

Chapter 6

ENROLLMENT, COURSE OFFERINGS, AND INSTRUCTIONAL PRACTICES IN MATHEMATICS PROGRAMS AT TWO-YEAR COLLEGES

This chapter reports estimated enrollment and instructional practices in mathematics and statistics courses at public two-year colleges in the United States in fall 2015. The data in this chapter has been rounded. Also included are total enrollment for these two-year colleges, average mathematics class size, trends in availability of mathematics courses, enrollment in mathematics courses offered outside of the mathematics programs, and services available to mathematics students. Many tables contain data from previous CBMS surveys (1975, 1980, 1985, 1990, 1995, 2000, 2005, and 2010) and hence allow for historical comparisons. Further analysis of many of the items discussed in this chapter can be found in Chapters 1 and 2 where they are discussed from a comprehensive point of view in comparison to similar data for four-year colleges and universities.

The Table display code in Chapter 6 is TYE, for "Two-Year Enrollment," because this chapter addresses issues related to enrollment.

In earlier CBMS surveys, computer courses taught outside two-year college mathematics departments, and the faculty who taught them, were considered part of the "mathematics program." By 1995, computer science and data processing programs at two-year colleges, for the most part, were organized separately from the mathematics program. Hence, in 1995, 2000, 2005, 2010 and again in this 2015 report, information about computer science courses and their faculty are not included in mathematics program data. In 1995, enrollment data were collected about computer courses taught within the mathematics program and can be found in those reports. But because such courses had become rare, the 2005, 2010 and 2015 surveys contained no specific data about these computer courses taught within the mathematics departments, though some, no doubt, were reported by mathematics programs under the "Other Courses" category. Furthermore, the enrollment tables that follow have been adjusted to eliminate all specific computer science enrollments that appeared in previous CBMS reports. This adjustment allows for a more accurate comparison of mathematics program enrollments over time. There are also instances where "na" will be displayed in a table, indicating that similar data was not collected or was not available.

In contrast to previous surveys, CBMS2005, CBMS2010 and CBMS2015 include data about public two-year colleges only. The two-year college data in this report were estimated from a stratified random sample of 222 institutions chosen from a sample frame of 1,019 public two-year colleges. Survey forms were returned by 108 colleges for the enrollment data and 11 more colleges for all other data (119 of 222 colleges = 54% of the sample). The Two-year College Committee instigated intense follow-up efforts to increase the survey return rate. For comparison purposes, the survey return rate for two-year colleges for CBMS2010 was 65% (388 of 593 colleges), CBMS2005 was 54% (130 of 241 colleges), CBMS2000 was 60% (179 of 300 colleges), and CBMS1995 was 65% (163 of 250 colleges). The return rate for all

institutions, two-year and four-year, in CBMS2015 was 66% (388 of 587 institutions). For more information on the sampling and projection procedures used in this survey, see Appendix II. A copy of the two-year college survey questionnaire for CBMS2015 may be found in Appendix V.

The terms " full-time permanent," " full-time continuing" and "other full-time" faculty occasionally are used in this chapter and other chapters. For a detailed explanation of these terms, see the first page of Chapter 7.

Highlights of Chapter 6

Enrollments, Class Size, and Course Offerings in Mathematics Programs

- From 2010 to 2015, public two-year colleges experienced an overall total enrollment decrease of 14%, an estimated total of 6,216,000 students, based on National Center for Education Statistics (NCES) projections for 2016. This decrease can be viewed in comparison with an overall increase at four-year colleges of 1%, an estimated total of 10,546,000 students. From 2005 to 2010, the overall total enrollment increase at public two-year colleges was 17%, compared with an overall enrollment increase at four-year colleges of 23%. Enrollment in two-year colleges in fall 2015 constituted about 37% of the total undergraduate enrollment in the United States, a four percent drop compared with 2010. For details, see the discussion before and after Tables S.1 in Chapter 1 and TYE.1 in this chapter.
- The fall 2015 enrollment in mathematics and statistics courses in mathematics programs at public two-year colleges was estimated to be 2,012,000 (SE 118,000) students. This total includes 94,000 (SE 23,000) dually enrolled students and 225,000 (SE 25,000) distance learning enrollments. Enrollment in mathematics and statistics at two-year colleges in fall 2015 constituted approximately 42% of the total mathematics and statistics undergraduate enrollment in postsecondary institutions. See Table S.1 in Chapter 1, and Tables TYE.2 and TYE.12 in this chapter.
- Table TYE.2 shows that two-year college mathematics and statistics on-campus and distance enrollment decreased 4% (1 SE) from 2010 to 2015 (the decrease was 5% (1 SE) when dual enrollment students are excluded in Table S.1). This can be compared to the growth from 2005 to 2010 of 21% (19% when dual enrollment students are excluded in Table S.1). During the same period, four-year institutions had an enrollment increase in mathematics courses of 13% (2 SEs, excluding dual enrollment), compared to the growth from 2005 to 2010 of 26%. See Tables S.1 in Chapter 1, and the discussion before Table TYE.2 in this chapter.
- Dual enrollment, where students enroll in a course that earns credit in high school and a two-year college, increased 16% (1 SE) from 2010 to 2015 to an estimated 94,000 (SE 23,000) students, compared with a 93% increase from 2005 to 2010 to

a total of 80,000 students. See Tables SP.16 and SP.17 in Chapter 2 and TYE.2 in this chapter.

- Approximately 41% of all two-year college mathematics and statistics enrollment in fall 2015 was in Precollege (remedial/developmental) courses, compared to 57% in fall 2010. See Table TYE.4.
- Enrollment in precollege mathematics courses (Arithmetic, Pre-algebra, Elementary and Intermediate Algebra, and Geometry) at two-year colleges was estimated to be 782,000 students (SE 65,000) in 2015. This represents a 32% (6 SEs) decrease from 2010 to 2015, compared to a 19% increase from 2005 to 2010. The increase from 2000 to 2005 was 26% and from 1995 to 2000 was 5%. Four-year college Precollege enrollment increased 21% (5 SEs) to an estimate of 253,000 (SE 26,000) students from 2010 to 2015, compared with 4% increase from 2005 to 2010. See Tables TYE.4 and E.2 in Chapter 2.
- Within the cohort of Precollege courses, all courses, except Geometry, showed a decrease in enrollment. Arithmetic/Basic Mathematics showed a 52% (5 SEs) decrease to 71,000 (SE 14,000) students from 2010 to 2015, compared with the 40% increase in enrollment seen from 2005 to 2010. A decreasing enrollment trend in Arithmetic was also present between 1990 and 2005.
- Pre-algebra courses showed a 44% (6 SEs) decrease to 127,000 (SE 16,000) students from 2010 to 2015, compared with the 65% increase in enrollment seen from 2005 to 2010. From 2010 to 2015, Elementary Algebra experienced a 35% (6 SEs) decrease to 277,000 (SE 27,000) students (13% increase in 2005 to 2010) and Intermediate Algebra a 13% (2 SEs) decrease to 299,000 (SE 30,000) students (2% increase in 2005 to 2010). See Table TYE.3 and the discussion before TYE.3 and TYE.11.
- The trend of an increasing enrollment in Precalculus level courses (College Algebra, Trigonometry, College and Trig, Mathematical Modeling, Elementary Functions) seen in 2010, continued in 2015 representing 23% of all mathematics enrollments, a total of 445,000 (SE 39,000) students, and a 21% (2 SEs) increase from 2010. The enrollment growth in this group grew 15% between 2005 and 2010 and 17% from 2000 to 2005. See Table TYE.4.
- Within the cohort of Precalculus level courses, College Algebra enrollment increased 27% (2 SEs) to 292,000 (SE 29,000) students, bypassing the number of students enrolled in Elementary Algebra (277,000; SE 27,000) and nearly reaching the number of students in Intermediate Algebra (299,000; SE 30,000) for the first time. Precalculus/Elem Functions/Analytic Geometry increased 35% (2 SEs) from 2010 to 2015 to a total of 87,000 (SE 13,000) students. See Table TYE.3.

