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A REMARK ON THE ULTRAPOWER ALGEBRA OF THE HYPERFINITE FACTOR

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ABSTRACT. On page 43 in [Adv. in Math. **50** (1983), pp. 27–48] Sorin Popa asked whether the following property holds: *If ω is a free ultrafilter on $\mathbb{N}$ and $\mathcal{R}_1 \subseteq \mathcal{R}$ is an irreducible inclusion of hyperfinite* II_1 *factors such that $\mathcal{R}' \cap \mathcal{R}^\omega \subseteq \mathcal{R}_1^\omega$ does it follows that $\mathcal{R}_1 = \mathcal{R}$?* In this short note we provide an affirmative answer to this question.

1. INTRODUCTION

Central sequences were introduced in [MvN36] as a tool to distinguish the hyperfinite II_1 factor $\mathcal{R}$ and the free group factor $L(\mathbb{F}_2)$. Later on, in the groundbreaking papers [MD69a, MD69b, MD69c] D. McDuff analyzed the ultrapower and central sequence algebras of II_1 factors to exhibit uncountably many nonisomorphic II_1 factors. In his celebrated work [Co76], A. Connes furthered the study of central sequence algebras and ultrapowers in his proof of *injective implies hyperfinite*, thereby underlining once again the importance of these objects. Since then, the study of ultrapowers and central sequences has played a central role in the theory of II_1 factors.

In 1967, at the Baton Rouge conference, R. V. Kadison asked a series of influential (yet unpublished!) questions. One of the questions asked whether all maximal amenable subalgebras of a II_1 factor are isomorphic to $\mathcal{R}$. In a seminal paper, [Po83], S. Popa obtained the striking result that the generator masa in $L(\mathbb{F}_2)$ is maximal amenable, thus answering negatively Kadison's question. In [Po83, Theorem 4.1] it was also shown that whenever $\mathbb{F}_n \curvearrowright X$ is a free, measure preserving action on a nonatomic probability space X, the $R_u = L^\infty(X) \rtimes \langle u \rangle$ is a maximal injective subalgebra of $\mathcal{M} = L^\infty(X) \rtimes \mathbb{F}_n$ (where u is a canonical generator of $\mathbb{F}_n$). The proof relied on showing that if $\mathcal{N} \supseteq R_u$ is an injective subalgebra satisfying $\mathcal{N}' \cap \mathcal{N}^\omega \subseteq R_u^\omega$, then $R_u = \mathcal{N}$. In turn this was shown using heavily the notion of *asymptotic orthogonality property* introduced in the same paper. This naturally led S. Popa to ask whether this phenomenon actually occurs in general: *Let $\mathcal{R}_1 \subseteq \mathcal{R}$ be a hyperfinite subfactor such that $\mathcal{R}_1' \cap \mathcal{R} = \mathbb{C}$ and $\mathcal{R}' \cap \mathcal{R}^\omega \subseteq \mathcal{R}_1^\omega$ for some free ultrafilter ω on $\mathbb{N}$. Does it follow that* $\mathcal{R}_1 = \mathcal{R}$? See [Po83, Section 4.5 Problem 2].

In this paper, we answer the aforementioned question in the affirmative (see Theorem 2.5). Thus the central sequence algebra of the hyperfinite II_1 factor cannot be absorbed by some nontrivial irreducible subfactor. Our approach relies upon an

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interplay between Popa's *deformation/rigidity theory*, subfactor theory, and some basic analysis of central sequences (e.g., Ocneanu's central freedom lemma). We believe that this general result may have future applications to maximal amenability questions.

