag{2.4}$$

since (2.4) implies (2.3) we obtain that φ is automatically continuous. Thus, we obtain a slight generalization of [8, Theorem 1.2], with a completely different proof. The main result in [8] was obtained via subharmonicity arguments, but we were not able to use the same type of idea in order to obtain Theorem 2.2 since we were not capable of building useful subharmonic functions defined in terms of the local spectrum at some fixed nonzero vector.

If we suppose that (2.5) holds instead of (2.4), then once again (2.3) is true and Theorem 2.2 gives the following completely new result.

Corollary 2.3. *Suppose that $x_0 \in X$ and $y_0 \in Y$ are nonzero vectors, and let $\varphi : \mathcal{L}(X) \to \mathcal{L}(Y)$ be a linear surjective map such that*

$$\sigma_{\varphi(T)}(y_0) \subseteq \sigma_T(x_0) \qquad (T \in \mathcal{L}(X)). \tag{2.5}$$

Then φ is continuous.

Thus, the statement of Theorem 2.1 holds even if we eliminate the continuity assumption on the linear map φ.

3. Proof of the main result

The main idea is to prove that under the hypothesis of Theorem 2.2 we have that φ is a spectral radius decreasing map, and then to use [2, Theorem 5.5.2]. We shall need the following auxiliary result.

Lemma 3.1. *Let $\varphi : \mathcal{L}(X) \to \mathcal{L}(Y)$ be a linear surjective map such that*

$$\sup\{|\lambda| : \lambda \in \sigma_p(\varphi(T)^*)\} \leq r(T) \qquad (T \in \mathcal{L}(X)). \tag{3.1}$$

(If the point spectrum of $\varphi(T)^*$ is empty, the supremum in (3.1) *is by convention zero.) Then*

$$r(\varphi(T)) \leq r(T) \qquad (T \in \mathcal{L}(X)). \tag{3.2}$$

Proof. Consider an arbitrary $T \in \mathcal{L}(X)$, and let $\lambda_0 \in \sigma(\varphi(T)^*)$ such that $|\lambda_0| = r(\varphi(T)^*)$. Then $|\lambda_0| = r(\varphi(T))$, and let us prove that $|\lambda_0| \leq r(T)$.

Consider $(\lambda_n)_{n\geq 1} \subseteq \mathbb{C}$ such that $|\lambda_n| > |\lambda_0|$ for $n \geq 1$ and $\lambda_n \to \lambda_0$. Then $(\lambda_n I_{Y'} - \varphi(T)^*)_n \subseteq \mathcal{L}(Y')$ is a sequence of invertible operators converging to a noninvertible one, namely $\lambda_0 I_{Y'} - \varphi(T)^* \in \mathcal{L}(Y')$. By [2, Theorem 3.2.11] we have that $\|(\lambda_n I_{Y'} - \varphi(T)^*)^{-1}\| \to +\infty$. That is,

$$\left\|((\lambda_n I_Y - \varphi(T))^*)^{-1}\right\| \to +\infty.$$

Since $(((\lambda_n I_Y - \varphi(T))^*)^{-1})_n \subseteq \mathcal{L}(Y')$ is unbounded, we use the Uniform Boundedness Principle to find $f \in Y'$ such that $(((\lambda_n I_Y - \varphi(T))^*)^{-1}(f))_n \subseteq Y'$ is unbounded. Using once more the Uniform Boundedness Principle, we find $y \in Y$ such that $((((\lambda_n I_Y - \varphi(T))^*)^{-1}(f))(y))_n \subseteq \mathbb{C}$ is unbounded. By passing to a subsequence, without loss of generality we may therefore suppose that, by denoting

$$\alpha_n = (((\lambda_n I_Y - \varphi(T))^*)^{-1}(f))(y) \in \mathbb{C} \qquad (n \geq 1),$$

we have that $(\alpha_n)_n \subseteq \mathbb{C}\backslash\{0\}$ and $|\alpha_n| \to +\infty$.

Since φ is surjective, we find $R \in \mathcal{L}(X)$ such that $\varphi(R) = y \otimes f \in \mathcal{L}(Y)$. Then $\varphi(R)^* = f \otimes K_Y(y)$, where $K_Y : Y \to Y''$ is the canonical injection into the bidual. Denoting $f_n = ((\lambda_n I_Y - \varphi(T))^*)^{-1}(f) \in Y'$ for each $n \geq 1$, we have

$$\begin{aligned}
&(\lambda_n I_{Y'} - (\varphi(T) + \varphi(R)/\alpha_n)^*)(f_n) \\
&= ((\lambda_n I_Y - \varphi(T))^* - \varphi(R)^*/\alpha_n))(((\lambda_n I_Y - \varphi(T))^*)^{-1}(f)) \\
&= f - (f \otimes K_Y(y))(((\lambda_n I_Y - \varphi(T))^*)^{-1}(f))/\alpha_n \\
&= f - f \cdot ((((\lambda_n I_Y - \varphi(T))^*)^{-1}(f))(y))/\alpha_n \\
&= 0.
\end{aligned}$$

