Chapter 1

PURPOSES AND PROCEDURES OF THE STUDY

In United States colleges and universities, departments in the mathematical sciences provide instruction that is a fundamental component of undergraduate education for students with extremely diverse educational interests and career goals. To help these departments, campus administrators, and national organizations in planning appropriate and effective programs, the Conference Board of the Mathematical Sciences (CBMS) has conducted a sequence of in-depth surveys describing current practices and trends in undergraduate mathematics education -- curricula, enrollments, instructional practices, and faculty characteristics.

Background and Purpose

The present study, based on questionnaire data collected from over 250 mathematical science departments in 1975-76, is a direct successor to three earlier studies conducted at five year intervals beginning in 1960-61. The first, by Clarence Lindquist for the U. S. Office of Education (USOE), surveyed graduate and undergraduate mathematical programs in four-year institutions [A]. In 1965-66 the CBMS Survey Committee repeated the undergraduate portion of the Lindquist study while expanding its coverage to include basic facts about faculty in the mathematical sciences [B]. The report of that 1965-66 survey also included data from a separate but related survey of two-year colleges, conducted in 1966-67. Then in 1970-71 the CBMS Committee conducted a comprehensive survey of two-year and four-year mathematical science programs and faculty characteristics [C].

The practices of each two-year college, four-year college, and university reflect unique institutional goals, traditions, and boundary conditions. But response to previous CBMS reports indicates the value of national perspective in making decisions regarding such questions as

--What new courses or major programs should be developed and what traditional courses or programs should be dropped?

--What are enrollment trends in various mathematical sciences specialties? What do these trends suggest about employment prospects and advising for undergraduates?

--What types of faculty expertise should be sought?

--What are emerging patterns of instructional staff utilization and how do they affect economic factors such as class size and faculty load?

In addition to these perennial broad concerns, individual CBMS surveys have focused on specific issues of timely importance such as

--What is the impact on undergraduate programs of changing secondary school mathematics curricula?

--How are technological innovations such as calculators and computers influencing curricula and enrollments?

--How have changing college admission standards affected the offerings and standards of mathematics departments?

--What are the age, education, and tenure profiles of mathematical science faculties, and how do they influence long term employment prospects for mathematics graduate students?

The present survey addressed each of these issues, as well as many others of current interest to the mathematical community.

Methodology

The balance of this chapter describes the sampling procedure, response patterns, and methods of estimation used in the 1975-76 study.

Sampling Procedure. The most interesting results of surveys repeated at regular intervals are patterns of change. To establish valid trends in undergraduate mathematics education, the sampling procedure of the 1975-76 survey followed, as closely as possible, that of the 1970-71 study.

The U. S. National Center for Education Statistics (NCES) report of 1974 opening Fall enrollment [D] listed 3,017 institutions of higher education. Of these, 478 graduate or special professional schools offer no systematic undergraduate mathematics instruction. Thus the population for the survey included the remaining 2,539 institutions of higher education in the 50 states and the District of Columbia. To obtain reliable data while imposing on a minimum number of respondents, the survey questionnaire was sent to a stratified random sample of 424 institutions.

In choosing the sample, institutions were stratified according to control and type:

A. Control
   1. Public
   2. Private

B. Type*
   1. University
   2. Four-year college or four-year branch of a university.
   3. Two-year college or two-year branch of a university or of a four-year college.

Then within each control/type category institutions were grouped into 212 zones of approximately equal total enrollment. The procedure for zone formation resulted in valuable additional stratification of the sample, generally placing institutions of similar size and geographical location in the same zone. From each zone two institutions were selected at random for the sample.

The zone formation method, equalizing total zone enrollments led to different sampling ratios for different size institutions.

*The list of responding institutions, given in Appendix B, is probably the most effective elaboration of these institution type definitions.

Within each control/type category larger institutions tended to be in zones with few members. Thus they were more likely to be sampled than colleges or universities in zones formed from many small institutions. Table 1.1 gives the number of institutions in each category of the population and the sample.