- Enrollment in all calculus-level courses (Mainstream Calculus I, II, and III and Non-mainstream Calculus I and II together) showed an 11% (1 SE) increase from 2010 to 2015 (total 152,000 students; SE 15,000), compared with the 29% increase between 2005 and 2010 and a 9% increase between 2000 and 2005. From 2010 to 2015, Mainstream Calculus I, II, and III experienced a 9% (1 SE) increase to a total of 119,000 students and Non-mainstream Calculus I and II increased 18% (1 SE) to 26,000 students. Calculus I had enrollment of 66,000 students and Non-mainstream Calculus I had enrollment of 26,000 students (each with SE 7,000). See Tables TYE.3 and TYE.4.
- Among college-level, transferable mathematics and statistics courses, notable enrollment increases occurred in Probability (833%; 28,000 students; SE 15,000), Finite Mathematics (124%; 40,000 with SE 19,000), and Elementary Statistics (87%; 251,000 students; SE 55,000). When Elementary Statistics and Probability are combined, the increase was 104% for a total of approximately 279,000 students. See Tables TYE.3 and TYE.3.1.
- With the exception of the precollege mathematics courses mentioned above, enrollment increased in 2015 compared with 2010 for every course except Introduction to Mathematical Modeling, Non-mainstream Calculus II, Mathematics for Elementary Teachers I and II, and “Other” mathematics courses. See Tables TYE.3, TYE.3.1 and TYE.3.2.
- Notable decreases in the percentage of two-year college mathematics programs teaching selected courses included Precollege courses, Introduction to Mathematical Modeling, Mainstream Calculus III, Finite Mathematics, Mathematics for Elementary School Teachers I and II. See Tables TYE.5 and TYE.6.
- The average size of classes taught on two-year college campuses was 22 (SE 2) students in 2015, compared to 24 students in 2010. The average section size decreased in Precollege level courses from 24 in 2010 to 19 (SE 4) in 2015. Average class size decreased in Precalculus level courses to 25 (SE 1) students and 26 (SE 5) students in Statistics and Probability. Average class size increased in Calculus level courses to 25 (SE 1) students in 2015, compared with 21 students in 2010. See Tables TYE.7 and TYE.8. For comparable four-year data, see Table E.12 in Chapter 3.
- The percentage of sections for all mathematics courses with an average size greater than 30 increased from 23% in 2010 to 25% (3 SEs) in 2015. The class size recommended by the American Mathematical Association of Two-Year Colleges (AMATYC) and the Mathematical Association of America (MAA) is 30 or less. See Tables TYE.7 and TYE.8. For comparable four-year data, see Tables E.12 and E.13 in Chapter 3.

- The average section size of all distance learning courses in fall 2015 was 21 (SE 1) students, with a range of 11-27 students. The percentage of distance learning courses with an average size greater than 30 was 17% (4 SEs). CBMS2010 data displayed an average section size of 22 students with a range of 17-28 and 10% of 2010 sections with a size greater than 30. See Tables TYE.7.1 and TYE.8.1 and CBMS2010 for historical data.
- Thirty-six percent (36%; 4 SEs) of mathematics class sections were taught by part-time faculty in 2015, down ten points from 2010. The percentage of sections taught by part-time faculty varied significantly by course type, with part-time faculty teaching 46% (10 SEs) of Precollege courses, 33% (3 SEs) of Precalculus courses, 15% (2 SEs) of Mainstream Calculus, 29% (10 SEs) of Non-mainstream Calculus, and 21% (5 SEs) of Statistics and Probability. See Table TYE.9.

Instructional Practices and Curricular Changes in Mathematics Programs; Redesign of Mathematics Programs

- For the first time, CBMS2015 asked questions the use of common department exams and homework management systems. Common department exams were most prevalent in Precollege level courses in 38-67% of sections. The use of homework management systems increased from 2010 to 2015 in the majority of courses and tended to be used in less in Calculus courses, Differential Equations, Linear Algebra and Discrete Mathematics. See Table TYE.10.
- Also for the first time, CBMS2015 asked questions about implementation of mathematics “Pathways,” defined to be “a redesign of a mathematics sequence that provides students with an alternative course or sequence to/through developmental mathematics and to/through a college-level mathematics or statistics course.” In fall 2015, 58% (5 SEs) of colleges reported having implemented a Pathways course sequence, enrolling a total of 193,000 students. Colleges sometimes implemented multiple Pathways courses including Foundations (51%; 7 SEs), Quantitative Reasoning/Literacy (59%; 8 SEs), Statistics (63%; 6 SEs) and Other (32%; 9 SEs). See Table TYE.11 and the discussion before TYE.11 in this chapter.
- Significant changes were reported by mathematics programs in two-year colleges in content, delivery methods, and instructional strategies in Precollege, College-Level Non-STEM, and statistics courses in a range of 5-46% (1-7 SEs) mathematics programs. Notable changes in content included students solving contextually-based problems and modeling. Colleges reported significant changes in Pre-college courses using emporium models, students completing prescribed models, and accelerated pace delivery methods. Notable changes in instructional strategies included use of computer programs or internet, group work, and active learning. These activities and percentages are listed in Table TYE.11.1. See the discussion before Table TYE.11.1 regarding Pathways and curricula redesign.

Distance Learning Courses and Practices

- Distance learning enrollment in mathematics and statistics grew to an estimated 225,000 (SE 25,000) students in fall 2015 and a total of 12% (1 SE) of all mathematics enrollments, increasing from 9% in fall 2010. The courses with the largest distance learning enrollment were College Algebra (38,000 students; SE 5,000), Elementary Algebra (38,000 students; SE 10,000), Intermediate Algebra (33,000 students; SE 5,000), and Elementary Statistics (31,000 students; SE 4,000). See Tables E.4 in Chapter 3 and TYE.2 and TYE.12 in this chapter.
- Precollege distance learning enrollments accounted for 11% of Precollege course enrollments in fall 2015. The number of students in Precollege distance learning courses increased 2% (89,000 students; SE ranged from 2,000 to 10,000 depending on the course) from 2010 to 2015, compared to the 135% increase from 2005 to 2010 (from 37,000 students to 87,000 students). See Tables TYE.12 and E.4 in Chapter 3.
- Distance learning increases were also experienced in fall 2015 in Precalculus courses (College Algebra, Trigonometry, College and Trig, Mathematical Modeling, Elementary Functions; 32%, 54,000 students; SE ranged from 1,000 to 5,000 depending on the course) and Elementary Statistics (35%, 31,000 students; SE 4,000). See Tables TYE.12 and E.4 in Chapter 3. A discussion about the use of distance learning by mathematics departments is included in Chapter 2 before Table SP.8 in Chapter 2.
- Other distance learning courses with the large percent of total enrollments were: Introduction to Mathematical Modeling (46%), Mathematics for Elementary School Teachers I and II (17% and 32% respectively), Business Math (21%), Elementary Statistics (17%), and Math for Liberal Arts (17%). Courses with percentage of enrollment in distance learning less than 2% were Geometry (0%), Mainstream Calculus II (1%), Differential Equations (1%), and Non-mainstream Calculus II (0%). Caution is needed when looking at percentages. While percentages may be large, total enrollments may be small. See Table TYE.12 for a listing of distance enrollments for all courses.
- Table TYE.12.1 presents data on various distance learning practices. For example, 58% (5 SEs) of responding colleges awarded transfer credit for distance learning courses; 67% (5 SEs) of responding colleges reported that instructional materials were created by a combination of faculty design and commercially produced materials; 69% (6 SEs) of distance learning courses are taught completely online; 97% (3 SEs) of responding colleges reported that the course outlines for distance courses were the same as face-to-face courses. For other practices, see Table TYE.12.1. A discussion about the use of distance learning practices by mathematics departments is included in Chapter 2 before Table SP.8 in Chapter 2.

- Forty percent (40%; 6 SEs) of responding colleges reported that a “significant challenge” of distance learning courses is that “student success rates in online courses are lower than face-to-face courses with similar content.” “Maintaining a level of rigor in distance learning mathematics courses equivalent to face-to-face courses” was reported as “somewhat of a challenge” by 41% (5 SEs) of responding colleges. See Table TYE.12.2.
- For the first time, CBMS2015, asked two-year and four-year mathematics departments if, during the academic years 2013-15, the department had offered a MOOC (massive open online course) for credit. Out of all the institutions surveyed, one four-year (bachelors-level) mathematics department, one (doctoral-level) statistics department, and two two-year colleges responded “yes”. The two-year colleges reported teaching courses in statistics, developmental mathematics, and college-level courses below, and above, calculus-level courses. Given the few responses, and large SEs, estimates of the percentage of departments offering MOOCs and the enrollments in MOOCs are not included in this report.

