

## Chapter 5

# First-Year Courses in Four-Year Colleges and Universities

The tables in this chapter explore the mathematics and statistics courses of four-year colleges and universities that generally are taught to beginning students. Tables S.6, S.7, S.8, S.9, S.13(A) and S.13(B) from Chapter 1, and Tables E.2, E.3, and E.5 from Chapter 3 are broken down by the level of department in this chapter to provide more information about the following courses, which tend to be the focus of the early college experience:

1. All introductory-level courses (Table FY.1)
2. College Algebra, Trigonometry, Precalculus (Tables FY.1, FY.2)
3. Introductory courses for pre-service elementary school teachers (Table FY.1)
4. Mainstream Calculus (Tables FY.3, FY.4)
5. Non-Mainstream Calculus (Table FY.5)
6. Elementary Statistics (Tables FY.6, FY.7, FY.8, and FY.9).

The introductory-level courses, listed in the 2010 Four-Year Mathematics Questionnaire (Appendix IV), are the same courses as in the 2005 survey: non-calculus courses for liberal arts students, Finite Mathematics, Business Mathematics, Mathematics for Elementary School Teachers, College Algebra, Trigonometry, Precalculus, Elementary Functions, Modeling, and “Other”. Mainstream Calculus courses are the calculus courses needed for the mathematics major, or for applications in the physical sciences or engineering. Other calculus courses, which tend to be for business, social science, or life science majors, are labeled Non-Mainstream Calculus. In past CBMS surveys the elementary statistics courses are the statistics (or probability and statistics) courses that have no calculus prerequisite. In the 2010 CBMS survey, an introductory course (for non-majors) with a calculus prerequisite was added to the questionnaire.

Beginning courses build the interest and skills that students need for further study of mathematics and the many other disciplines that use mathematics or statistics. These courses constitute a substantial portion of four-year mathematics and statistics departments’ course enrollments. Hence, these courses merit the careful consideration of the mathematical sciences community. The issues addressed in this chapter are the course enrollments, the appointment type of the

course instructors, and the methods used in teaching these courses.

**Standard errors:** As the estimates produced from the survey data are broken down more finely, the estimates are made over smaller sets of departments, and the standard errors typically increase, sometimes to magnitudes that make the estimates rather uncertain. This phenomenon occurs particularly in the masters-level mathematics and statistics departments, which are smaller in number and possibly less homogeneous than the other levels of departments. Standard errors for all CBMS2010 tables can be found in Appendix VII.

**Enrollments:** (Tables FY.1, FY.3, FY.5, FY.6, FY.9, and Appendix I)

Table E.2 in Chapter 3 presented total enrollments, including distance-learning enrollments, in the first-year courses discussed in this chapter. The tables presented in this chapter do not include distance-learning enrollments. For comparison, Tables A.1, A.2, and A.3 in Appendix I give enrollments (with distance learning included) for fall 2000, 2005, and 2010 for each of the courses in the four-year mathematics and statistics questionnaires. Appendix I also gives the enrollments with distance learning excluded for fall 2010, except for advanced courses (where distance-learning enrollments were not gathered). Unless presented in some table in CBMS2005, the fall 2010 enrollments without distance learning are not comparable to enrollments in the 2005 or earlier CBMS survey reports. In the discussion that follows, we present enrollments without distance-learning enrollments whenever these are available for some preceding years; we use enrollments with distance learning included when necessary to compare to previous years.

### Introductory courses:

- Of the introductory mathematics courses, the course titled “College Algebra” has the largest course enrollments (excluding distance-learning enrollments) for each level of department in fall 2010. The introductory mathematics course with the second highest enrollment in fall 2010 at doctoral-level mathematics departments is Precalculus, and at masters-level and bachelors-level depart-

ments the course is Mathematics for the Liberal Arts. See Table FY.1.

- The sum of the enrollments (including the distance-learning enrollments) in the courses listed on the four-year mathematics department CBMS questionnaire as “Finite Mathematics” and “Mathematics for the Liberal Arts” were 133,000 in 1995, 168,000 in 2000, and 217,000 in 2005, but only 209,000 in 2010. The Finite Mathematics enrollments were down 34% over 2005, while the Mathematics for the Liberal Arts enrollments were up 20% from 2005 to 2010. See Appendix I, Table A.1.

### **College Algebra, Trigonometry, Precalculus:**

- The total enrollments in the cluster of the four courses that were listed on the questionnaire as College Algebra, Trigonometry, College Algebra and Trigonometry, and Precalculus (Elementary Functions) have been generally rising, except in the 2005 CBMS survey, where they showed a decline. The total (non-distance-learning) enrollments in these four courses at all four-year mathematics departments (combined) were roughly 368,000 in fall 1995, 386,000 in 2000, 352,000 in 2005, and 431,000 in 2010 (Table FY.1). Hence, there has been a 22% increase in total enrollment in these four courses since 2005 and a 17% increase since 1995. In fall 2010, the sum of the enrollments in these four classes represented 21% of all doctoral-level undergraduate enrollments, 22% of masters-level undergraduate enrollments, and 24% of bachelors-level enrollments (in all cases, distance-learning enrollments are excluded). See Table FY.1.

