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ON NON-ABELIAN LUBIN-TATE THEORY AND ANALYTIC COHOMOLOGY

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Abstract. We prove that the p-adic local Langlands correspondence for $\mathrm{GL}_2(\mathbb{Q}_p)$ appears in the étale cohomology of the Lubin-Tate tower at infinity. We use global methods using recent results of Emerton on the local-global compatibility, and hence our proof applies to local Galois representations which come via a restriction from global pro-modular Galois representations.

1. Introduction

The p-adic Langlands program, started by Christophe Breuil and developed largely by Laurent Berger, Pierre Colmez, Gabriel Dospinescu, Matthew Emerton, Mark Kisin and Vytautas Paskunas, has as a goal to establish a correspondence between p-adic Galois representations and representations of p-adic reductive groups on p-adic Banach spaces. It has (and will have) many applications, for example the Fontaine-Mazur conjecture (see [Em2]). Unfortunately, at the moment the p-adic correspondence is constructed (mostly by Colmez) only for $\mathrm{GL}_2(\mathbb{Q}_p)$, and it seems hard to generalise it to other groups because of many algebraic obstacles: in particular, there are many more representations of GL_n (or even $\mathrm{GL}_2(F)$ for a finite extension $F/\mathbb{Q}_p$) than Galois representations, and it is hard to decide how the correspondence should be defined by purely representation-theoretic means as shown by Breuil and Paskunas in [BP].

Hence it seems natural to try to find the correspondence in some appropiate cohomology groups as was done for the classical Langlands correspondence. We are interested in the p-adic completed cohomology of Shimura varieties and Rapoport-Zink spaces, which are natural objects to consider in the context of the Langlands program. This paper might be seen as a sequel to [Cho], where we have studied the mod p étale cohomology of the Lubin-Tate tower. Here we turn to the study of the p-adic completed cohomology. We refer the reader to [Cho] for a more detailed introduction to this circle of ideas.

In this paper we show a result analogous to the one obtained in [Cho], namely that the p-adic local Langlands correspondence for $\mathrm{GL}_2(\mathbb{Q}_p)$ appears in the étale

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cohomology of the Lubin-Tate tower at infinity $\mathcal{M}_{LT,\infty}$. Let E be a finite extension of $\mathbb{Q}_p$. Our main theorem is:

Theorem. *Let* $\rho : G_{\mathbb{Q}} = \mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q}) \to \mathrm{GL}_2(E)$ *be a pro-modular representation. Assume that* $\bar{\rho}_p = \bar{\rho}_{|G_{\mathbb{Q}_p}}$ *is absolutely irreducible. Then we have a* $\mathrm{GL}_2(\mathbb{Q}_p) \times G_{\mathbb{Q}_p}$*-equivariant injection*

$$B(\rho_p) \otimes_E \rho_p \hookrightarrow H^1(\mathcal{M}_{LT,\infty}, E)$$

where $B(\rho_p)$ *is the admissible* $\mathrm{GL}_2(\mathbb{Q}_p)$*-representation attached to* ρ_p *by the* p*-adic local Langlands correspondence.*

The methods we use are partly those of [Cho] (localisation at a supersingular representation, use of the local-global compatibility of Emerton), though we approach them differently by working in the setting of adic spaces (we have worked with Berkovich spaces in [Cho]). This gives us more freedom as we can work directly at the infinite level (modular curves at the infinite level, Lubin-Tate tower at the infinite level) thanks to the work of Scholze on perfectoid spaces ([Sch1], [Sch3], [SW]). In this way, we do no longer need to pass to the limit in the cohomology, as working at the infinite level is the same as working with completed cohomology (see Chapter IV of [Sch3] for torsion coefficients). We prove our main result for local Galois representations ρ_p which are restrictions of some global pro-modular (a notion from [Em2]) representations ρ and such that the mod p reduction $\bar{\rho}_p$ is absolutely irreducible. We need these assumptions in order to be able to use the main result of [Em2].

After this article was written, more work on the subject appeared. In [Sch5] Scholze proved that cohomology groups of the Lubin-Tate tower for certain sheaves (related to representations of GL_n) are admissible. In our work we consider only trivial coefficients which give non-admissible cohomology groups. Compare with Remark 1.2 in [Sch5]. Other important work was done by Dospinescu and Le Bras in [DB]. They have computed H^0 of the Drinfeld tower in terms of the p-adic local Langlands correspondence.

2. Modular curves at infinity

In this section we review the geometric background which we use. We describe modular curves (and their compactifications) at the infinite level and we deal with the ordinary locus and the supersingular locus. We will use the language of adic spaces for which the reader should consult [Hu2] and [Sch1].

We let E be a finite extension of $\mathbb{Q}_p$ with the ring of integers $\mathcal{O}$ and the residue field $k = \mathcal{O}/\varpi$ where ϖ is a uniformiser. This is our coefficient field.

2.1. **Geometry of modular curves.** We denote open modular curves over $\mathbb{C}$ for an open compact subset $K \subset \mathrm{GL}_2(\mathbb{A}_f)$ by

$$Y(K) = \mathrm{GL}_2(\mathbb{Q})\backslash(\mathbb{C}\backslash\mathbb{R}) \times \mathrm{GL}_2(\mathbb{A}_f)/K.$$

There is a canonical algebraic model of it over $\mathbb{Q}$. We fix some complete and algebraically closed extension C of $\mathbb{Q}_p$. Let $\mathcal{O}_C$ be the ring of integers of C. We consider modular curves as adic spaces over $\mathrm{Spa}(C, \mathcal{O}_C)$, which we may do after base-changing each $Y(K)$.