Placement and Opportunities Available to Students

- Ninety-four percent (94%; 3 SEs) of two-year college mathematics programs offered diagnostic or placement testing available. Seventy-eight percent (78%) of those colleges required placement tests of first-time enrollees in fall 2015, compared to 100% in fall 2010. See Table TYE.13.
- Opportunities offered to students that displayed increases in CBMS2015 included honors sections, mathematics clubs and contests, programs to encourage women and minorities in mathematical studies, Outreach in K-12 schools, undergraduate student research and independent studies in mathematics. These are described in Tables SP.12 and SP.13 in Chapter 2 and Table TYE.13 in this chapter.
- The collection of Precollege, Statistics, Business and Technical Mathematics courses taught "outside" the mathematics program showed a 15% (1 SE) decrease from 2010 to 2015. These "outside" mathematics enrollments totaling about 129,000 (SE 24,000) students, at 32% (5 SEs) colleges, are not included in Table TYE. 2. See the discussion before Tables TYE.3 and TYE.5 and also the discussion before Tables TYE.14, TYE.15 and TYE.16.

Topics of Special Interest in CBMS2015

- In each CBMS survey cycle, certain topics of special interest are chosen for data collection and comprehensive analysis from both two-year and four-year institutions. Special topics for two-year and four-year institutions are discussed in Chapters 2 and 6 of this report. Additional questions were added in CBMS2015 regarding the offering of Massive Open Online Courses (MOOCs), and distance learning courses and practices (Tables SP.8-SP.10 in Chapter 2 and Tables

TYE.12, TYE.12.1, and TYE.12.2 in Chapter 6). Pre-service education of teachers (Tables SP.2, SP.3, and SP.4 in Chapter 2) and data on dual enrollment courses and faculty (Table SP.16 in Chapter 2) are discussed at the end of this chapter. Questions regarding mathematics Pathways and course redesign (Tables TYE.11 and TYE.11.1 in Chapter 6) were asked of two-year college respondents.

Enrollment, Class Size, and Course Offerings in Mathematics Programs

Number of two-year-college students

Approximately 6,216,000 students were enrolled in public two-year colleges in fall 2015 with 61% of students attending part-time. This estimate is based on an overall 2016 enrollment projection for public two-year colleges by the National Center for Educational Statistics (NCES). These enrollments constitute a 14% enrollment decrease from 2010-2015 for public two-year colleges. NCES projections indicated about a 1% increase in four-year college enrollments in the same time period and totaled 10,546,000 students. Enrollment in two-year colleges in fall 2015 constituted about 37% of the total undergraduate enrollment in the United States, a four percent drop compared with 2010. Data from the NCES indicated over 96% of two-year college enrollment in 2015 was at public institutions. See Tables TYE.1 and S.1 in Chapter 1.

Table TYE.1 (public and private) and Figure TYE.1.1 go here

Category	2010	2011	2012	2013	2014	2015	2016
Public	5,800,000	5,600,000	5,400,000	5,200,000	5,000,000	4,800,000	4,600,000
Private	400,000	400,000	400,000	400,000	400,000	400,000	400,000
Total	6,200,000	6,000,000	5,800,000	5,600,000	5,400,000	5,200,000	5,000,000

Enrollment trends in mathematics programs

Enrollment in mathematics and statistics courses in mathematics programs at public two-year colleges was estimated to be 2,012,000 (SE 118,000) students in 2015. The 2,012,000 enrollments in mathematics includes approximately 225,000 (SE 25,000) students enrolled in distance learning courses and 94,000 (SE 23,000) dual-enrollment students and represents a decrease of 4% (1 SE) since 2010, compared to an increase of 19% from 2005 and 2010. The 4% enrollment decrease in mathematics and statistics courses from 2010 to 2015 is consistent with the decrease in two-year institutional enrollment mentioned above and with the decrease in the number of full-time mathematics faculty discussed in Chapter 7.

Dual enrolled students are high school students who take courses taught either in high school or a two-year college campus and receive course credit at the both the high school and at the two-year college. The estimated 94,000 dual enrollment students in

mathematics represented almost 5% of total mathematics and statistics enrollments in fall 2015. The estimated 225,000 students in distance learning mathematics courses represented 12% of total mathematics and statistics enrollments in fall 2015. See Table TYE.2.

Table S.1 in Chapter 1 presents data on both two-year and four-year institutions' overall and mathematics and statistics enrollments, excluding dual enrollments. The estimated total of 1,918,000 (SE 115,000) two-year college enrollment shown in Table S.1 is a 5% (1 SE) decrease from fall 2010 to fall 2015. Two-year college mathematics and statistics enrollment (excluding dual enrollment) comprised 42% of all postsecondary mathematics and statistics enrollments in fall 2015. See Table S.1 in Chapter 1 and Tables SP.16 and SP.17 in Chapter 2 and Table TYE.2.

The fall 2015 enrollments in mathematics and statistics courses represent the second decrease in enrollment since CBMS began collecting data in 1985. From 1995 to 2010, mathematics and statistics enrollments had increased a total of 125% to a total of 2,105,000, with a decrease of 7% from 1995 to 2000. See Table S.1.1 In Chapter 1 and Table TYE.2.1.

It is difficult to draw specific conclusions about the reasons for the decrease in institutional and mathematics enrollment in two-year colleges in fall 2015. However, the reader may consider several economic and national factors that may have played a part in the decrease. Two-year colleges saw enrollment increases in fall 2010, given a downturn in the U.S. economy. In response to a more positive economic situation preceding fall 2015, two-year college enrollments decreased across the country. Other factors that may have influenced mathematics enrollments include national degree completion and "Guided Pathways" initiatives, changes in State legislation regarding decrease funding for developmental education and high school graduation requirements, and implementation of multiple placement measures/procedures. More discussion about trends in specific course enrollment and implementation of mathematics "Pathways" can be found before Tables TYE.3, TYE.11, and TYE.11.1.

Two-year college mathematics and statistics enrollment from 2010 to 2015 can be considered in light of the pattern in the nation's four-year colleges and universities. Between 2010 and 2015, mathematics and statistics enrollment (excluding dual enrollments) at two-year colleges decreased 5% (1 SE) and four-year mathematics and statistics enrollment increased 13% (2 SEs). See Table S.1 in Chapter 1.

In addition to the tables that follow in this chapter, the reader should consult Chapter 1 of the current report. Chapter 1 contains a detailed analysis of mathematics and statistics department enrollments at both two-year and four-year colleges from 1995 to 2015.

Table TYE.2 and Figure TYE.2.1 go here

Enrollment trends in course groups and in specific courses

Tables TYE.3 and TYE.4 report mathematics enrollments in two-year colleges. Table TYE.3 reports enrollment in individual mathematics courses. Table TYE.4 reports enrollment for categories of courses. Table TYE.4 is constructed from Table

TYE.3 and reports headcounts and percentages from 1995 through 2015 for the following course groupings: Precollege, Precalculus, Calculus, Statistics and Remaining Courses. Each category consists of five or more specific courses from Table TYE.3.

In fall 2015, over 782,000 (SE 65,000) students enrolled in Precollege mathematics courses (Arithmetic, Pre-algebra, Elementary and Intermediate Algebra, and Geometry). Enrollment in these courses comprised 41% of mathematics program enrollment. This percentage had been at 57% since 2000. These percentages are calculated from Table TYE.4, which does not include the 94,000 students in dual enrollment courses. Precollege enrollment has varied over time as follows: down by 5% from 1995 to 2000, up 26% from 2000 to 2005, and up 19% from 2005 to 2010. Fall 2015 is the second time that Precollege enrollment showed a decrease, which was 32% (6 SEs). This two-year college decrease is contrasted to the 21% (5 SEs) increase of four-year college Precollege enrollment, a total of 253,000 (SE 26,000) students from 2010 to 2015. See Table E.2 in Chapter 3.