2. Proof of the main result

Popa intertwining techniques. To study the structural theory of von Neumann algebras, S. Popa has introduced the following notion of intertwining subalgebras which has been very instrumental in the recent developments in the classification of von Neumann algebras [Po06, Va10, Io17]. Given (not necessarily unital) inclusions $\mathcal{P}, \mathcal{Q} \subseteq \mathcal{M}$ of von Neumann subalgebras, one says that *a corner of* $\mathcal{P}$ *embeds into* $\mathcal{Q}$ *inside* $\mathcal{M}$ and writes $\mathcal{P} \prec_{\mathcal{M}} \mathcal{Q}$ if there exist nonzero projections $p \in \mathcal{P}, q \in \mathcal{Q}$, a $*$-homomorphism $\theta : p\mathcal{P}p \to q\mathcal{Q}q$ and a nonzero partial isometry $v \in q\mathcal{M}p$ so that $\theta(x)v = vx$, for all $x \in p\mathcal{P}p$. The partial isometry v is also called an *intertwiner* between $\mathcal{P}$ and $\mathcal{Q}$. If we moreover have that $\mathcal{P}p' \prec_{\mathcal{M}} \mathcal{Q}$, for any nonzero projection $p' \in \mathcal{P}' \cap 1_{\mathcal{P}}\mathcal{M}1_{\mathcal{P}}$ (equivalently, for any nonzero projection $p' \in \mathcal{Z}(\mathcal{P}' \cap 1_{\mathcal{P}}\mathcal{M}1_{\mathcal{P}})$), then we write $\mathcal{P} \prec^{s}_{\mathcal{M}} \mathcal{Q}$.

Then in [Po03, Theorem 2.1 and Corollary 2.3] Popa developed a powerful analytic method to identify intertwiners between arbitrary subalgebras of tracial von Neumann algebras.

Theorem 2.1 ([Po03])**.** *Let* $(\mathcal{M}, \tau)$ *be a separable tracial von Neumann algebra and let* $\mathcal{P}, \mathcal{Q} \subseteq \mathcal{M}$ *be* (*not necessarily unital*) *von Neumann subalgebras. Then the following are equivalent:*

i) $\mathcal{P} \prec_{\mathcal{M}} \mathcal{Q}$.
ii) *For any group* $\mathcal{G} \subset U(\mathcal{P})$ *such that* $\mathcal{G}'' = \mathcal{P}$ *there is no sequence* $(u_n)_n \subset \mathcal{G}$ *satisfying* $\|E_{\mathcal{Q}}(xu_ny)\|_2 \to 0$ *for all* $x, y \in \mathcal{M}$.

In order to show our main result we need the following technical result on intertwining.

Lemma 2.2. *Let* ω *be a free ultrafilter on* $\mathbb{N}$. *Let* $\mathcal{N} \subseteq \mathcal{M}$ *be an inclusion of hyperfinite* II_1 *factors such that* $\mathcal{M} \nprec_{\mathcal{M}} \mathcal{N}$. *Then we have* $\mathcal{M}' \cap \mathcal{M}^\omega \nprec_{\mathcal{M}^\omega} \mathcal{N}^\omega$. *In particular if* $\mathcal{M}' \cap \mathcal{M}^\omega \subseteq \mathcal{N}^\omega$, *then* $\mathcal{M} \prec_{\mathcal{M}} \mathcal{N}$.

Proof. Since $\mathcal{M}$ is hyperfinite there exists an ascending sequence of algebras $\mathcal{M}_n \subseteq \mathcal{M}$ satisfying $\mathcal{M}_n \cong M_{2^n}(\mathbb{C})$, $\overline{\bigcup_n \mathcal{M}_n}^{sot} = \mathcal{M}$, and $\mathcal{M} = \mathcal{M}_n \bar{\otimes} (\mathcal{M}_n' \cap \mathcal{M})$ for all n. Next we briefly argue that $\mathcal{M}_n' \cap \mathcal{M} \nprec_{\mathcal{M}} \mathcal{N}$ for all n. Assuming otherwise, by [Va10, Lemma 2.5] there exists a nonzero projection $e \in (\mathcal{M}_n' \cap \mathcal{M})' \cap \mathcal{M} = \mathcal{M}_n$ such that $(\mathcal{M}_n' \cap \mathcal{M})e \prec^{s}_{\mathcal{M}} \mathcal{N}$. Also, since $\mathcal{M}_n$ is finite dimensional, then $[e\mathcal{M}e : (\mathcal{M}_n' \cap \mathcal{M})e] < \infty$ and hence $\mathcal{M} \prec_{\mathcal{M}} (\mathcal{M}_n' \cap \mathcal{M})e$. Using [Va10, Remark 3.8] we would get that $\mathcal{M} \prec_{\mathcal{M}} \mathcal{N}$, a contradiction.