We let $X(K)$ be the compactification of $Y(K)$, which we also consider as an adic space over $\mathrm{Spa}(C, \mathcal{O}_C)$. We will work with modular curves at the infinite level. We recall Scholze's results. We use $\sim$ in the sense of Definition 2.4.1 in [SW].

Theorem 2.1. *For any sufficiently small level $K^p \subset \mathrm{GL}_2(\mathbb{A}_f^p)$ there exist adic spaces $Y(K^p)$ and $X(K^p)$ over* $\mathrm{Spa}(C, \mathcal{O}_C)$ *such that*

$$Y(K^p) \sim \varprojlim_{K_p} Y(K_pK^p),$$

$$X(K^p) \sim \varprojlim_{K_p} X(K_pK^p)$$

where K_p runs over open compact subgroups of $\mathrm{GL}_2(\mathbb{Q}_p)$.

Proof. See Theorem III.1.2 in [Sch3]. □

In what follows we will write $Y = Y(K^p)$ and $X = X(K^p)$, having fixed one tame level K^p throughout the text.

For the maximal compact open subgroup $\mathrm{GL}_2(\mathbb{Z}_p)$ we can define the supersingular locus $Y(\mathrm{GL}_2(\mathbb{Z}_p)K^p)_{\mathrm{ss}}$ (respectively, the ordinary locus $Y(\mathrm{GL}_2(\mathbb{Z}_p)K^p)_{\mathrm{ord}}$) as the inverse image under the reduction of the set of supersingular points (resp. closure of the inverse image of the ordinary locus) in the special fiber of $Y(\mathrm{GL}_2(\mathbb{Z}_p)K^p)$. Then for any compact open subgroup $K_p \subset \mathrm{GL}_2(\mathbb{Z}_p)$, we define $Y(K_pK^p)_{\mathrm{ss}}$ (resp. $Y(K_pK^p)_{\mathrm{ord}}$) as the pullback of $Y(\mathrm{GL}_2(\mathbb{Z}_p)K^p)_{\mathrm{ss}}$ (resp. $Y(\mathrm{GL}_2(\mathbb{Z}_p)K^p)_{\mathrm{ord}}$). Hence $Y(K_pK^p)_{\mathrm{ord}}$ is the complement of $Y(K_pK^p)_{\mathrm{ss}}$ and hence a closed subspace of $Y(K_pK^p)$. We define similarly the supersingular locus $X(K_pK^p)_{\mathrm{ss}}$ and the ordinary locus $X(K_pK^p)_{\mathrm{ord}}$ of $X(K_pK^p)$. Using the pullback from the finite level, we define also $X_{\mathrm{ss}}, Y_{\mathrm{ss}}, X_{\mathrm{ord}}, Y_{\mathrm{ord}}$ at the infinite level. The reader may consult the discussion in [Sch3] which appears after Theorem III.1.2.

Theorem 2.2. *There exist adic spaces Y_{ss}, Y_{ord} and X_{ss}, X_{ord} over* $\mathrm{Spa}(C, \mathcal{O}_C)$ *such that*

$$Y_{\mathrm{ss}} \sim \varprojlim_{K_p} Y(K_pK^p)_{\mathrm{ss}},$$

$$Y_{\mathrm{ord}} \sim \varprojlim_{K_p} Y(K_pK^p)_{\mathrm{ord}}$$

and similarly for X_{ss} and X_{ord}. Here K_p runs over open compact subgroups of $\mathrm{GL}_2(\mathbb{Q}_p)$.

Proof. Follows from Proposition 2.4.3 in [SW]. □

One of the main results of [Sch3] (Theorem III.1.2) is the construction of the Hodge-Tate period map π_{HT} which is a $\mathrm{GL}_2(\mathbb{Q}_p)$-equivariant morphism

$$\pi_{\mathrm{HT}} : X \to (\mathbb{P}^1)^{ad}$$

where $(\mathbb{P}^1)^{ad}$ is the adic projective line over $\mathrm{Spa}(C, \mathcal{O}_C)$. This morphism commutes with Hecke operators away from p for the trivial action of these Hecke operators on $(\mathbb{P}^1)^{ad}$. Moreover, the decomposition of X into the supersingular and the ordinary locus can be seen at the flag variety level. Namely, we have (see the discussion after Theorem III.1.2 in [Sch3])

$$X_{\mathrm{ord}} = \pi_{\mathrm{HT}}^{-1}(\mathbb{P}^1(\mathbb{Q}_p)),$$

$$X_{\mathrm{ss}} = \pi_{\mathrm{HT}}^{-1}((\mathbb{P}^1)^{ad} \backslash \mathbb{P}^1(\mathbb{Q}_p)).$$

We let

$$j : X_{\mathrm{ss}} \hookrightarrow X$$

denote the open immersion and we put

$$i : X_{\mathrm{ord}} \to X.$$

For any injective étale sheaf I on X we have an exact sequence of global sections

$$0 \to \Gamma_{X_{\mathrm{ord}}}(X, I) \to \Gamma(X, I) \to \Gamma(X_{\mathrm{ss}}, j^* I) \to 0$$

which gives rise to the exact sequence of étale cohomology for any étale sheaf F on X (take an injective resolution $I^\bullet$ of F and apply the above exact sequence to it):

$$\cdots \to H^0(X_{\mathrm{ss}}, j^* F) \to H^1_{X_{\mathrm{ord}}}(X, F) \to H^1(X, F) \to H^1(X_{\mathrm{ss}}, j^* F) \to \cdots .$$