Within the Precollege courses, each course, except Geometry, experienced a decrease in enrollment from 2010 to 2015: Arithmetic & Basic Mathematics (down 52%; 5 SEs); Pre-algebra (down 44%; 6 SEs); Elementary Algebra (down 35%; 6 SEs); Intermediate Algebra (down 13%; 2 SEs); and Geometry (up 44%; 1 SE). See Tables TYE.3 and TYE.3.2 for enrollment in individual courses.

Approximately 129,000 (SE 24,000) students were enrolled in mathematics and statistics courses managed by departments "outside" the mathematics department (Developmental Education Division, Occupational Programs, Business or Other Divisions) in fall 2015, a decrease of 15% from 2010 to 2015. About one-third (32%; 5 SEs) of two-year colleges responding to the survey conducted part of their Precollege (remedial) mathematics program outside of the mathematics program in an alternate structure such as a developmental studies division or learning laboratory. These courses accounted for 101,000 students and 78% of the mathematics enrollment outside of the mathematics departments. These enrollments are not included in Tables TYE.3 and TYE.4. See the discussion for Tables TYE.14, TYE.15 and TYE.16 later in this chapter.

Precalculus level courses (College Algebra, Trigonometry, College Algebra & Trigonometry, Introduction to Mathematical Modeling, Precalculus), 445,000 (SE 39,000) students, accounted for 23% of 2015 enrollment, five percentage points up from enrollment reported in 2010. Precalculus courses, together with Precollege courses, accounted for 64% of mathematics and statistics enrollment at public two-year colleges in fall 2015, a decrease from 2010 of 11%. See Table TYE.4.

Within the cohort of Precalculus level courses, College Algebra enrollment increased 27% (2 SEs) to 292,000 (SE 29,000) students, bypassing the number of students enrolled in Elementary Algebra (277,000; SE 27,000) and nearly reaching the number of students in Intermediate Algebra (299,000; SE 30,000) for the first time. Other specific course enrollment changes in Precalculus level courses include Trigonometry (up 13% with 1 SE), College Algebra and Trigonometry combined (up 28% with 1 SE), Introduction to Mathematical Modeling (down 89% with 16 SEs), and

Precalculus/Elem Functions/Analytic Geometry (up 35% with 2 SEs). See Tables TYE.3, TYE.3.1 and TYE.3.2 for enrollment in individual courses.

All calculus-level courses, Mainstream and Non-mainstream Calculus together, in Tables TYE.3 and TYE.4 displays an 11% (1 SE) increase in fall 2015 enrollment and a total of 152,000 (SE 15,000) students. When Differential Equations is included with Calculus courses, the increase is 10% from 2010 to 2015. Calculus I had enrollment of 66,000 students and Non-mainstream Calculus I had enrollment of 26,000 students (each with SE 7,000). Specific course group changes include: Mainstream Calculus I, II and III (9% with 1 SE); Non-mainstream Calculus I and II (18% with 1 SE); and Differential Equations (17% with 1 SE). See Tables TYE.3, TYE.3.1 and TYE.4.

In reading the enrollment tables, the reader is reminded that Mainstream Calculus consists of those calculus courses that lead to more advanced mathematics courses and usually is required of majors in mathematics, the physical sciences, and engineering. Non-mainstream Calculus includes the calculus courses most often taught for biology, behavioral science, and business majors. Additional calculus enrollment data and analysis can also be found in Chapter 1.

In reviewing this list of percentages of changes from 2010 to 2015, one needs to consider the actual number of students enrolled and standard error (SE) of a statistic. Table TYE.3 lists enrollment estimates in mathematics courses, rounded to the nearest thousands. Percentages can be misleading: an 822% increase in Probability enrollment represented a change of 25,000 students of a total enrollment of 28,000 (SE 15,000) students, while a 27% increase in College Algebra represented a change of 62,000 students of a total of 292,000 students (SE 29,000). Tables TYE.3.1 and TYE.3.2 list the percentage change for each course, computed before rounding enrollment estimates.

Summarizing the enrollment trends in mathematics course categories (see Table TYE.4), the trend in enrollments from fall 2010 to 2015 for courses offered within a two-year college mathematics department was upward in every category except Precollege level:

- Precollege level courses enrolled 368,000 less students in 2015 than in 2010 representing a 32% (6 SEs) decrease.
- Precalculus courses enrolled 77,000 more students in 2015 than in 2010 representing a 21% (2 SEs) increase.
- Mainstream and Non-mainstream Calculus and Differential Equations enrolled 15,000 more students in 2015 than in 2010 representing a 11% (1 SE) increase.
- Elementary Statistics and Probability enrolled 143,000 more students in 2015 than in 2010 representing a 104% (2 SEs) increase.
- Of special note is the 12% (1 SE) increase in the "Remaining" category of 28,000 students which included Linear Algebra, Discrete Mathematics,

Probability, Finite Mathematics, Mathematics for Elementary School Teachers and Business and Technical Mathematics.

In addition to considering the factors listed above related to the decrease in total mathematics and statistics enrollment in 2015, several factors may have impacted enrollment in individual course categories or courses in two-year colleges. Implementation of mathematics "Pathways," defined as a redesign of a mathematics sequence that provides students with an alternative course or sequence to/through developmental mathematics and to/through a college-level mathematics or statistics course, may be related to decreased enrollments in traditional Precollege courses at some colleges and increased enrollment in College Algebra, Quantitative Literacy, Mathematics for Liberal Arts and Statistics courses. In addition, changes in placement policies are affecting the number of students who were previously placed into Precollege courses. Mathematics Pathways have been designed and implemented to create appropriate career course paths that decrease the number of developmental courses that students are required to take and increase students' enrollment and success in a college-level mathematics and path to graduation. If the goals of Pathways are achieved, enrollments in precollege mathematics courses should decrease and enrollments in college-level mathematics courses should increase. Table TYE.11 shows that 58% (5 SEs) of responding college implemented a Pathways course sequence. Table TYE.11.1 presents information about the changes in content, delivery methods, and instructional strategies between 2010 and 2015.

Table TYE.3, TYE.3.1 (new), TYE.3.2 (new), Table TYE.4, and Figure TYE.4.1 go here.

TABLE TYE.3.1 Enrollment in thousands in mathematics and statistics courses (not including dual enrollments; including distance enrollments) in mathematics programs at two-year colleges in fall 2010 and 2015 for those courses showing percentage increases from 2010 to 2015.

Course Number	Type of course	2010	2015	Percentage change 2015-2010*
	Precollege level			
5	Geometry (High School level)	6	8	44%
	Precalculus level			
6	College Algebra (above Intermediate Algebra)	230	292	27%
7	Trigonometry	45	51	13%
8	College Algebra & Trigonometry (combined)	11	13	28%
10	Precalculus/Elem Functions/Analytic Geometry	64	87	35%
	Calculus level ¹			
11	Mainstream Calculus I	65	66	0.4%
12	Mainstream Calculus II	29	34	17%
13	Mainstream Calculus III	15	19	22%
14	Non-mainstream Calculus I	20	26	30%
16	Differential Equations	6	7	19%
	Other mathematics courses			
17	Linear Algebra	5	7	45%
18	Discrete Mathematics	2	5	126%
19	Elementary Statistics (with or w/o Probability)	134	251	87%
20	Probability (with or w/o Statistics)	3	28	833%
21	Finite Mathematics	18	40	124%
22	Mathematics for Liberal Arts	91	95	4%
25	Other Mathematics Courses for Teacher Preparation ³	1	1	29%
26	Business Mathematics (not transferable)	16	16	5%
27	Business Mathematics (transferable)	4	10	145%
28	Technical Math (non-calculus-based)	17	21	24%
29	Technical Math (calculus-based)	1	3	415%
Total all Two-year College math courses in Tables TYE.3.1 and 3.2		2024	1918	-5%

*Percentages were computed on enrollment values before rounding.

Note: 0 means fewer than 500 enrollments and na means not available. Round-off may make column sums seem inaccurate.

¹ Mainstream calculus is for mathematics, physics, science & engineering. Non-mainstream calculus is for biological, social, and management sciences.

² In 2005 and earlier surveys there was a single course listed as *Mathematics for Elementary School Teachers*.

³ This course was not listed in 2005 and earlier surveys.

⁴ In 2005 and earlier surveys there was a single course listed as *Other Mathematics Courses*.