Since $f_n \in Y'$ is nonzero, this gives

$$\lambda_n \in \sigma_p((\varphi(T) + \varphi(R)/\alpha_n)^*) = \sigma_p((\varphi(T + R/\alpha_n))^*).$$

Using (3.1), we then have

$$|\lambda_n| \leq r(T + R/\alpha_n) \qquad (n \geq 1).$$

This gives

$$\limsup_{n\to\infty} |\lambda_n| \leq \limsup_{n\to\infty} r(T + R/\alpha_n),$$

that is,

$$|\lambda_0| \leq r(T),$$

since the facts that the spectral radius is upper semicontinuous [2, Theorem 3.4.2] and $|\alpha_n| \to +\infty$ give $\limsup_{n\to\infty} r(T + R/\alpha_n) \leq r(T)$. □

We prove now that under the hypothesis of Theorem 2.2, we may apply Lemma 3.1 to the map φ.

Theorem 3.2. *Let $\varphi : \mathcal{L}(X) \to \mathcal{L}(Y)$ be a map as in the statement of Theorem* 2.2. *Then* (3.1) *holds.*

Proof. We shall use some of the ideas from the proof of [6, Lemma 2.2].

Consider an arbitrary $\lambda \in \sigma_p(\varphi(T)^*)$. There exists then $f \in Y'$ nonzero such that $(\varphi(T)^*)(f) = \lambda f$, and therefore

$$(\varphi(T) - \lambda I_Y)(Y) \subseteq \ker f.$$

If $f(y_0) \neq 0$, then $y_0 \notin (\varphi(T) - \lambda I_Y)(Y)$. Using (1.1) we have $|\lambda| \leq \Gamma_{\varphi(T)}(y_0)$, and (2.3) gives

$$|\lambda| \leq r(T).$$

Suppose now that $f(y_0) = 0$. Thus $y_0 \in \ker f$, and let $Z \subseteq \ker f$ be a closed subspace such that $\ker f = Z \oplus (\mathbb{C}y_0)$. Fix $\widetilde{y} \in Y$ such that $f(\widetilde{y}) \neq 0$, and consider $g \in Y'$ such that $g = 0$ on Z, but $g(y_0) \neq 0$ and $g(\widetilde{y}) \neq 0$. For $|\mu| < 1/(\|\widetilde{y}\|\,\|g\|)$, consider

$$A_\mu := I_Y - \mu(\widetilde{y} \otimes g) \in \mathcal{L}(Y).$$

Since for all such μ we have $\|\mu(\widetilde{y} \otimes g)\| = |\mu|\,\|\widetilde{y}\|\,\|g\| < 1$, then $A_\mu \in \mathcal{L}(Y)$ is invertible, with

$$A_\mu^{-1} = \sum_{j=0}^{\infty} (\mu(\widetilde{y} \otimes g))^j.$$

Define the complex-valued analytic function

$$h(\mu) = \sum_{j=0}^{\infty} \mu^{j+1} g(\widetilde{y})^j \qquad (|\mu| < 1/(\|\widetilde{y}\|\,\|g\|)).$$

Then one can easily check that

$$A_\mu^{-1} = I_Y + h(\mu)(\widetilde{y} \otimes g) \qquad (|\mu| < 1/(\|\widetilde{y}\|\,\|g\|)).$$

Putting $W_\mu = A_\mu \varphi(T) A_\mu^{-1} \in \mathcal{L}(Y)$, then for all $|\mu| < 1/(\|\widetilde{y}\|\,\|g\|)$ we have

$$\begin{aligned} W_\mu &= (I_Y - \mu(\widetilde{y} \otimes g))\varphi(T)(I_Y + h(\mu)(\widetilde{y} \otimes g)) \\ &= \varphi(T) - \mu(\widetilde{y} \otimes g)\varphi(T) + h(\mu)\varphi(T)(\widetilde{y} \otimes g) \\ &\quad - \mu h(\mu)(\widetilde{y} \otimes g)\varphi(T)(\widetilde{y} \otimes g). \end{aligned}$$

Also,

$$\begin{aligned}\overline{(W_\mu - \lambda I_Y)(Y)} &= \overline{A_\mu(\varphi(T) - \lambda I_Y)A_\mu^{-1}(Y)} = \overline{A_\mu(\varphi(T) - \lambda I_Y)(Y)} \\ &= A_\mu(\overline{(\varphi(T) - \lambda I_Y)(Y)}) \subseteq A_\mu(\ker f) \\ &= A_\mu(Z \oplus (\mathbb{C}y_0)).\end{aligned}$$

Suppose, for a contradiction, that for some $0 < |\mu| < 1/(\|\widetilde{y}\| \, \|g\|)$ we have that $y_0 \in \overline{(W_\mu - \lambda I_Y)(Y)}$. Then $y_0 \in A_\mu(Z \oplus (\mathbb{C}y_0))$. Since $g = 0$ on Z, then A_μ is the identity on Z, and therefore $y_0 = y + \alpha A_\mu(y_0)$ for some $y \in Z$ and $\alpha \in \mathbb{C}$. Thus $y_0 = y + \alpha(y_0 - \mu g(y_0)\widetilde{y})$, which gives

$$\alpha\mu g(y_0)\widetilde{y} = (\alpha - 1)y_0 + y \in \ker f.$$

Since μ and $g(y_0)$ are nonzero and $\widetilde{y} \notin \ker f$, we then have $\alpha = 0$. This gives $y_0 = y \in Z$, arriving at the desired contradiction.