Fix $(s_n)_n \subseteq \mathbb{N}$ a sequence that tends to ∞. Next we claim that for every finite set $F \subset \mathcal{M}^\omega$ there exists a unitary $v^\omega \in \prod_{n\to\omega} (\mathcal{M}_{s_n}' \cap \mathcal{M})$ such that $E_{\mathcal{N}^\omega}(x^\omega v^\omega y^\omega) = 0$ for all $x^\omega, y^\omega \in F$. This relies on the usage of the analytic criterion from Popa's intertwining techniques, i.e., part ii) of Theorem 2.1. Since for every $n \in \mathbb{N}$ we have $\mathcal{M}_{s_n}' \cap \mathcal{M} \nprec \mathcal{N}$ there exists a unitary $v_n \in \mathcal{M}_{s_n}' \cap \mathcal{M}$ such that $\|E_{\mathcal{N}}(x_nv_ny_n)\|_2 \leq n^{-1}$ for all $x^\omega = (x_n)_n, y^\omega = (y_n)_n \in F$. Letting $v^\omega = (v_n)_n \in \prod_{n\to\omega} \mathcal{M}_{s_n}' \cap \mathcal{M} \subset \mathcal{M}' \cap \mathcal{M}^\omega$ the previous inequalities show that $E_{\mathcal{N}^\omega}(x^\omega v^\omega y^\omega) = 0$ for all $x^\omega, y^\omega \in F$, as desired.

Assume by contradiction $\mathcal{M}' \cap \mathcal{M}^\omega \prec_{\mathcal{M}^\omega} \mathcal{N}^\omega$. Thus one can find projections $0 \neq p^\omega \in \mathcal{M}' \cap \mathcal{M}^\omega$,$0 \neq q^\omega \in \mathcal{N}^\omega$, a partial isometry $0 \neq w^\omega \in \mathcal{M}^\omega$, and a unital $*$-homomorphism $\phi : p^\omega(\mathcal{M}' \cap \mathcal{M}^\omega)p^\omega \to q^\omega \mathcal{N}^\omega q^\omega$ such that

$$\phi(x)w^\omega = w^\omega x \text{ for all } x \in p^\omega(\mathcal{M}' \cap \mathcal{M}^\omega)p^\omega. \tag{1}$$

Since $p^\omega \in \mathcal{M}^\omega = \overline{\bigcup_n \mathcal{M}_n}^{sot}$ there exists a sequence $(t_n)_n \subseteq \mathbb{N}$ that tends to ∞ for which $p^\omega \in \prod_{n\to\omega} \mathcal{M}_{t_n}$. Using our claim for the sequence t_n and the set $F = \{w^\omega, (w^\omega)^*\}$ one can find a unitary $u^\omega \in \prod_{n\to\omega}(\mathcal{M}'_{t_n} \cap \mathcal{M}) \subseteq \mathcal{M}' \cap \mathcal{M}^\omega$ such that $E_{\mathcal{N}^\omega}(w^\omega u^\omega (w^\omega)^*) = 0$. Using this in combination with (1) and $p^\omega u^\omega = u^\omega p^\omega$ we further get that $0 = \|E_{\mathcal{N}^\omega}(w^\omega p^\omega u^\omega p^\omega (w^\omega)^*)\|_2 = \|\phi(p^\omega u^\omega p^\omega)E_{\mathcal{N}^\omega}(w^\omega (w^\omega)^*)\|_2 = \|\phi(u^\omega p^\omega)E_{\mathcal{N}^\omega}(w^\omega (w^\omega)^*)\|_2 = \|E_{\mathcal{N}^\omega}(w^\omega (w^\omega)^*)\|_2$. This implies that $E_{\mathcal{N}^\omega}(w^\omega (w^\omega)^*) = 0$ and hence $w^\omega = 0$, which is a contradiction. $\square$

Remark. Theorem 2.1 also holds without separability assumptions if one uses nets instead of sequences. So the second part of the proof of Lemma 2.2 can be directly deduced from Theorem 2.1 applied in $\mathcal{M}^\omega$. The authors would like to thank the anonymous referee for pointing this out.