TABLE TYE.3.2 Enrollment in thousands in mathematics and statistics courses (not including dual enrollments; including distance enrollments) in mathematics programs at two-year colleges in fall 2010 and 2015 for those courses showing percentage decreases from 2010 to 2015.

Course Number	Type of course	2010	2015	Percentage change 2015-2010*
	Precollege level			
1	Arithmetic & Basic Mathematics	146	71	-52%
2	Pre-algebra	226	127	-44%
3	Elementary Algebra (High School level)	428	277	-35%
4	Intermediate Algebra (High School level)	344	299	-13%
	Precalculus level			
9	Introduction to Mathematical Modeling	18	2	-88%
	Calculus level ¹			
15	Non-mainstream Calculus II	2	0*	-97%
	Other mathematics courses			
23	Mathematics for Elementary School Teachers I ²	21	12	-45%
24	Mathematics for Elementary School Teachers II ³	8	3	-58%
30	Other Mathematics Courses (not transferable) ⁴	33	31	-6%
31	Other Mathematics Courses (transferable) ³	14	12	-17%
Total all Two-year College math courses in Tables TYE.3.1 and 3.2		2024	1918	-5%

*Percentages were computed on enrollment values before rounding.

Note: 0 means fewer than 500 enrollments and na means not available. Round-off may make column sums seem inaccurate. Enrollment in non-Mainstream Calculus II was 60 students.

¹ Mainstream calculus is for mathematics, physics, science & engineering. Non-mainstream calculus is for biological, social, and management sciences.

² In 2005 and earlier surveys there was a single course listed as *Mathematics for Elementary School Teachers*.

³ This course was not listed in 2005 and earlier surveys.

⁴ In 2005 and earlier surveys there was a single course listed as *Other Mathematics Courses*.

Trends in availability of courses in mathematics programs

Tables TYE.5 and TYE.6 should be considered together and represent the availability of fall 2010 and 2015 course offerings and percentage of two-year college mathematics programs teaching individual courses. The increases and decreases displayed in these tables mirror the increases and decreases in student enrollment presented in Tables TYE.3, TYE.3.1, TYE.3.2, and TYE.4.

In considering the availability of courses, the reader also should note that 32% (5 SEs) of two-year colleges in fall 2015 reported some or all of the Precollege (Arithmetic, Elementary Algebra, and Intermediate Algebra) mathematics courses at the college were organized separately from the mathematics department, totaling 129,000 (SE 24,000) students. This represents a 3% increase reported in 2010. See Table TYE.16. These "outside" courses are not included below in Tables TYE.3, TYE.4, TYE.5 and TYE.6 in

reporting the availability of particular courses. The "outside" headcount enrollment is estimated in Tables TYE.14 and TYE.15 and also includes Business Mathematics, Statistics & Probability, and Technical Mathematics.

Table TYE.5 reports that the percentage of two-year college mathematics programs offering a course titled Arithmetic/Basic Mathematics course in 2015 was 36% (5 SEs), a decline from 50% in 2010. From 2010 to 2015, the percentage of mathematics programs offering a Pre-algebra course, which generally included arithmetic and basic algebra skills, dropped from 49% to 44% (5 SEs).

Table TYE.5 also shows the availability of Elementary Algebra within mathematics programs decreased in 2015 to 75% (5 SEs) from 82% in 2010. Intermediate Algebra, which is roughly equivalent to the second year of high school algebra, was offered in 72% (5 SEs) of mathematics departments in fall 2015, down from 88% in 2005 and 79% in 2010. CBMS2010 reported a sharp decrease from 19% in fall 2005 to 7% in fall 2010 and CBMS2015 reported a slight increase to 8% (2 SEs) in the percentage of two-year colleges offering high school level Geometry courses.

Data for courses directly preparatory for calculus are also presented in Table TYE.5. In fall 2015, the percentage of colleges offering a separate College Algebra course increased by three points to 79% (4 SEs). The percentage of colleges offering a separate Trigonometry course was up two points to 57% (5 SEs). The course College Algebra & Trigonometry (combined) experienced an eight-point increase to 20% (4 SEs) of colleges offering the course. Precalculus/Elementary Functions experienced a one percentage point increase in availability from 2010 to 2015 to 54% (6 SEs).

Comparing fall 2010 to fall 2015, the percentage of colleges offering the first semester of Mainstream Calculus rose one point to 80% (6 SEs), 66,000 students (7 SEs). The availability of Mainstream Calculus II was up four points to 65% (4 SEs). Mainstream Calculus III decreased by two points to 54% (4 SEs). In fall 2015, enrollment increased 30% to a total of 26,000 (SE 7,000) students in Non-mainstream Calculus I with 26% (4 SEs) of reporting colleges offering the course. See Tables TYE.3 and TYE.5.

Introductory Mathematical Modeling was a new course first surveyed in 2000. In that year, 12% of colleges reported offering the course. In fall 2005, this percentage had dropped to 5%. In 2010, while 9% of colleges reported offering the course, the actual total enrollment was 18,000. In fall 2015, five percent (5%, 3 SEs) of responding colleges reported offering this course with an enrollment of 2,000 students.

The CBMS1995 survey noted that many students at two-year colleges could not complete lower division mathematics requirements in certain majors because essential courses such as Linear Algebra, Mathematics for Liberal Arts, and Mathematics for Elementary School Teachers were offered at fewer than half of two-year college mathematics programs. Availability of those courses has had ups and downs since then. Comparing fall 2015 to fall 2010 course offerings, the percentage of colleges offering Linear Algebra increased five points to 25% (4 SEs), while Mathematics for Elementary School Teachers I decreased by 14% (5 SEs). Mathematics for Liberal Arts showed an 18% (5 SEs) increase in departments

offering the course in the fall 2015, following the 12% decrease from fall 2005 to 2010. See Table TYE.5.

Availability of other courses important to baccalaureate degrees in science, technology, engineering, mathematics, and computer science, such as Differential Equations, Discrete Mathematics, Elementary Statistics, and Finite Mathematics, is reported in Table TYE.6. An increase in colleges offering these courses is seen in all courses except Finite Mathematics (decrease of 4%; 4 SEs) and Mathematics for Elementary School Teachers (decrease of 14%; 5 SEs). Elementary Statistics (with or without Probability) increased by ten points to a total of 83% (6 SEs) of two-year college mathematics programs teaching Statistics. See the discussion about Teacher Preparation at the end of this chapter.

Tables TYE.5 and TYE.6 go here.

Trends in average section size

The downward trend in the average number of students per on campus class section in two-year college mathematics courses exhibited in 1990 through 2005, shifted slightly upward in 2010 and downward again in 2015. The average class size in fall 2015 was 22 (SE 2) students, compared with 24 students in fall 2010. The Precollege and Precalculus course categories had average class size of 19 (SE 4) and 25 (SE 1) students, respectively in 2015. Calculus classes (Mainstream and Non-mainstream Calculus) had average class size of 25 (SE 1) students. Statistics and Probability had average class size of 26 (SE5), about 4 students above the overall average of 22. See Table TYE.7. For a closer examination of individual course average section sizes in 2015, see Table TYE.8 displaying a range of 10 – 35 average section sizes of on-campus courses.

In 2005, the lower cut-off of 30 students per class was chosen to make data for two-year colleges directly comparable to that collected for four-year institutions and to coincide with the recommendation from the Mathematical Association of America (MAA) and endorsement by the American Mathematical Association of Two-Year Colleges (AMATYC) that undergraduate class size not exceed 30 students. In fall 2015, 75% of all class sections in two-year colleges met the goal of the two professional societies of class size less than or equal to 30 (25% of colleges with class size >30; 3 SEs; see Table TYE.7). At four-year colleges and universities, the average class size for freshman/sophomore level courses through calculus ranged from 12 - 37 students, depending on course type. At PhD-granting institutions, these numbers ranged from 21 - 55 students. See Tables E.12 in Chapter 3 for four-year institutional data.