Therefore, $y_0 \notin \overline{(W_\mu - \lambda I_Y)(Y)}$ for $0 < |\mu| < 1/(\|\widetilde{y}\| \, \|g\|)$, and in particular $y_0 \notin (W_\mu - \lambda I_Y)(Y)$ for all such μ. Then (1.1) gives

$$|\lambda| \le \Gamma_{W_\mu}(y_0) \qquad (0 < |\mu| < 1/(\|\widetilde{y}\| \, \|g\|)).$$

Now since φ is surjective, we find $A, B, C \in \mathcal{L}(X)$ such that $\varphi(A) = (\widetilde{y} \otimes g)\varphi(T)$, $\varphi(B) = \varphi(T)(\widetilde{y} \otimes g)$ and $\varphi(C) = (\widetilde{y} \otimes g)\varphi(T)(\widetilde{y} \otimes g)$. For $0 < |\mu| < 1/(\|\widetilde{y}\| \, \|g\|)$, since (2.3) holds for $T - \mu A + h(\mu)B - \mu h(\mu)C \in \mathcal{L}(X)$ we have

$$\begin{aligned}\Gamma_{W_\mu}(y_0) &= \Gamma_{\varphi(T - \mu A + h(\mu)B - \mu h(\mu)C)}(y_0) \\ &\le r(T - \mu A + h(\mu)B - \mu h(\mu)C).\end{aligned}$$

Therefore

$$|\lambda| \le r(T - \mu A + h(\mu)B - \mu h(\mu)C) \qquad (0 < |\mu| < 1/(\|\widetilde{y}\| \, \|g\|)).$$

Since $h(0) = 0$, taking the limsup with $\mu \to 0$ in the last inequality and using the fact that the spectral radius is upper semicontinuous we obtain that $|\lambda| \le r(T)$. ☐

Using now Theorem 3.2 and Lemma 3.1, by [2, Theorem 5.5.2] we obtain that the statement of Theorem 2.2 holds. Then Corollary 2.3 follows directly from Theorem 2.2.

Acknowledgment

The author wishes to express his thanks to the referee for carefully reading the paper and for giving valuable suggestions which greatly improved the original version of the article.

References

[1] Pietro Aiena, *Fredholm and local spectral theory, with applications to multipliers*, Kluwer Academic Publishers, Dordrecht, 2004. MR2070395

[2] Bernard Aupetit, *A primer on spectral theory*, Universitext, Springer-Verlag, New York, 1991. MR1083349

[3] Abdellatif Bourhim and Javad Mashreghi, *A survey on preservers of spectra and local spectra*, Invariant subspaces of the shift operator, Contemp. Math., vol. 638, Amer. Math. Soc., Providence, RI, 2015, pp. 45–98, DOI 10.1090/conm/638/12810. MR3309349

[4] Abdellatif Bourhim and Vivien G. Miller, *Linear maps on $M_n(\mathbb{C})$ preserving the local spectral radius*, Studia Math. **188** (2008), no. 1, 67–75, DOI 10.4064/sm188-1-4. MR2430550

[5] Abdellatif Bourhim and Thomas Ransford, *Additive maps preserving local spectrum*, Integral Equations Operator Theory **55** (2006), no. 3, 377–385, DOI 10.1007/s00020-005-1392-2. MR2244195

[6] Janko Bračič and Vladimír Müller, *Local spectrum and local spectral radius of an operator at a fixed vector*, Studia Math. **194** (2009), no. 2, 155–162, DOI 10.4064/sm194-2-3. MR2534182

[7] Matej Brešar and Peter Šemrl, *Linear maps preserving the spectral radius*, J. Funct. Anal. **142** (1996), no. 2, 360–368, DOI 10.1006/jfan.1996.0153. MR1423038

[8] Constantin Costara, *Automatic continuity for linear surjective mappings decreasing the local spectral radius at some fixed vector*, Arch. Math. (Basel) **95** (2010), no. 6, 567–573, DOI 10.1007/s00013-010-0191-4. MR2745466

[9] Manuel González and Mostafa Mbekhta, *Linear maps on $M_n(\mathbb{C})$ preserving the local spectrum*, Linear Algebra Appl. **427** (2007), no. 2-3, 176–182, DOI 10.1016/j.laa.2007.07.005. MR2351350

[10] Kjeld B. Laursen and Michael M. Neumann, *An introduction to local spectral theory*, London Mathematical Society Monographs. New Series, vol. 20, The Clarendon Press, Oxford University Press, New York, 2000. MR1747914

Faculty of Mathematics and Informatics, Ovidius University of Constanţa, Mamaia Boul. 124, 900527, Constanţa, Romania

E-mail address: cdcostara@univ-ovidius.ro