Table 1.1

SAMPLING AND RESPONSE IN MATHEMATICS DEPARTMENTS

Control/Type	Population	Sample	Respondents	Rate of Response
1. Public Universities	95	48	36	75%
2. Private Universities	65	28	15	54%
3. Public Colleges	407	86	50	58%
4. Private Colleges	862	98	62	63%
5. Public 2-Year Colleges	897	146	81	55%
6. Private 2-Year Colleges	213	18	11	61%
	2,539	424	255	60%

After sample institutions were chosen, appropriate questionnaires were sent to heads of all mathematical science departments listed under the sample institutions in the 1976 Mathematical Sciences Administrative Directory [E]. Every university and four-year college in the sample had a mathematics department, so for these schools the sample of mathematics departments had the same structure as the sample of institutions. Mathematics programs in two-year colleges are often under the direction of broad departments or divisions such as Mathematics and Engineering, Mathematics and Physical Science, Mathematics and Natural Science, or Mathematics and Computer Science. Questionnaires for two-year colleges were addressed to the person in charge of the mathematics program.

In the 424 sample institutions there were 48 separate departments of computer science, 32 separate departments of statistics, and 25 other special mathematical science departments such as operations research, applied mathematics, or mathematics education. Questionnaires were sent to each of these departments. Table 1.2

shows the distribution of computer science, statistics, and other mathematical science departments in the sample.

Table 1.2

SAMPLING AND RESPONSE IN COMPUTER SCIENCE, STATISTICS, AND OTHER MATHEMATICAL SCIENCES

Control/Type	Institutions in Sample	Departments in Sample	Departments Responding
Computer Science			
1. Public Universities	48	34	16
2. Private Universities	28	8	2
3. 4-Year Colleges	184	6	5
4. 2-Year Colleges	164	0	0
Statistics			
1. Public Universities	48	21	12
2. Private Universities	28	6	3
3. 4-Year Colleges	184	5	2
4. 2-Year Colleges	164	0	0
Other Mathematical Sciences			
1. Public Universities	48	14	3
2. Private Universities	28	8	3
3. 4-Year Colleges	184	3	3
4. 2-Year Colleges	164	0	0

The sample and response sizes indicated in Table 1.2 are very small for reliable extrapolation to national figures, except in two special categories of departments. The number and distribution of responses seemed to justify inclusion of the categories "university computer science departments" and "university statistics departments" in subsequent analyses (combining public and private universities). Information from other types of institutions and other mathematical science departments was pooled with the appropriate mathematics department figures, making the resulting "composite departments" comparable to comprehensive mathematical science departments at other institutions.

Estimation Procedures. To facilitate interpretation of the data and comparison with results of preceding surveys, data presented in this report are estimates of national totals for institutions of higher education rather than totals for responding institutions or estimates of the sample. To arrive at national estimates it was necessary to multiply response totals by appropriate weighting factors to compensate for sampling and nonresponse. Sampling rates and response rates were different for each type of institution and each type of mathematical science department, so the weighting factors were determined separately for each of these groups and for each survey question.

Since sampling was accomplished by selecting two institutions each from zones including several institutions, the natural procedure for creating national estimates from responses would be

1)
$$\text{Zone Data Estimate} = \frac{\text{Number of institutions in zone}}{\text{Number of respondents in zone}} \times \text{Response Data}$$

2)
$$\text{Control/Type Category Data Estimate} = \text{Sum of Zone Data Estimates}$$

Because the number of respondents in each zone was 0, 1, or 2, this method of weighting seemed dangerously sensitive to nonresponses. Thus in practice the responses from similar zones were clustered before extrapolation to national estimates.

For example, the Fall 1975 national enrollment in mathematics for elementary school teachers was estimated to be 79,000 students. Calculation of this estimate began with data from public universities. The 95 institutions in this control/type category were partitioned into 5 clusters according to total enrollment.