Given the increasing enrollments in distance learning courses (see Table.TYE.12), CBMS2010 and CBMS2015 collected data on the average section size of distance learning classes. As reported in Tables TYE 7.1 and 8.1, average section size for all distance learning courses was 21 (SE 1) students, ranging from 9 - 22 students, with 17% (4 SEs) of departments having an average size greater than 30. Average sections sizes in Precollege distance courses (course numbers 1-5) ranged from 18 - 23 students. Precalculus (course numbers 6-10) average section sizes ranged from 13 -

23 students. Mainstream Calculus and Non-mainstream Calculus average section sizes ranged from 11 - 17 students. Comparing the section sizes of distance learning by course category to face-to-face section sizes, distance learning section size was less than or equal to face-to-face in courses, except Intermediate Algebra, Introduction to Mathematical Modeling, and Technical Mathematics. See Tables 7.1 and 8.1.

Table TYE.7 AND 7.1 and Table TYE.8 AND 8.1 go here

Trends in the use of part-time faculty

In fall 2015, sixty-seven percent (67%; 20,247 persons) of those who taught mathematics courses in two-year colleges were part-time faculty (Table TYF.1 in Chapter 1). However, this is a statement that requires some explanation. The relevant issue, as seen in the faculty data in Table TYF.1 in Chapter 7, is who is included in the various categories. When faculty of every sort are included, such as part-time faculty paid by third parties and full-time (permanent, continuing, and other) faculty, part-time faculty in fall 2015 made up the 67% of the total mathematics faculty. The comparable figure in 2010 was 70%. If the 2,359 (SE 528) third-party-payee part-time faculty members are excluded, 65% of the faculty had part-time status in fall 2010. The comparable figure for 2010 was 68%. See Table TYF.1.

Though making up about two-thirds (67%) of total faculty by headcount, part-time faculty taught slightly more than one-third (36%; 4 SEs) of mathematics program class sections in fall 2015, down ten percentage points from 2010 (46%). See Table TYE.9. For historical reference, in fall 2000, 46% of class sections were taught by part-time faculty. In fall 1995, this figure was 38%.

Concerning the instructional issue of which types of courses are taught most often by part-time faculty, the pattern in fall 2015 continued from fall 2010. Once again in fall 2015, it was more likely that a part-time faculty member was teaching a course below calculus, than a calculus course. In particular, forty-six percent (46%; 10 SEs) of all precollege level sections were taught by part-time faculty, down twelve points compared with 2010. Fifteen percent (15%; 2 SEs) of Mainstream Calculus sections were taught by part-time faculty, up four points from 2010. Twenty-nine percent (29% with 10 SEs) of Non-mainstream Calculus sections were taught by part-time faculty, up two points from 2010. See Tables TYE.9 and TYE.9.1.

Table TYE.9 and Figures TYE.9.1 go here.

Instructional Practices and Curricular Changes in Mathematics Programs

Reflecting on historical CBMS survey data regarding instructional practices displayed in Table TYE.10, CBMS2005 presented the percentage of class sections in mathematics courses at public two-year colleges that employed the instructional

practices of using graphic calculators, writing assignments, computer assignments group projects, online resource systems, and standard lecture methods. At that time, the predominant instructional method was the standard lecture format. Reflecting changes in mathematics instruction practices, CBMS2010 responders were asked to report on faculty use of computer algebra systems, commercially produced electronic instructional packages, and the standard lecture method.

In CBMS2015, responders were asked to report on sections with common department exams and the use of homework management systems (Table TYE.10). Historical data is not available on instructional practices as each CBMS survey focuses on specific practices at the time of each survey.

Regarding the 2015 data collected, the following observations can be made from data in TYE.10:

- Common Department exams were most prevalent in Precollege level courses with a range of 45 – 76% and in 40 – 65% of Statistics and Probability sections of on-campus sections.
- The use of Homework Management systems was prevalent in most courses, particularly Precollege level, Mainstream Calculus, and Statistics and Probability.

In Table TYE.10, the reader will note the small number of percentages in some categories and with the number of sections taught in each modality totaling more than 100% for every course. A possible reason for the incomplete data may be that department chairs (or persons completing the survey) were not always sure which instructional practice is used by instructors, and/or that it was difficult to collect such data. In spite of the gaps, the writers of this summary felt that the data in the table should be presented as collected. This situation was also experienced in the 2010 survey data.

Data and analysis on how first-year courses were taught at four-year institutions can be found in Chapter 5 of this report in Tables FY.2 through FY.10. Additional Information about instructional strategies employed at four-year institutions can be found in Chapter 1, Tables S.6-S.8 and Table SP.26 in Chapter 2.

Table TYE.10 goes here

Redesign of mathematics programs and Pathways

Strategies to improve success/completion rates and to update the curriculum were a result of nationwide discussions starting in 2009. Colleges experimented with accelerated, as well as slower-paced precollege courses, implemented learning communities, and created summer boot camps in Beginning and Intermediate Algebra. Some colleges began to rethink the curriculum, questioning historically traditional topics, wondering what to emphasize and de-emphasize, and considering new topics more relevant to how people use mathematics. These efforts and discussions led to curricular programs called mathematics "Pathways." By fall 2015, mathematics Pathway courses and course sequences could be found in many two- and

four-year colleges and deemed as an important topic to be surveyed in CBMS2015. In this survey, Pathways is defined to be "a redesign of a mathematics sequence that provides students with an alternative course or sequence to/through developmental mathematics and to/through a college-level mathematics or statistics course." These curricular changes often involved revisions of course prerequisites in those courses. Availability of Pathways courses and sequences may be the cause of decreased enrollments in traditional Precollege courses at some colleges and increased enrollment in College Algebra, Quantitative Literacy, Mathematics for Liberal Arts and Statistics courses. See Tables TYE.3 and TYE.4.

Table TYE.11 reports that 58% (5 SEs) of responding two-year colleges implemented a Pathways course sequence in fall 2015. Some colleges implemented multiple courses and more than 193,000 students enrolled in Pathways courses. The following Pathways courses were implemented in the given percentage of mathematics departments: Foundations (51%; 7 SEs), Quantitative Reasoning/Literacy (59%; 8 SEs), Statistics (63%; 6 SEs), and Other courses (32%; 9 SEs).

Significant changes between 2010 and 2015 were found in the areas of content, delivery methods and instructional strategies in Precollege, College-level Non-STEM, STEM, and Statistics courses as presented in Table TYE.11.1. Many of these changes were the result of the redesign efforts mentioned above. Changes in content including students collecting and analyzing data, solving contextually-based problems, focus on quantitative reasoning and less symbol manipulation were reported in a range of 8 – 38% of courses in various courses. Alternative delivery methods, such as emporium models, modules, flipped classrooms, accelerated or slower pace courses were most prevalent in Precollege level courses. Group work, handheld devices, computer programs and the internet, spreadsheets, guided questioning and active learning strategies were reported in 5 – 46% of responding colleges.

The possible implementation of Pathways programs/courses at four-year institutions was not surveyed in CBMS2015. Table SP.26 in Chapter 2 reports that 58% (6 SEs) of four-year mathematics and statistics departments implemented inquiry-based classes, 58% (4 SEs) flipped classes, 66% (5 SEs) activity based lessons, and 86% (3 SEs) used technology to develop conceptual understanding.

While the impact of mathematics Pathways needs to be studied as implementation and improvements continue across the country, possible decreases observed in CBMS2015 Precollege enrollment data and increases in College Algebra, Precalculus, and Statistics, might be related to Pathways initiatives and/or other curricular changes.

Table TYE.11 and TYE.11.1 go here

Distance learning courses and MOOCs

In CBMS2015, as in 2010 and 2005, distance learning was defined as "a course in which the majority of instruction occurs with the instructor and the students separated by time and/or place." The CBMS2005 survey inquired about the number of course sections taught via distance. In 2015 and 2010 data about distance learning courses was collected including information about both course enrollment and number of

class sections. This change was motivated by the fact that distance-learning sections are not bound by room-size limits and can vary dramatically depending on local administrative practice. The comments that precede Table E.4 in Chapter 3 discuss the survey questions in CBMS2015 about distance learning for both four-year and two-year colleges. Additional discussion and tables about distance learning enrollments and instructional strategies for both two-year and four-year institutions are included in Chapter 2, Tables SP.8 - SP.10.