By specialising F to a constant sheaf $\mathcal{O}/\varpi^s\mathcal{O}$ $(s > 0)$ we get an exact sequence

$$\begin{aligned}\cdots \to H^0(X_{\mathrm{ss}}, \mathcal{O}/\varpi^s\mathcal{O}) \to H^1_{X_{\mathrm{ord}}}(X, \mathcal{O}/\varpi^s\mathcal{O}) \to H^1(X, \mathcal{O}/\varpi^s\mathcal{O})\\ \to H^1(X_{\mathrm{ss}}, \mathcal{O}/\varpi^s\mathcal{O}) \to \cdots .\end{aligned}$$

We can obtain an analogous exact sequence for analytic cohomology, which we review later. In what follows we will be interested in the p-adic completed cohomology, introduced by Emerton in [Em1]. We define

$$H^i(X, E) = \left(\varprojlim_s H^i_{et}(X, \mathcal{O}/\varpi^s\mathcal{O}) \right) \otimes_{\mathcal{O}} E.$$

Using the fact that $X \sim \varprojlim_{K_p} X(K_p K^p)$ and Theorem 7.17 in [Sch1], we have

$$H^i(X, E) = \left(\varprojlim_s \varinjlim_{K_p} H^i_{et}(X(K_p K^p), \mathcal{O}/\varpi^s\mathcal{O}) \right) \otimes_{\mathcal{O}} E,$$

which is precisely the p-adic completed cohomology of Emerton. We use similar definitions for X_{ss} and X_{ord}.

2.2. **Ordinary locus.** We recall the decomposition of the ordinary locus, which implies that representations arising from the cohomology are induced from a Borel subgroup. This is a classical and well-known result, but we shall give it a short proof using recent results of Scholze and the fact that we are working at the infinite level. We have given a different proof in section 2.2 of [Cho].

Proposition 2.3. *The étale (and also analytic) cohomology of X_{ord} is induced from a Borel subgroup $B(\mathbb{Q}_p)$ of upper-triangular matrices in $\mathrm{GL}_2(\mathbb{Q}_p)$,*

$$H^i_{X_{\mathrm{ord}}}(X, F) = \mathrm{Ind}_{B(\mathbb{Q}_p)}^{\mathrm{GL}_2(\mathbb{Q}_p)} W(F),$$

where $F = \mathcal{O}/\varpi^s\mathcal{O}$ is an étale constant sheaf on X_{ord} and $W(F)$ is a certain cohomology space defined below in the proof which depends on F and admits an action of $B(\mathbb{Q}_p)$. The induction appearing above is the smooth induction.

Proof. Recall that $X_{\mathrm{ord}} = \pi_{\mathrm{HT}}^{-1}(\mathbb{P}^1(\mathbb{Q}_p))$, where π_{HT} is the Hodge-Tate period map. Let $\infty = \left(\begin{smallmatrix}1\\0\end{smallmatrix}\right) \in \mathbb{P}^1(\mathbb{Q}_p)$. The stabilizer of ∞ is equal to the Borel subgroup $B(\mathbb{Q}_p)$ of upper-triangular matrices in $\mathrm{GL}_2(\mathbb{Q}_p)$. We have

$$\begin{aligned} H^i_{X_{\mathrm{ord}}}(X, F) &= H^i(X_{\mathrm{ord}}, i^! F) = H^0(\mathbb{P}^1(\mathbb{Q}_p), R^i \pi_{\mathrm{HT},*}(i^! F)) \\ &= \mathrm{Ind}_{B(\mathbb{Q}_p)}^{\mathrm{GL}_2(\mathbb{Q}_p)} H^0(\{\infty\}, R^i \pi_{\mathrm{HT},*}(i^! F)) \end{aligned}$$

where the second isomorphism follows from the continuity of π_{HT}. Those are all smooth spaces, because $H^i(X_{\mathrm{ord}}, i^! F)$ is smooth (by Theorem 2.2 and recalling that $F = \mathcal{O}/\varpi^s\mathcal{O}$). $\square$

2.3. **Supersingular locus.** Let us denote by $\mathcal{M}_{LT,K_p}$ the Lubin-Tate space for $\mathrm{GL}_2(\mathbb{Q}_p)$ at the level K_p, where K_p is a compact open subgroup of $\mathrm{GL}_2(\mathbb{Q}_p)$. See section 6 of [SW] for a definition. We just recall that this is a deformation space for p-divisible groups with additional data and it is a local analogue of modular curves. We view it as an adic space over $\mathrm{Spa}(C, \mathcal{O}_C)$.

Once again, we would like to pass to the limit and work with the space at infinity.

Theorem 2.4. *There exists a perfectoid space $\mathcal{M}_{LT,\infty}$ over $\mathrm{Spa}(C, \mathcal{O}_C)$ such that*

$$\mathcal{M}_{LT,\infty} \sim \varprojlim_{K_p} \mathcal{M}_{LT,K_p}$$

where K_p runs over compact open subgroups of $\mathrm{GL}_2(\mathbb{Q}_p)$.

Proof. This is Theorem 6.3.4 from [SW]. One defines $\mathcal{M}_{LT,\infty}$ as a deformation functor of p-divisible groups with a trivialization of Tate modules. $\square$

To compare X and $\mathcal{M}_{LT,\infty}$ (hence their cohomology groups) we use the p-adic uniformisation of Rapoport-Zink at the infinite level. Let us denote by D the quaternion algebra over $\mathbb{Q}$ which is ramified exactly at p and ∞. The p-adic uniformisation of Rapoport-Zink states

Proposition 2.5. *We have an isomorphism of adic spaces*

$$X_{\mathrm{ss}} \sim \varprojlim_{K_p} D^{\times}(\mathbb{Q})\backslash \left(\mathcal{M}_{LT,K_p} \times \mathrm{GL}_2(\mathbb{A}_f^p)\right)/K_pK^p.$$

This isomorphism is equivariant with respect to the action of the Hecke algebra of level K^p.