Proposition 2.3. *Let $\mathcal{N} \subseteq \mathcal{M}$ be* II_1 *factors such that $\mathcal{N}' \cap \mathcal{M} = \mathbb{C}1$. Then $\mathcal{M} \prec_{\mathcal{M}} \mathcal{N}$ if and only if $[\mathcal{M} : \mathcal{N}] < \infty$.*

Proof. Suppose $\mathcal{M} \prec_{\mathcal{M}} \mathcal{N}$. Thus one can find nonzero projections $p \in \mathcal{M}$,$q \in \mathcal{N}$, a nonzero partial isometry $v \in q\mathcal{M}p$, and a unital $*$-homomorphism $\phi : p\mathcal{M}p \to q\mathcal{N}q$ such that

$$\phi(x)v = vx \text{ for all } x \in p\mathcal{M}p. \tag{2}$$

Denote by $\mathcal{B} := \phi(p\mathcal{M}p) \subseteq q\mathcal{N}q$ and notice that by (2) we have $vv^* \in \mathcal{B}' \cap q\mathcal{M}q$ and $v^*v \in p\mathcal{M}p' \cap p\mathcal{M}p$. Since $\mathcal{M}$ is a factor we have $v^*v = p$. Moreover, by restricting q if necessary, we can assume without any loss of generality that the support projection of $E_{\mathcal{N}}(vv^*)$ equals q. Equation (2) also implies that $\mathcal{B}vv^* = vp\mathcal{M}pv^* = vv^*\mathcal{M}vv^*$. Since $\mathcal{M}$ is a factor, this further gives that $vv^*(\mathcal{B}' \cap q\mathcal{M}q)vv^* = (\mathcal{B}vv^*)' \cap vv^*\mathcal{M}vv^* = vv^*(\mathcal{M}' \cap \mathcal{M})vv^* = \mathbb{C}vv^*$. Since $\mathcal{B}' \cap q\mathcal{N}q \subseteq \mathcal{B}' \cap q\mathcal{M}q$, then there exists a nonzero projection $r \in \mathcal{B}' \cap q\mathcal{N}q$ such that $r(\mathcal{B}' \cap q\mathcal{N}q)r = \mathcal{B}r' \cap r\mathcal{N}r = \mathbb{C}r$. Since $q = s(E_{\mathcal{N}}(vv^*))$ one can check that $rv \neq 0$. Thus replacing $\mathcal{B}$ by $\mathcal{B}r$, $\phi(\cdot)$ by $\phi(\cdot)r$, q by r, and v by the partial isometry from the polar decomposition of rv then the intertwining relation (2) still holds with the additional assumption that $\mathcal{B}' \cap q\mathcal{N}q = \mathbb{C}q$. In particular we have that $E_{q\mathcal{N}q}(vv^*) = cq$ where c is a positive scalar.

Since $\mathcal{B} \subseteq q\mathcal{N}q \subseteq q\mathcal{M}q$ is an inclusion of II_1 factors, $\mathcal{N} \subseteq \mathcal{M}$ is irreducible, and $\mathcal{B}vv^* = vv^*\mathcal{M}vv^*$ then it follows from [Jo81, Corollary 3.1.9] and [PP86, Corollary 1.8] that $\langle q\mathcal{N}q, vv^*\rangle \subseteq q\mathcal{M}q$ is the basic construction of $\mathcal{B} \subseteq q\mathcal{N}q$. This entails that $\mathcal{B} \subseteq q\mathcal{N}q$ is finite index and moreover $vv^*\langle q\mathcal{N}q, vv^*\rangle vv^* = \mathcal{B}vv^* = vv^*\mathcal{M}vv^*$. Since $\langle q\mathcal{N}q, vv^*\rangle$ is a factor then $\langle q\mathcal{N}q, vv^*\rangle = q\mathcal{M}q$ and consequently $q\mathcal{N}q \subseteq q\mathcal{M}q$ has finite index. Thus $\mathcal{N} \subseteq \mathcal{M}$ also has finite index.