Using enrollment data, not section counts, the fall 2015 data for two-year colleges (Table TYE.12 and Table E.4 in Chapter 5) reported that almost 12% (1 SE) mathematics students enrolled via distance (225,000 students of the total 1,918,000 students; SE 25,000), an increase of three points from 2010. Comparing 2015 to 2010, two-year colleges had increases in the number of students enrolled in distance learning courses in all Precollege courses, College Algebra, Precalculus, Mainstream Calculus I, II, and III, Statistics, and Mathematics for Liberal Arts.

Elementary Algebra and College Algebra had the largest student enrollment in fall 2015 distance learning enrollment of 38,000 students each (SE 10,000 and 5,000 respectively). Intermediate Algebra was next largest with 33,000 (SE 5,000) students, followed by Statistics with 31,000 (SE 4,000). Largest distance learning percentage of individual course enrollment in courses with greater than 10,000 students was reported in Mathematics for Liberal Arts (19%, 3 SEs), Arithmetic (13%; 5 SEs), Elementary Algebra (13%; 3 SEs), and College Algebra (13%; 1 SE). See Table TYE.12.

As reported in Tables TYE 7.1 and 8.1, the total average section size for all distance learning courses was 21 (SE 1) students, ranging from 11 to 27 students. Sections sizes in Precollege courses (course numbers 1-5) ranged from 18-23 students and averaged 23 (SE 1) students. Precalculus (course numbers 6-10) average section sizes ranged from 13 - 23 students and averaged 20 (SE 1) students. Mainstream Calculus and Non-mainstream Calculus section sizes ranged from 11 - 24 students and averaged 19 (SE 4) students. The percentage of distance learning courses with an average section size greater than 30 was 17% (4 SEs). Comparing the section sizes of distance learning courses to face-to-face section sizes, distance learning section size was less than the face-to-face courses, except for Intermediate Algebra, Introduction to Mathematical Modeling, Discrete Mathematics, and Technical Mathematics. See Tables 7.1 and 8.1.

CBMS2010 also collected data on characteristics of distance learning courses and programs in two-year colleges (see Table TYE.12.1 and Tables SP.9 and SP.10 in Chapter 2). Eighty-seven percent (87%; 4 SEs) of mathematics departments taught distance learning courses with 69% (6 SEs) of those courses taught completely online. Ninety-seven percent (97%; 3 SEs) of mathematics programs used the same course outlines for distance learning as face-to-face classes. Instructional materials were a combination of materials created by faculty and commercially produced products in 67% (5 SEs) of the departments. Distance learning students took the majority of tests at monitored testing sites at 47% (5 SEs) of the reporting colleges. Transfer credit for distance learning courses not taught by faculty at the reporting

institution was awarded at 58% (5 SEs) of reporting colleges. Distance Learning instructors are evaluated in the same way that face-to-face instructors are evaluated at 93% (3 SEs) of responding colleges. See Table TYE.12.1.

A more detailed discussion about trends in distance learning enrollment in four-year institutions can be found in Table E.4 in Chapter 3 and in the discussion in Chapter 2 proceeding Tables SP.9 and SP.10. At four-year mathematics departments in fall 2010, the percentage of distance learning enrollments in Precollege level, College Algebra/Trigonometry/Pre-Calculus, Calculus I, and Statistics were 4%, 3%, 0.6%, and 6% respectively. In 2015, while the number of students enrolled in distance learning in four-year mathematics departments was less than at two-year colleges, data showed that percentage of distance learning enrollments in Precollege level, College Algebra/Trigonometry/Pre-Calculus, Calculus I, and Statistics were 3%, 5%, 3%, and 7% respectively in four-year mathematics departments.

Distance learning delivery and course design can present unique challenges for departments. "Maintaining a level of rigor in distance learning mathematics courses equivalent to courses offered face-to-face" was reported to be "somewhat of a challenge" by 41% (5 SEs) of two-year colleges and a "very significant challenge" by 17% (5 SEs). Forty percent (40%; 6 SEs) of colleges stated that "student success rates in online distance mathematics courses are lower than face-to-face courses" with similar content presented a "very significant challenge" to the department. See Table TYE.12.2.

The 2015 survey asked two-year and four-year mathematics departments if, during the academic years 2013-15, the department had offered a MOOC (massive open online course) for credit. Out of all the institutions surveyed, one four-year (bachelors-level) mathematics department, one (doctoral-level) statistics department, and two two-year colleges responded "yes". The two-year colleges reported teaching courses in statistics, developmental mathematics, and college-level courses below, and above, calculus-level courses. The four-year mathematics department taught one or more courses that were college-level, but below calculus, and also statistics. The statistics department taught a course that required previous statistical knowledge. Given the few responses, and large SEs, estimates of the percentage of departments offering MOOCs and the enrollments in MOOCs are not included in this report. That is, given the rarity of such MOOCs, a different sample might show a different distribution of courses and different statistics.

Table TYE.12 AND TYE.12.1 and TYE.12.2 (new) and TYE.12.3 (deleted) goes here.

Placement testing, Math Clubs, independent study, honors programs, programs for minorities, programs for women, and outreach projects in K-12 schools

Table TYE.13 reported that diagnostic or placement/diagnostic testing was available in 94% (4 SEs) of two-year colleges in fall 2015, up four points from 2010. Seventy-eight percent (78%; 4 SEs) of these colleges usually required such testing mandatory for first-time students, and 79% (4 SEs) of the colleges responding periodically assess

the effectiveness of their placement tests. Advising by a member of the mathematics faculty occurred in 49% (6 SEs) of responding colleges, up seven points from 2010.

Tables TYE.13 in this chapter and Tables SP.14 and SP.15 in Chapter 2, report specific outside-of-class opportunities for two-year college mathematics students. Notable increases in participation occurred in opportunities for students to participate in various activities: mathematics clubs (32% in 2015; 5 SEs, compared to 31% in 2010) and lectures/colloquia not part of mathematics clubs (21% in 2015; 4 SEs, compared to 16% in 2010), undergraduate research activities (17% in 2015; 3 SEs, compared to 14% in 2010). Participation in mathematics contests was down one point to 40% (5 SEs) of colleges. Independent studies in mathematics increased five points to 41% (6 SEs). Since 1995, honors sections in mathematics programs have gone up and down, from 17% in 1995 to 20% in 2000 to 24% in 2005, back down to 20% in 2010 and up to 28% (4 SEs) in 2015. Special programs to encourage minorities in mathematics were reported in 15% (of two-year colleges in 2005 and down to 11% in 2010, and back up to 15% (3 SEs) in 2015. Special programs to encourage women in mathematics increased nine points to 15% (3 SEs) in 2015.

Chapter 2 of this report also contains a comparison of academic services and other opportunities available to both four-year college students and to two-year college students in fall 2015. See Tables SP.12 and SP.13 in that chapter. In 2015, K-12 outreach opportunities increased again, up twelve points from 2010 to 46% (4 SEs), even though enrollment in the course Mathematics for Elementary School Teachers had decreased (see Table TYE.3). Similarly, opportunities for involvement with K-12 schools increased in four-year colleges from up one point to 50% (4 SEs) in 2015. Additional discussion about teacher training in two-year colleges appears at the end of this chapter and in Chapter 2, Tables SP.2, SP.3, and SP.12.

Table TYE.13 goes here.

CBMS2015 and CBMS2010 did not attempt to survey the habits of mathematics students related to academic services or the amount of time spent by faculty in these areas. Data of this kind has been collected by other entities. One resource is the Community College Survey of Student Engagement (CCSSE), conducted under the auspices of the Center for Community College Student Engagement Leadership Program at The University of Texas at Austin since 2001. The 2016 CCSSE Survey collected data from 701 colleges in 46 states, the District of Columbia, three Canadian provinces, plus Micronesia, Guam, and the Marshall Islands. Additional information can be found at <http://www.ccsse.org/survey/reports/2016/overview.cfm>.

Mathematics Courses Taught Outside of the Mathematics Programs

Two-year colleges have a long history of offering mathematics courses in instructional units outside of the mathematics program. Tables TYE.14, TYE.14.1, TYE.15 and TYE.16 give the enrollment in mathematics courses offered outside of mathematics programs. These enrollments were estimated by mathematics program department chairs. Thus, the estimates may not be as accurate as the numbers given for enrollment within mathematics programs. These enrollments are not included in course enrollment data in earlier tables in CBMS2015.