Proof. The uniformisation at finite level is proved in [RZ]. We adify their construction and pass to the limit using Theorem 2.2. $\square$

3. On admissible representations

Having recalled the geometric results, we now pass to the results about representations of $\mathrm{GL}_2(\mathbb{Q}_p)$. We review and prove some facts about Banach admissible representations. Then we recall recent results of Paskunas which allow us to consider the localisation functor.

3.1. **General facts and definitions.** We start with general facts about admissible representations. In our definitions, we will follow [Em4]. As before, let E be a finite extension of $\mathbb{Q}_p$ with ring of integers $\mathcal{O}$, a uniformiser ϖ and the residue field k. Let $C(\mathcal{O})$ denote the category of complete Noetherian local $\mathcal{O}$-algebras having finite residue fields. Let us consider $A \in C(\mathcal{O})$. We let G be any connected reductive group over $\mathbb{Q}_p$. The following definitions make sense in more generality, in particular for parabolic subgroups of G, which we use below in Lemma 3.6. However, we work with G connected reductive to be in the same setting as [Em4].

Definition 3.1. Let V be a representation of G over A. A vector $v \in V$ is smooth if v is fixed by some open subgroup of G and v is annihilated by some power $\mathfrak{m}^i$ of the maximal ideal of A. Let V_{sm} denote the subset of smooth vectors of V. We say that a G-representation V over A is smooth if $V = V_{sm}$.

A smooth G-representation V over A is admissible if $V^H[\mathfrak{m}^i]$ (the $\mathfrak{m}^i$-torsion part of the subspace of H-fixed vectors in V) is finitely generated over A for every open compact subgroup H of G and every $i \geq 0$.

Definition 3.2. We say that a G-representation V over A is ϖ-adically continuous if V is ϖ-adically separated and complete, $V[\varpi^\infty]$ is of bounded exponent, and $V/\varpi^i V$ is a smooth G-representation for any $i \geq 0$.

Definition 3.3. A ϖ-adically admissible representation of G over A is a ϖ-adically continuous representation V of G over A such that the induced G-representation on $(V/\varpi V)[\mathfrak{m}]$ is smooth admissible over $A/\mathfrak{m}$.

This definition implies that for every $i \geq 0$, the G-representation $V/\varpi^i V$ is smooth admissible. See Remark 2.4.8 in [Em4].

Definition 3.4. We call a G-representation V over E Banach admissible if there exists a G-invariant lattice $V^\circ \subset V$ over $\mathcal{O}$ such that V° is ϖ-adically admissible as a representation of G over $\mathcal{O}$.

Proposition 3.5. *The category of ϖ-adically admissible representations of G over A is abelian and, moreover, a Serre subcategory of the category of ϖ-adically continuous representations.*

Proof. The category is anti-equivalent to the category of finitely generated augmented modules over certain completed group rings. See Proposition 2.4.11 in [Em4]. □

Now, we will prove an analogue of Lemma 13.2.3 from [Bo] in the $l = p$ setting. We will later apply this lemma to the cohomology of the ordinary locus to force its vanishing after localisation at a supersingular representation of $\mathrm{GL}_2(\mathbb{Q}_p)$. We have proved it already in the mod p setting as Lemma 3.3 in [Cho].

Lemma 3.6. *For any smooth admissible representation (π, V) of the parabolic subgroup $P \subset G$ over A, the unipotent radical U of P acts trivially on V.*

Proof. Let L be a Levi subgroup of P so that $P = LU$. Let $v \in V$ and let $K_P = K_L K_U$ be a compact open subgroup of P such that $v \in V^{K_P}$. We choose an element z in the centre of L such that

$$z^{-n} K_P z^n \subset \cdots \subset z^{-1} K_P z \subset K_P \subset z K_P z^{-1} \subset \cdots \subset z^n K_P z^{-n} \subset \cdots$$

and $\bigcup_{n \geq 0} z^n K_P z^{-n} = K_L U$. For every n and m, modules $V^{z^{-n} K_P z^n}[\mathfrak{m}^i]$ and $V^{z^{-m} K_P z^m}[\mathfrak{m}^i]$ are of the same length for every $i \geq 0$, as they are isomorphic via $\pi(z^{n-m})$. We naturally have an inclusion $V^{z^{-n} K_P z^n}[\mathfrak{m}^i] \subset V^{z^{-m} K_P z^m}[\mathfrak{m}^i]$ and hence we get an equality $V^{z^{-n} K_P z^n}[\mathfrak{m}^i] = V^{z^{-m} K_P z^m}[\mathfrak{m}^i]$. By smoothness, there exists i such that $v \in V[\mathfrak{m}^i]$. Thus we have $v \in V^{K_P}[\mathfrak{m}^i] = V^{z^{-n} K_P z^n}[\mathfrak{m}^i] = V^{K_L U}[\mathfrak{m}^i]$, which is contained in $V^U[\mathfrak{m}^i]$. □

Lemma 3.7. *For any ϖ-adically admissible representation (π, V) of the parabolic subgroup $P \subset G$ over A, the unipotent radical U of P acts trivially on V.*

Proof. By the remark above, each $V/\varpi^i V$ is admissible, and hence the preceding lemma applies, so that U acts trivially on each $V/\varpi^i V$. But $V = \varprojlim_i V/\varpi^i V$; hence U acts trivially on V. □

Later on, we will need the following result.

Lemma 3.8. *Let $V = \mathrm{Ind}_P^G W$ be a parabolic induction. If V is a ϖ-adically admissible representation of G over A, then W is a ϖ-adically admissible representation of P over A.*

Proof. This follows from Theorem 4.4.6 in [Em4]. □

3.2. **Localisation functor.** Let π be a supersingular representation of $\mathrm{GL}_2(\mathbb{Q}_p)$ over k. Recall that supersingular representations correspond to irreducible two-dimensional Galois representations under the local Langlands correspondence modulo p. See [Be].