If $[\mathcal{M} : \mathcal{N}] < \infty$, then $\mathcal{M} \prec_{\mathcal{M}} \mathcal{N}$ follows easily from the fact $\mathcal{M}_{-1}e_{-1} = e_{-1}\mathcal{M}e_{-1}$, where $\mathcal{M}_{-1}$ denotes the downward basic construction, and $e_{-1} \in \mathcal{M}$ is the corresponding Jones' projection, as in [Jo81, Corollary 3.1.9]. Note that one does not need $\mathcal{N}' \cap \mathcal{M} = \mathbb{C}1$ for this direction. $\square$

For further use we recall next a result due to A. Ocneanu. For a proof the reader may consult [EK98, Lemma 15.25].

Lemma 2.4 (Ocneanu's central freedom lemma)**.** *Let $\mathcal{R} \subseteq \mathcal{P} \subseteq \mathcal{Q}$ be separable finite von Neumann algebras, with $\mathcal{R}$ the hyperfinite factor. If ω is a free ultrafilter on $\mathbb{N}$, then we have the following relation*:

$$(\mathcal{R}' \cap \mathcal{P}^\omega)' \cap \mathcal{Q}^\omega = \mathcal{R} \vee (\mathcal{P}' \cap \mathcal{Q})^\omega.$$

With these results at hand we are now ready to answer affirmatively Popa's question from [Po83].

Theorem 2.5. *Let $\mathcal{N} \subseteq \mathcal{M}$ be hyperfinite* II_1 *factors such that $\mathcal{N}' \cap \mathcal{M} = \mathbb{C}$. If $\mathcal{M}' \cap \mathcal{M}^\omega \subseteq \mathcal{N}^\omega$, then $\mathcal{N} = \mathcal{M}$.*

Proof. First we notice that from Lemma 2.2 and Proposition 2.3 it follows that $[\mathcal{M} : \mathcal{N}] < \infty$. Since $\mathcal{M}' \subseteq \mathcal{N}'$ and $\mathcal{M}' \cap \mathcal{M}^\omega \subseteq \mathcal{N}^\omega$ then $\mathcal{M}' \cap \mathcal{M}^\omega = \mathcal{M}' \cap \mathcal{M}^\omega \cap \mathcal{N}' \subseteq \mathcal{N}^\omega \cap \mathcal{N}'$. So we get the following inclusions:

$$\mathcal{M}' \cap \mathcal{M}^\omega \subseteq \mathcal{N}' \cap \mathcal{N}^\omega \subseteq \mathcal{N}' \cap \mathcal{M}^\omega. \tag{3}$$

Since $\mathcal{M}, \mathcal{N}$ are McDuff, then $\mathcal{M}' \cap \mathcal{M}^\omega$ and $\mathcal{N}' \cap \mathcal{N}^\omega$ are von Neumann algebras of type II_1. Also, since $\mathcal{M}' \cap \mathcal{M}^\omega \subseteq \mathcal{N}' \cap \mathcal{M}^\omega$ then it follows that $\mathcal{N}' \cap \mathcal{M}^\omega$ is type II_1 as well. Also since $\mathcal{M}$ is hyperfinite then applying Lemma 2.4 for $\mathcal{R} = \mathcal{P} = \mathcal{Q} = \mathcal{M}$ we get $(\mathcal{M}' \cap \mathcal{M}^\omega)' \cap \mathcal{M}^\omega = \mathcal{M}$ and hence $(\mathcal{M}' \cap \mathcal{M}^\omega)' \cap (\mathcal{N}' \cap \mathcal{M}^\omega) = \mathcal{N}' \cap \mathcal{M} = \mathbb{C}$. In particular, this implies that all algebras displayed in (3) are in fact II_1 factors. Next we show the following relations:

$$[\mathcal{N}' \cap \mathcal{M}^\omega : \mathcal{M}' \cap \mathcal{M}^\omega] = [\mathcal{M} : \mathcal{N}] = [\mathcal{N}' \cap \mathcal{M}^\omega : \mathcal{N}' \cap \mathcal{N}^\omega]. \tag{4}$$