Cluster	Number of Institutions	Average Enrollment
A	14	39,000
B	15	28,000
C	20	22,000
D	22	18,000
E	24	10,000

Of the 24 institutions in cluster E, 6 were in the sample, 3 of these responded to the survey and provided the requested data on mathematics enrollments. For the question on mathematics for elementary school teachers the 3 institutions reported Fall 1975 enrollments of 590. Thus the estimate for all institutions in cluster E was calculated as (24/3) x 590 = 4,720. Similar estimates for each of the other clusters were summed to get a nationa figure for public universities. Then the procedure was repeated for private universities, public and private four-year colleges, and public and private two-year colleges.

Accuracy of Enrollment Estimates. Confidence in the results of any questionnaire survey depends on the quality of the sample, the rate of response, and, most important, on the extent to which respondents are representative of the population as a whole. In designing the survey sample, the number of institutions chosen in each control/type category was determined by the desire to have 95% confidence that absolute error in estimates would not exceed 4.5%. Several empirical tests of the estimation procedure confirm that the precision requirement has been met. For example, it is known that total Fall 1974 enrollment in the 897 public two-year colleges was 3,273,265 [D]. The estimation procedures described above, when applied to known enrollments of respondent two-year colleges, led to an estimated national figure of 3,399,504, an over-estimate of 3.9%. The complete set of such estimation checks appears in Table 1.3.

Table 1.3

COMPARISON OF ESTIMATED AND ACTUAL DEGREE CREDIT ENROLLMENTS IN ALL INSTITUTIONS

Control/Type	Estimated Enrollment	Actual Enrollment	Error
1. Public University	2,014,661	2,006,723	+ .4%
2. Private University	713,751	695,583	+2.6%
3. Public Four-Year College	2,655,810	2,625,266	+1.2%
4. Private Four-Year College	1,335,225	1,284,302	+4.0%
5. Public Two-Year College	3,399,504	3,273,265	+3.9%
6. Private Two-Year College	114,875	111,585	+2.9%

In a few cases respondents were not uniformly distributed throughout the sample. For example, in one cluster of 101 public colleges the eight responses were from institutions about 50% larger than the cluster average. In this case appropriate adjustment in weighting factors led to better estimates.

Given the above checks on estimation procedures, one might still quite reasonably ask 'Do the patterns of mathematics enrollments and faculty characteristics in non-respondent institutions differ in significant ways from those completing the survey questionnaire?' Responses were received from mathematical science departments in 10% of all U. S. institutions of higher education, institutions which have 20% of all higher education enrollments. However, the overall questionnaire response rate was only 60% of the sample (as low as 54% for private universities).

In contrast to more common opinion surveys, the CBMS questionnaire asked each responding department to assemble, often from disparate sources, detailed information about its program and staff. Comments from many respondents suggest that timing of the survey (calling for Fall data well after the Spring semester had begun) made completion of the questionnaire particularly troublesome. This factor in low response does not seem likely to have caused distortion in the actual respondent data.

In every control/type category response rates for the 1975-76 survey were lower than in previous CBMS efforts. But this decline seems consistent with an acknowledged pattern in all survey research -- as individuals and institutions face sharply increased numbers of such survey requests, more and more become non-respondents. Again, this factor does not seem to undermine, in any obvious way, the data patterns established by actual respondents.

The most reliable check on validity of response data is to sample the non-respondents and compare the results of this collection with the original respondents. The survey committee identified fourteen non-respondents institutions, concentrating on control/type categories and geographical regions notable by under-representation in the respondents, and mailed special requests for response to the mathematical science departments in those institutions. Ultimately, ten of these non-respondents did complete the questionnaire and the findings were compared with estimates based on the first collection of responses. In general the original estimates were supported, but wherever this second round suggested modification of estimates or cautions on interpretation, the results have been included.

Structure of the Report

Universities, four-year colleges, and two-year colleges are increasingly part of higher education systems with complex interrelationships of instructional program, course enrollments, and faculty characteristics. Changes in any aspect of one institution have implications for and are often caused by changes in the others. The survey data and analyses of this study are presented in two main parts: Part I, devoted to universities and four-year colleges, and Part II, to two-year institutions. However, there are frequent cross-references, and clear understanding of undergraduate education in the mathematical sciences requires careful consideration of the entire document.