In [Pa], Paskunas has proved the following result (Proposition 5.32).

Proposition 3.9. *We have a decomposition*

$$\mathrm{Rep}^{adm}_{\mathrm{GL}_2(\mathbb{Q}_p),\xi}(\mathcal{O}/\varpi^s\mathcal{O}) = \mathrm{Rep}^{adm}_{\mathrm{GL}_2(\mathbb{Q}_p),\xi}(\mathcal{O}/\varpi^s\mathcal{O})_{(\pi)} \oplus \mathrm{Rep}^{adm}_{\mathrm{GL}_2(\mathbb{Q}_p),\xi}(\mathcal{O}/\varpi^s\mathcal{O})^{(\pi)}$$

where $\mathrm{Rep}^{adm}_{\mathrm{GL}_2(\mathbb{Q}_p),\xi}(\mathcal{O}/\varpi^s\mathcal{O})$ *is the (abelian) category of smooth admissible* $\mathcal{O}/\varpi^s\mathcal{O}$*-representations admitting a central character* ξ *and* $\mathrm{Rep}^{adm}_{\mathrm{GL}_2(\mathbb{Q}_p),\xi}(\mathcal{O}/\varpi^s\mathcal{O})_{(\pi)}$ *(resp.* $\mathrm{Rep}^{adm}_{\mathrm{GL}_2(\mathbb{Q}_p),\xi}(\mathcal{O}/\varpi^s\mathcal{O})^{(\pi)}$*) is the subcategory of it consisting of representations* Π *such that all irreducible subquotients of* Π *are (resp. are not) isomorphic to* π*.*

We denote the projection

$$\mathrm{Rep}^{adm}_{\mathrm{GL}_2(\mathbb{Q}_p),\xi}(\mathcal{O}/\varpi^s\mathcal{O}) \mapsto \mathrm{Rep}^{adm}_{\mathrm{GL}_2(\mathbb{Q}_p),\xi}(\mathcal{O}/\varpi^s\mathcal{O})_{(\pi)}$$

by

$$V \mapsto V_{(\pi)}$$

and we refer to it as the localisation functor with respect to π. The existence of a central character follows from the work [DS] for irreducible representations. In what follows, we will ignore the central character ξ in our notation, though whenever we localise we mean that we first localise the representation at ξ and then we project as above.

4. p-adic Langlands correspondence and analytic cohomology

In this section we show that the p-adic local Langlands correspondence for $\mathrm{GL}_2(\mathbb{Q}_p)$ appears in the étale cohomology of the Lubin-Tate tower at infinity.

4.1. **p-adic Langlands correspondence.** For this section we refer the reader to [Be] (for the Colmez functor) and [Pa] (for equivalence of categories). We recall that Colmez has constructed a covariant exact functor $\mathbb{V}$,

$$\mathbb{V} : \mathrm{Rep}_{\mathcal{O}}(\mathrm{GL}_2(\mathbb{Q}_p)) \to \mathrm{Rep}_{\mathcal{O}}(G_{\mathbb{Q}_p}),$$

which sends $\mathcal{O}$-representations of $\mathrm{GL}_2(\mathbb{Q}_p)$ to $\mathcal{O}$-representations of $G_{\mathbb{Q}_p} = \mathrm{Gal}(\bar{\mathbb{Q}}_p/\mathbb{Q}_p)$. Moreover this functor is compatible with deformations and induces an equivalence of categories when restricted to appropiate subrepresentations. We call the inverse of this functor the p-adic local Langlands correspondence and we denote it by $B(\cdot)$. For our applications we will only need the fact that for p-adic continuous representations $\rho : G_{\mathbb{Q}_p} \to \mathrm{GL}_2(E)$, $B(\rho)$ is a Banach admissible E-representation. Furthermore, when ρ is irreducible, $B(\rho)$ is topologically irreducible.

Let $\bar{\rho} : G_{\mathbb{Q}_p} \to \mathrm{GL}_2(k)$ be the reduction of ρ which we assume to be irreducible. Let π be the supersingular representation of $\mathrm{GL}_2(\mathbb{Q}_p)$ over k which corresponds to $\bar{\rho}$ by the mod p local Langlands correspondence, that is, $\mathbb{V}(\pi) = \bar{\rho}$. Then one knows that $B(\rho)$ is an object of the category $\mathrm{Rep}^{adm}_{\mathrm{GL}_2(\mathbb{Q}_p)}(E)_{(\pi)}$ defined above.

4.2. **Étale cohomology.** We recall the results of Emerton on the p-adic completed cohomology and then we prove that certain p-adic Banach representations appear in the étale cohomology of $\mathcal{M}_{LT,\infty}$. From now on we work in the global setting. Let $\rho : G_{\mathbb{Q}} = \mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q}) \to \mathrm{GL}_2(E)$ be a continuous Galois representation. We assume that it is unramified outside some finite set $\Sigma = \Sigma_0 \cup \{p\}$. Moreover we assume that its reduction $\bar{\rho}$ is modular (that is, isomorphic to the reduction of a Galois representation associated to some automorphic representation on $\mathrm{GL}_2(\mathbb{Q})$) and $\bar{\rho}_p = \bar{\rho}_{|G_{\mathbb{Q}_p}}$ is absolutely irreducible.

Let us recall that we have introduced spaces X, Y depending on the tame level K^p. We now assume that K^p is unramified outside Σ. We shall factor K^p as $K^p = K_{\Sigma_0}K^{\Sigma_0}$. Let $\mathbb{T}_\Sigma = \mathcal{O}[T_l, S_l]_{l \notin \Sigma}$ be the commutative $\mathcal{O}$-algebra with T_l, S_l formal variables indexed by $l \notin \Sigma$. This is a standard Hecke algebra which acts on modular curves by correspondences.