### **Introductory mathematics courses for pre-service elementary teachers:**

- Non-distance-learning enrollments in introductory courses in mathematics departments designed for pre-service elementary teachers continued an increasing trend. In fall 1995, the enrollment was roughly 59,000, in 2000 it was 68,000, in 2005 it was 72,000, and in 2010 it rose to 80,000, up 36% since 1995 and 11% over 2005. See Table FY.1.

### **Mainstream Calculus:**

- Mainstream Calculus I had (non-distance-learning) enrollment in fall 2010 of roughly 233,000, up 16% from fall 2005 (Chapter 1, Table S.6) and up 23% from fall 2000 (CBMS2005, Chapter 1, Table S.7). Most of the enrollment gains took place at the masters- and bachelors-level departments (masters-level Mainstream Calculus I enrollment was up 37%, and bachelors-level Mainstream

Calculus I enrollment was up 31% from 2005 to 2010). See Table FY.3.

- Mainstream Calculus II had (non-distance-learning) enrollment in fall 2010 of roughly 128,000. The CBMS2005 survey had reported enrollments of 85,000, and the 2000 survey reported enrollments of 87,000. Hence, in fall 2010, the enrollment in Mainstream Calculus II was up 51% over 2005. Most of the enrollment growth occurred at masters- and bachelors-level departments. See Table FY.3.

### **Non-Mainstream Calculus:**

An error in the 2010 four-year mathematics department CBMS survey instrument clouds the interpretation of the data for Non-Mainstream Calculus. The questionnaire asked for enrollments in Non-Mainstream Calculus I (broken down by lecture/recitation sections, classes with 30 or fewer students, and classes with enrollments larger than 30), followed by a request for “Non-Mainstream Calculus I, II, III, etc.” enrollments (not broken down by various section sizes). The intention had been to combine all Non-Mainstream Calculus enrollments above Non-Mainstream Calculus I, and hence, Non-Mainstream Calculus I should not have been included in the second list of courses. From other data provided, it was clear that some departments listed Non-Mainstream Calculus I enrollments in both rows, and reviewing the data, with some follow-up correspondence with some of the departments, the data were interpreted as best as could be.

- With the above caveats, Table FY.5 shows that Non-Mainstream Calculus I enrollment (not including distance-learning courses) was 99,000 in fall 2010, compared to 108,000 in fall 2005 (according to CBMS2005 Table FY.6), with almost the entire decline occurring at bachelors-level departments. Given the number of students obtaining credit for AP Mainstream Calculus I (see Chapter 3, Table E.15) and the rise in Mainstream Calculus I enrollments, perhaps it is not surprising that Non-Mainstream Calculus I enrollments would decline, particularly at the bachelors-level institutions. See Table FY.5.
- The 2010 survey data, interpreted as explained, showed that the Non-Mainstream Calculus II, III, etc. enrollment (excluding distance-learning courses) of roughly 22,000 in fall 2010 was double the fall 2005 enrollment (excluding distance learning courses) in Non-Mainstream Calculus II (CBMS2005, Table S.8). Comparing enrollments that include distance learning (since those were the only enrollments for these courses that are broken down by level of department in the 2005 report) that appear in Appendix I, Table A.1, almost all of the growth occurred at the masters- and

bachelors-level departments. The rise in these enrollments may be due to the broadened description of Non-Mainstream Calculus II to include other courses, and it is also possible that some departments entered their Non-Mainstream Calculus I enrollment in the Non-Mainstream Calculus I, II, III, etc. row (as we noted, Non-Mainstream Calculus I enrollments were lower in 2010 than in 2005), though some departments verified that their Non-Mainstream Calculus II, III, etc. enrollments actually were larger than their Non-Mainstream Calculus I enrollments. More clarity in the statistics for Non-Mainstream Calculus courses should come with the 2015 survey. See Table FY.5.

### Elementary Statistics:

The 2010 four-year mathematics CBMS questionnaire listed four elementary statistics courses: (F1) Introductory Statistics (no calculus prerequisite), (F2) Introductory Statistics (calculus prerequisite, for non-majors), (F3) Probability and Statistics (no calculus prerequisite), and (F4) other introductory probability and statistics courses. Course F2 was included in the CBMS survey for the first time in 2010.

- Total (including distance-learning) enrollments in elementary probability and statistics courses taught in mathematics departments of four-year colleges and universities (the sum of courses F1, F2, F3, and F4 from the four-year mathematics questionnaire) have increased to roughly 231,000 in fall 2010, up 56% over 2005 (CBMS2005, Appendix I, Table A.2). Without including the course F2 enrollments, the sum of the enrollments (including distance learning) for courses F1, F3, and F4 in mathematics departments was roughly 205,000 in 2010, up 39% from 2005.
- Table FY.6 presents the (non-distance-learning) enrollments in Introductory Statistics (no calculus prerequisite, course F1) and Probability and Statistics (no calculus prerequisite, the sum of courses F3 and F4), which both are significantly up in 2010 over 2005 at the doctoral- and bachelors-level departments. In addition to the enrollments in these courses, Appendix I, Table A.2 shows that course F2, Introductory Statistics (with a calculus prerequisite, for non-majors), enrolled an additional 23,000 students (non-distance-learning), producing a total elementary probability and statistics enrollment (not including distance-learning courses) in four-year mathematics departments of 218,000 students, just below the Mainstream Calculus I enrollments. See Table FY.6 and Appendix I, Table A.2.