To this end let $\{m_i\}_{1 \le i \le n+1}$ be an orthonormal basis of $\mathcal{M}$ over $\mathcal{N}$. Then, by [Po02, Lemma 3.1] it follows that the map $\Phi(x) = [\mathcal{M} : \mathcal{N}]^{-1} \sum_i m_i x m_i^*$ implements the conditional expectation from $\mathcal{N}' \cap \mathcal{M}^\omega$ onto $\mathcal{M}' \cap \mathcal{M}^\omega$. In addition, the index of Φ is majorized by $[\mathcal{M} : \mathcal{N}]$. Thus, we get

$$[\mathcal{N}' \cap \mathcal{M}^\omega : \mathcal{M}' \cap \mathcal{M}^\omega] \le [\mathcal{M} : \mathcal{N}]. \tag{5}$$

Now by [PP86, Proposition 1.10] we have $[\mathcal{M} : \mathcal{N}] = [\mathcal{M}^\omega : \mathcal{N}^\omega]$. Set $c = [\mathcal{N}' \cap \mathcal{M}^\omega : \mathcal{N}' \cap \mathcal{N}^\omega]^{-1}$ and $\lambda = [\mathcal{M}^\omega : \mathcal{N}^\omega]^{-1}$ and from (3) and (5) we infer that $c \ge \lambda$.

Denote by $E_{\mathcal{N}' \cap \mathcal{M}^\omega}$ the conditional expectation from $\mathcal{M}^\omega$ onto $\mathcal{N}' \cap \mathcal{M}^\omega$ and notice that $E_{\mathcal{N}' \cap \mathcal{M}^\omega} \circ E_{\mathcal{N}^\omega} = E_{\mathcal{N}^\omega} \circ E_{\mathcal{N}' \cap \mathcal{M}^\omega}$. Let $\mathcal{N}_n \subseteq \mathcal{N}$ such that $\mathcal{N}_n \cong \mathcal{M}_{2^n}(\mathbb{C})$, $\overline{\bigcup_n \mathcal{N}_n}^{sot} = \mathcal{N}$, and $\mathcal{N} = \mathcal{N}_n \bar{\otimes} (\mathcal{N}_n' \cap \mathcal{N})$. Since $\mathcal{N}_n' \cap \mathcal{N} \subseteq \mathcal{N}_n' \cap \mathcal{M}$ is an inclusion of II_1 factors of index λ then one can find projections $e_n \in \mathcal{N}_n' \cap \mathcal{M}$ such that $E_{\mathcal{N}_n' \cap \mathcal{N}}(e_n) = \lambda$ for all n. This implies that $E_{\mathcal{N}}(e_n) = \lambda$ for all n.

Altogether, these give $e^\omega = (e_n)_n \in \mathcal{N}' \cap \mathcal{M}^\omega$ and $E_{\mathcal{N}^\omega}(e^\omega) = \lambda$. Thus using [PP86, Theorem 2.2] we get that $\lambda \ge c$ and hence $\lambda = c$. Summarizing,

$$[\mathcal{N}' \cap \mathcal{M}^\omega : \mathcal{N}' \cap \mathcal{N}^\omega] = [\mathcal{M} : \mathcal{N}]. \tag{6}$$

Altogether, relations (5)-(6) conclude relation (4). In turn (4) shows that $[\mathcal{N}' \cap \mathcal{N}^\omega : \mathcal{M}' \cap \mathcal{M}^\omega] = 1$ and hence

$$\mathcal{M}' \cap \mathcal{M}^\omega = \mathcal{N}' \cap \mathcal{N}^\omega. \tag{7}$$

To finish the proof, we use Lemma 2.4. Indeed setting $\mathcal{R} = \mathcal{P} = \mathcal{N}$ and $\mathcal{Q} = \mathcal{M}$ in Lemma 2.4 we get $(\mathcal{N}' \cap \mathcal{N}^\omega)' \cap \mathcal{M}^\omega = \mathcal{N}$, as $\mathcal{N}' \cap \mathcal{M} = \mathbb{C}$. Also letting $\mathcal{R} = \mathcal{P} = \mathcal{Q} = \mathcal{M}$ in Lemma 2.4 we have $(\mathcal{M}' \cap \mathcal{M}^\omega)' \cap \mathcal{M}^\omega = \mathcal{M}$. Therefore using (7) we get $\mathcal{N} = \mathcal{M}$, as desired. ☐

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