To the modular Galois representation $\bar{\rho} : G_{\mathbb{Q}} \to \mathrm{GL}_2(E)$ we can associate the maximal Hecke ideal $\mathfrak{m}$ of $\mathbb{T}_\Sigma$ which is generated by ϖ (uniformiser of $\mathcal{O}$) and elements $T_l + a_l$ and $lS_l - b_l$, where l is a place of $\mathbb{Q}$ which does not belong to Σ, $X^2 + \bar{a}_l X^1 + \bar{b}_l$ is the characteristic polynomial of $\bar{\rho}(\mathrm{Frob}_l)$ and a_l, b_l are any lifts of $\bar{a}_l, \bar{b}_l$ to $\mathcal{O}$.

We let $\pi_{\Sigma_0}(\rho) = \bigotimes_{l \in \Sigma_0} \pi_l(\rho_l)$ be the tensor product of E-representations of $\mathrm{GL}_2(\mathbb{Q}_l)$ ($l \in \Sigma_0$) associated to $\rho_l = \rho_{|G_{\mathbb{Q}_l}}$ by the generic version of the l-adic local Langlands correspondence (see [EH]).

We assume that ρ is pro-modular in the sense of Emerton (see [Em2]). Let $\mathfrak{p}$ be the prime ideal of $\mathbb{T}_\Sigma$ associated to ρ (similarly as we have associated $\mathfrak{m}$ to $\bar{\rho}$). We have an obvious inclusion $\mathfrak{p} \subset \mathfrak{m}$. We remark that pro-modularity is a weaker condition than modularity and it can be seen as saying that ρ is a Galois representation associated to some p-adic Hecke eigensystem coming from the completed Hecke algebra (the projective limit over finite level Hecke algebras). Recall that we have assumed that $\bar{\rho}_p = \bar{\rho}_{|G_{\mathbb{Q}_p}}$ is absolutely irreducible. This permits us to state the main result of [Em2] as

Theorem 4.1. *Let $\rho : G_{\mathbb{Q}} \to \mathrm{GL}_2(E)$ be a continuous Galois representation which is pro-modular and such that $\bar{\rho}_p$ is absolutely irreducible. Then we have a $G_{\mathbb{Q}} \times \mathrm{GL}_2(\mathbb{Q}_p) \times \prod_{l \in \Sigma_0} \mathrm{GL}_2(\mathbb{Q}_l)$-equivariant isomorphism of Banach admissible E-representations:*

$$H^1(Y, E)[\mathfrak{p}] \simeq \rho \otimes_E B(\rho_p) \otimes_E \pi_{\Sigma_0}(\rho)^{K_{\Sigma_0}}.$$

We recall that the cohomology group on the left is the p-adic completed cohomology of Emerton

$$H^1(Y, E) = \left(\varprojlim_s \varinjlim_{K_p} H^1_{et}(Y(K^p K_p), \mathcal{O}/\varpi^s \mathcal{O}) \right) \otimes_{\mathcal{O}} E$$

where K_p runs over compact open subgroups of $\mathrm{GL}_2(\mathbb{Q}_p)$.

Let us remark that the Galois action of $G_{\mathbb{Q}_p}$ arises on Y, X, $\mathcal{M}_{LT,\infty}$ (which we treat as adic spaces over $\mathrm{Spa}(C, \mathcal{O}_C)$) from the Galois action on the corresponding model over $\bar{\mathbb{Q}}_p$.

We also have a similar theorem for the compactification.

Theorem 4.2. *With assumptions as in the theorem above, we have an isomorphism of Banach admissible K-representations:*

$$H^1(X,E)_{\mathfrak{m}} \simeq H^1(Y,E)_{\mathfrak{m}}.$$

In particular,

$$H^1(X,E)[\mathfrak{p}] \simeq \rho \otimes_E B(\rho_p) \otimes_E \pi_{\Sigma_0}(\rho)^{K_{\Sigma_0}}.$$

Proof. We have assumed that $\bar{\rho}_p$ is absolutely irreducible and hence $\bar{\rho}$ is absolutely irreducible, which implies that $\mathfrak{m}$ is a non-Eisenstein ideal. Now the theorem follows as in the proof of Proposition 7.7.13 of [Em3]. □

We now come back to the exact sequence which we obtained earlier:

$$\cdots \to H^0(X_{\mathrm{ss}}, \mathcal{O}/\varpi^s\mathcal{O}) \to H^1_{X_{\mathrm{ord}}}(X, \mathcal{O}/\varpi^s\mathcal{O}) \to H^1(X, \mathcal{O}/\varpi^s\mathcal{O}) \\ \to H^1(X_{\mathrm{ss}}, \mathcal{O}/\varpi^s\mathcal{O}) \to \cdots .$$