In fall 2015, the total enrollment in a collection of mathematics courses taught outside the department was reported to be 129,000 (SE 24,000) students, a 15% (1 SE) decrease from 2010 to 2015, after a 19% decrease from 2005 to 2010. Seventy-eight percent (78%, 101,000 students) of those enrollments was in Precollege courses (Arithmetic/Pre-algebra, Elementary and Intermediate Algebra), similar to 2010. Statistics and Probability, Business Mathematics, and Technical Mathematics comprised the remaining 22% of the enrollment taught outside of mathematics departments (28,000 students with 9,000 SE). See Tables TYE.14.

Eighty percent (80%) of the courses listed above were taught in a developmental education department or division (103,000 students) outside of the mathematics department. Arithmetic and Elementary Algebra and Technical Mathematics were taught within Occupational Programs and Elementary Statistics/Probability and Business Mathematics were taught in Business divisions. See Table TYE.15.

The largest component of the outside mathematics enrollment described above was in Precollege developmental courses. The structure of Precollege course offerings within a particular college is determined by the institution's philosophy concerning developmental education. A student might have taken all developmental courses (mathematics, reading, and writing) in a self-contained unit devoted to developmental studies, or developmental courses were offered as part of the disciplinary curriculum.

The earliest CBMS survey for which "outside" Precollege mathematics enrollment data are available on a course-by-course basis was in 1990. The following percentages are obtained by using Table TYE.3 and Table TYE.14, tracing the pattern of enrollment outside the mathematics program from 1990 to 2015 in Arithmetic, Elementary Algebra and Intermediate Algebra as a *percentage of the total enrollment* in the course.

	1990	1995	2000	2005	2010	2015	
Arithmetic/Pre-algebra		18%	19%	17%	20%	13%	19%
Elementary Algebra	13%	12%	12%	15%	9%	13%	
Intermediate Algebra	9%	4%	4%	7%	8%	9%	

Looking only at percentages of total enrollment is part of the story. From 2010 to 2015, actual enrollment changes in Arithmetic/Prealgebra, Elementary Algebra and Intermediate Algebra of -10,000, -2,000, and -2,000 students, respectively, along with overall enrollment decreases in these courses, further highlight the downturn in Precollege enrollments in fall 2015.

Fluctuations in the numbers of outside the mathematics department enrollment may be influenced by the fact that the mathematics department chairs, who do not manage these outside programs, were responsible for estimating the numbers.

Table TYE.16 shows 32% (5 SEs) of colleges reported some part of their precollege mathematics program was administered separately from the mathematics program, up from 29% in 2010, but the similar to 2005.

Table TYE.14 and Figure TYE.14.1, Table TYE.15 and TYE.16 go here.

Topics of Special Interest for Mathematics Programs

In each CBMS survey cycle, certain topics of special interest are chosen for data collection and comprehensive analysis across both two-year and four-year colleges or for two-year or four-year institutions individually. Special topics for two-year and four-year institutions are discussed in Chapter 2 and/or 6 of this report. Additional questions were added in CBMS2015 regarding the various options available in mathematics Pathways and course redesign (Tables TYE.11 and TYE.11.1 in Chapter 6), and distance learning courses and practices (Tables SP.8-SP.10 in Chapter 2 and Tables TYE.12, TYE.12.1, and TYE.12.2 in Chapter 6). Pre-service education of teachers (Tables SP.2, SP.3, and SP.4 in Chapter 2) and data on dual enrollment courses and faculty (Table SP.19 in Chapter 2) are discussed below.

Scope and organization of pre-service mathematics education for K-8 teachers

CBMS2015 continued an inquiry begun in 2000 about the level of involvement of two-year college mathematics programs in the mathematical education of future mathematics teachers. These data are reported primarily among the special topics in Chapter 2, Tables SP.2 and SP.3.

In the last two decades, involvement in teacher education at two-year colleges has been active as more students turned to them to take required mathematics and education courses. Enrollment in the Mathematics for Elementary Teachers course fall 2010 and 2005 survey data confirm this involvement. However, fall 2015 saw student enrollment drop to 12,000 (SE 2,000; down from 21,000 students in 2010) in the course Mathematics for Elementary School Teachers I and a decrease of 5,000 students to 3,000 (SE 1,000) in fall 2015 in the second course, Mathematics for Elementary School Teachers II. See Table TYE.3 in Chapter 6.

Table TYE.5 shows that 41% (5 SEs) of two-year colleges offered the course Mathematics for Elementary School Teachers I in fall 2015, compared to 55% of two-year colleges in fall 2010. For the five-year CBMS intervals beginning in 1990 through 2015, the percentages of two-year colleges teaching the Mathematics for Elementary School Teachers I course are successively 32%, 43%, 49%, 59%, 55% and 41%. The historical growth, and now decrease in 2015, in offerings for this course and other selected courses at two-year colleges, for the five-year CBMS intervals (2000-2015), is reported in TYE.6. As expected, a decrease in fall 2015 is reported in the percentage of colleges in Mathematics for Elementary School Teachers II from 27% in 2010 to 17% (4 SEs) in 2015.

Table SP.2 (Chapter 2) reports on “organized” programs at two-year colleges in which students can obtain their entire mathematics course requirement for teacher licensure. Although 2015 data present decreasing numbers, these data confirm that two-year colleges are involved in teacher education primarily at the K-8 level, though future secondary school teachers often take their lower division mathematics courses at two-year colleges. The single largest component is the program for pre-service elementary

school teachers reported by 28% of two-year colleges in 2015, with a decrease from 41% in 2010. Pre-service middle school licensure-oriented programs reported a ten-point decrease to 14% of colleges. Between 5% and 16% of two-year colleges reported programs at the elementary or middle school levels for retraining by career switchers moving into teaching. Compared with 2010, all categories of Table SP.2 showed decreases in percentages of responding colleges.

Table SP.3 (Chapter 2) reports on other involvements two-year college mathematics programs have with K-8 teacher education. Thirty-five percent (35%) report that a faculty member is assigned to coordinate mathematics education for future K-8 teachers. About 55% of the reporting colleges designate special mathematics courses for future (preservice) K-8 teachers and 19% offer a special mathematics course for preservice secondary teachers. Among mathematics departments, 9% offer mathematics pedagogy courses for future K-8 teachers and 6% of colleges offer such pedagogy courses outside the mathematics department.

The conclusion from Chapter 2 is that, given the large number of two-year colleges in the United States, even when the percentage of colleges involved in the education of future K-8 teachers is small and enrollments decreased in fall 2015, the impact of two-year colleges on the next generation of K-8 teachers is important.

Dual Enrollment and Credentials and supervision of dual enrollment faculty

Dual enrollment in CBMS2015 is defined as “a credit structure that allows high school students to receive simultaneous high school and college credit for courses that were taught at a high school by a high school teacher.” Data in Chapter 2 (Tables SP.16 and SP.17) show how that by fall 2015, 94,000 (SE 23,000) students were dually enrolled, a 16% (1 SE) increase from 2010. Of special note in fall 2015 is the 86% increase of dual enrollment in College Algebra from fall 2010 to fall 2015. Precalculus experienced a 43% decrease dual enrollment from fall 2010 to fall 2015. Dual enrollment in Calculus decreased 42%, in contrast to dual enrollment in Statistics that increased 66% in fall 2015. Dual enrollment in “other” courses also decreased. Table SP.16 also includes data for spring 2015 enrollments. See Table TYE.3.1.

In some cases, a faculty member teaching a dual enrollment course was classified as a part-time faculty member at the two-year college that awarded college credit for the course, even though the salary was paid completely by a third party, e.g., the local school district. In 2015, two-year and four-year institutions assigned and paid their own faculty to teach courses in a high school that awards both high school and college credit in 44% (6 SEs) and 6% (2 SEs) of departments respectively. See Table SP.17 in Chapter 2.

As reported in Tables TYF.24 and TYF.25 in Chapter 7, among all survey respondents (including respondents from colleges that do not have dual enrollment arrangements), seven percent (7%; 3 SEs) of mathematics program heads in two-year colleges saw dual enrollment courses as a “major” problem, down four points from 2010. Another 36% (5

SEs) found dual enrollment arrangements “somewhat of a” problem, up twenty points from 2010.