By Theorem 2.1.5 of [Em1], we get that $H^1(X,\mathcal{O}/\varpi^s\mathcal{O})$ is a smooth admissible $\mathcal{O}/\varpi^s\mathcal{O}$-representation of $\mathrm{GL}_2(\mathbb{Q}_p)$. Moreover, $H^0(X_{\mathrm{ss}},\mathcal{O}/\varpi^s\mathcal{O})$ is also a smooth admissible $\mathcal{O}/\varpi^s\mathcal{O}$-representation of $\mathrm{GL}_2(\mathbb{Q}_p)$ as $X_{\mathrm{ss}}(K_pK^p)$ has only a finite number of connected components for each K_p and K^p. The category of smooth admissible $\mathcal{O}/\varpi^s\mathcal{O}$-representations is a Serre subcategory of the category of smooth (not necessarily admissible) $\mathcal{O}/\varpi^s\mathcal{O}$-representations (see Proposition 2.4.11 of [Em4]). Hence, as $H^1_{X_{\mathrm{ord}}}(X,\mathcal{O}/\varpi^s\mathcal{O})$ is smooth, we infer that it is also smooth admissible. By Proposition 2.4 we get that $H^1_{X_{\mathrm{ord}}}(X,\mathcal{O}/\varpi^s\mathcal{O})$ is induced from some representation $W(\mathcal{O}/\varpi^s\mathcal{O})$ of the Borel $B(\mathbb{Q}_p)$. We deduce from Lemma 3.8 (Theorem 4.4.6 in [Em4]) that $W(\mathcal{O}/\varpi^s\mathcal{O})$ is a smooth admissible $\mathcal{O}/\varpi^s\mathcal{O}$-representation of $B(\mathbb{Q}_p)$. Thus, we can apply to it Lemma 3.7. If π is any supersingular k-representation of $\mathrm{GL}_2(\mathbb{Q}_p)$, it implies that

$$H^1_{X_{\mathrm{ord}}}(X,\mathcal{O}/\varpi^s\mathcal{O})_{(\pi)} = 0$$

because by Proposition 2.3 the representation $H^1_{X_{\mathrm{ord}}}(X,\mathcal{O}/\varpi^s\mathcal{O})$ is a smooth induction of an admissible representation; hence no supersingular representation appears as a subquotient of it.

Localising the exact sequence above at some supersingular k-representation π we get an injection

$$H^1(X,\mathcal{O}/\varpi^s\mathcal{O})_{(\pi)} \hookrightarrow H^1(X_{\mathrm{ss}},\mathcal{O}/\varpi^s\mathcal{O}).$$

By passing to the limit with s we get an injection

$$H^1(X,E)_{(\pi)} \hookrightarrow H^1(X_{\mathrm{ss}},E).$$

We can now prove our main theorem.

Theorem 4.3. *Let $\rho : G_{\mathbb{Q}} = \mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q}) \to \mathrm{GL}_2(E)$ be a pro-modular representation. Assume that $\bar{\rho}_p = \bar{\rho}_{|G_{\mathbb{Q}_p}}$ is absolutely irreducible. Then we have a $\mathrm{GL}_2(\mathbb{Q}_p) \times G_{\mathbb{Q}_p}$-equivariant injection*

$$B(\rho_p) \otimes_E \rho_p \hookrightarrow H^1(\mathcal{M}_{LT,\infty},E).$$

Proof. Let π be the mod p representation of $\mathrm{GL}_2(\mathbb{Q}_p)$ corresponding to $\bar{\rho}_p$ by the mod p local Langlands correspondence. It is a supersingular representation by our assumption that $\bar{\rho}_p$ is absolutely irreducible. Let $\mathfrak{p}$ be the prime ideal of $\mathbb{T}_\Sigma$

associated to ρ, where $\Sigma = \Sigma_0 \cup \{p\}$ is some finite set which contains p and all the primes at which ρ is ramified. As above we have

$$H^1(X, E)_{(\pi)} \hookrightarrow H^1(X_{\mathrm{ss}}, E)$$

and hence also

$$H^1(X, E)_{(\pi)}[\mathfrak{p}] \hookrightarrow H^1(X_{\mathrm{ss}}, E)[\mathfrak{p}].$$

Theorem 4.2 implies that (we keep track only of $G_{\mathbb{Q}_p}$-action instead of $G_{\mathbb{Q}}$)

$$B(\rho_p) \otimes_E \rho_p \otimes_E \pi_{\Sigma_0}(\rho)^{K_{\Sigma_0}} \hookrightarrow H^1(X_{\mathrm{ss}}, E)[\mathfrak{p}].$$

Let K'_{Σ_0} be a compact open subgroup of $\prod_{l\in\Sigma_0} \mathrm{GL}_2(\mathbb{Q}_l)$ for which we have $\dim \pi_{\Sigma_0}(\rho)^{K'_{\Sigma_0}} = 1$ where the dimension is over E. Such a subgroup always exists by classical results of Casselman (see [Cas]). Remark that we are using here the generic local Langlands correspondence as explained in [EH]: to generic representations we associate the same representation as in the classical correspondence up to the twist by the determinant and for non-generic ones we take Steinberg representations induced from a parabolic subgroup. This allows us to still appeal to [Cas] for the conclusion. Hence we have

$$B(\rho_p) \otimes_E \rho_p \hookrightarrow H^1(X_{\mathrm{ss}}, E)[\mathfrak{p}]^{K'_{\Sigma_0}}.$$

By the Kunneth formula and Proposition 2.5 (the p-adic uniformisation of Rapoport-Zink) we get that

$$H^1(X_{\mathrm{ss}}, E) = \left(H^1(\mathcal{M}_{LT,\infty}, E)\widehat{\otimes}_E \mathcal{S}\right)^{D^\times(\mathbb{Q}_p)}$$

where we have denoted by $\mathcal{S}$ the p-adic quaternionic forms of level K^p

$$\widehat{H}^0(D^\times(\mathbb{Q})\backslash D^\times(\mathbb{A})/K^p, E) = \left(\varprojlim_s \varinjlim_{K_p} H^0(D^\times(\mathbb{Q})\backslash D^\times(\mathbb{A})/K_pK^p, \mathcal{O}/\varpi^s\mathcal{O})\right)\otimes_{\mathcal{O}} E$$

where K_p runs over compact open subgroups of $D^\times(\mathbb{Q}_p)$. As $\mathrm{GL}_2(\mathbb{Q}_p)$ and $G_{\mathbb{Q}_p}$ act on $H^1(X_{\mathrm{ss}}, E)$ through $H^1(\mathcal{M}_{LT,\infty}, E)$ we conclude by the preceding discussion that

$$B(\rho_p) \otimes_E \rho_p \hookrightarrow H^1(\mathcal{M}_{LT,\infty}, E)$$

as wanted. □

4.3. **Cohomology with compact support.** We show that the cohomology with compact support of the Lubin-Tate tower does not contain any p-adic representations which reduce to mod p supersingular representations. Recall we have morphisms

$$j : X_{\mathrm{ss}} \hookrightarrow X$$

and

$$i : X_{\mathrm{ord}} \to X$$

which give an exact sequence for any étale sheaf F on X:

$$0 \to j_! j^* F \to F \to i_* i^* F \to 0.$$

This leads to an exact sequence of the cohomology

$$\begin{aligned} \cdots \to H^0(X_{\mathrm{ord}}, \mathcal{O}/\varpi^s\mathcal{O}) \to H^1_c(X_{\mathrm{ss}}, \mathcal{O}/\varpi^s\mathcal{O}) &\to H^1(X, \mathcal{O}/\varpi^s\mathcal{O}) \\ &\to H^1(X_{\mathrm{ord}}, \mathcal{O}/\varpi^s\mathcal{O}) \to \cdots . \end{aligned}$$

Because $H^1(X, \mathcal{O}/\varpi^s\mathcal{O})$ is smooth admissible as an $\mathcal{O}/\varpi^s\mathcal{O}$-representation of $\mathrm{GL}_2(\mathbb{Q}_p)$ (by the result of Emerton) and $H^0(X_{\mathrm{ord}}, \mathcal{O}/\varpi^s\mathcal{O})$ is smooth admissible as an $\mathcal{O}/\varpi^s\mathcal{O}$-representation of $\mathrm{GL}_2(\mathbb{Q}_p)$ because at each finite level X_{ord} has a finite number of connected components, we infer that also $H^1_c(X_{\mathrm{ss}}, \mathcal{O}/\varpi^s\mathcal{O})$ is smooth admissible (as the category of admissible $\mathcal{O}/\varpi^s\mathcal{O}$-representations is a Serre subcategory of smooth $\mathcal{O}/\varpi^s\mathcal{O}$-representations). Passing to the limit with s, we infer that $H^1_c(X_{\mathrm{ss}}, E)$ is Banach admissible over E. This means that we can localise $H^1_c(X_{\mathrm{ss}}, E)$ at supersingular representations.

Let π be a supersingular k-representation of $\mathrm{GL}_2(\mathbb{Q}_p)$, where k is the residue field of K. Observe that if $H^1_c(X_{\mathrm{ss}}, E)_{(\pi)} \neq 0$, then also its reduction $H^1_c(X_{\mathrm{ss}}, k)_{(\pi)}$ would be non-zero. But Theorem 8.2 in [Cho] states that $H^1_c(X_{\mathrm{ss}}, k)_{(\pi)} = 0$. Hence we get

Theorem 4.4. *For any supersingular k-representation π of $\mathrm{GL}_2(\mathbb{Q}_p)$ we have*

$$H^1_c(X_{\mathrm{ss}}, E)_{(\pi)} = 0.$$

In particular

$$H^1_c(\mathcal{M}_{LT,\infty}, E)_{(\pi)} = 0.$$

Proof. The first part follows from the preceding discussion; the second part follows from the Rapoport-Zink uniformisation. ☐

This theorem implies that for any continuous $\rho_p : G_{\mathbb{Q}_p} \to \mathrm{GL}_2(E)$ which has an absolutely irreducible reduction $\bar{\rho}_p : G_{\mathbb{Q}_p} \to \mathrm{GL}_2(k)$, the $\mathrm{GL}_2(\mathbb{Q}_p)$-representation $B(\rho_p)$ associated to ρ_p by the p-adic local Langlands correspondence does not appear in $H^1_c(\mathcal{M}_{LT,\infty}, E)$. Nevertheless, we believe that it appears in $H^2_c(\mathcal{M}_{LT,\infty}, E)$, though we could not prove it.

4.4. **Final remarks.** Observe that our proof of Theorem 4.3 depends on the global data as we have to start with a global pro-modular Galois representation ρ. As our result is completely local, it is natural to ask whether the same thing holds for any absolutely irreducible Galois representation ρ_p of $G_{\mathbb{Q}_p}$ which is not necessarily a restriction of some global ρ. In fact, it is also natural to ask which absolutely irreducible Galois representations ρ_p of $G_{\mathbb{Q}_p}$ are restrictions of some global ρ. We couldn't determine whether it is always the case or not. Observe that (essentially) any global two-dimensional representation ρ of $G_{\mathbb{Q}}$ which is absolutely irreducible and odd is pro-modular (Theorem 1.2.3 in [Em2]), and it looks plausible that for any absolutely irreducible ρ_p we can find such a global ρ with $\rho_{|G_{\mathbb{Q}_p}} \simeq \rho_p$.

Another natural problem is to try to prove Theorem 4.3 without assuming that $\bar{\rho}_p$ is absolutely irreducible. This would require a more careful study of the cohomology of the ordinary locus.

The most pertinent problem is whether one can reconstruct $B(\rho_p)$ from either the p-adic completed or the analytic cohomology of the Lubin-Tate tower and hence give a different proof of the p-adic local Langlands correspondence. This might be useful in trying to prove the existence of the p-adic correspondence for groups other than $\mathrm{GL}_2(\mathbb{Q}_p)$ as well as Theorem 4.3 for Galois representations ρ_p not necessarily coming from global Galois representations.

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