The 2010 four-year statistics department questionnaire listed five elementary statistics courses. Listed courses for non-majors/minors were (E1)

Introductory Statistics (no calculus prerequisite) and (E2) Introductory Statistics (calculus prerequisite, not for majors). Other listed introductory courses were (E3): Statistics for Pre-service Elementary or Middle School Teachers, (E4): Statistics for Pre-service Secondary School Teachers, and (E5): Other elementary-level statistics courses.

- The 2010 CBMS survey was the first survey in which an introductory statistics course for non-majors/minors with a calculus prerequisite was listed on the CBMS statistics questionnaire, and in fall 2010, this course enrolled (not including distance-learning enrollments) roughly 16,000 students, compared to roughly 56,000 in the introductory course without a calculus prerequisite (Table FY.9). The enrollment of 56,000 in the introductory statistics course without a calculus prerequisite represents a 33% increase over the 2005 non-distance-learning enrollment in that course (see CBMS2005, Table FY.10, p. 131). See Table FY.9.
- When all introductory statistics department enrollments (including distance-learning enrollments) for courses E1 through E5 are combined, statistics departments had a total enrollment of roughly 81,000 students in introductory statistics courses for non-majors/minors, a 50% increase from the enrollment of roughly 54,000 in 2005 (CBMS2005, Appendix I, Table A.2). This enrollment in statistics department introductory courses was a little more than one-third of the enrollment in all of the elementary probability and statistics courses in four-year mathematics departments. See Table FY.9 and Appendix I, Table A.2.

### Appointment Type of First-Year Course Instructors (Tables FY.1, FY.3, FY.5, FY.6, FY.9)

In Chapter 3, the appointment type of course instructors was considered for various course categories; in this chapter, the appointment type of instructors in first-year courses is considered, and these data are broken down by the level of the department. For the CBMS2010 survey, faculty at four-year institutions were split into four categories: tenured, tenure-eligible, and permanent faculty (TTE), other full-time faculty (OFT) who are full-time but not TTE, part-time faculty, and graduate teaching assistants (GTAs). A course was to be reported as being taught by a GTA if and only if the GTA was the “instructor of record” for the course. GTAs who ran discussion or recitation sections as part of a lecture/recitation course were not included in this category.

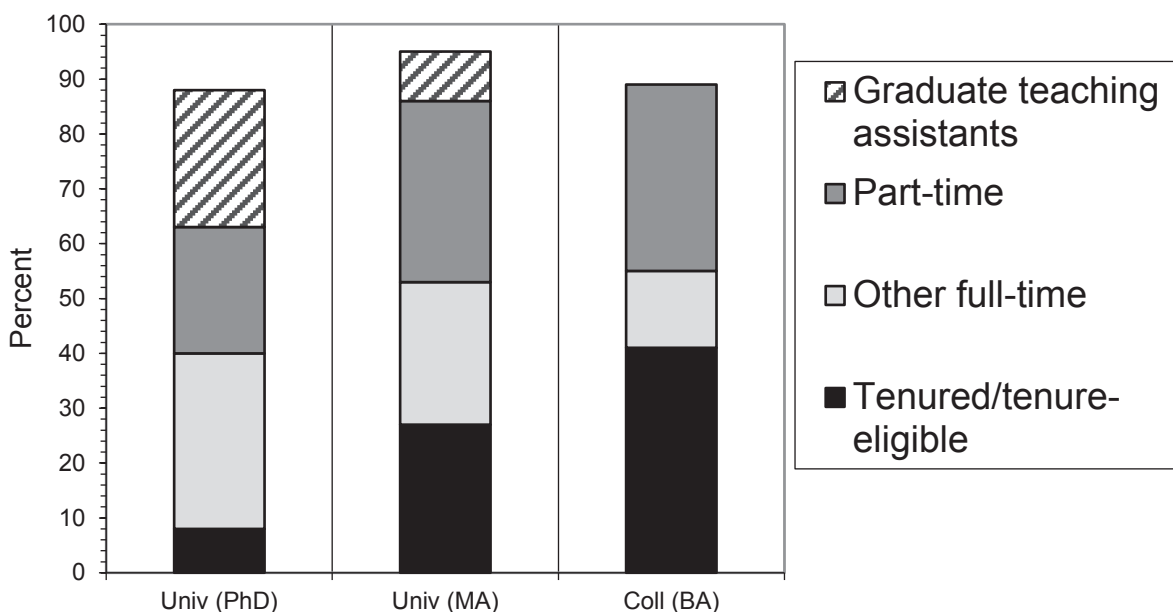
In past CBMS surveys, the TTE category was labeled “tenured/tenure-eligible” on the survey questionnaire without the word “permanent”, but in the instructions, departments at institutions that did not recognize

**TABLE FY.1** Percentage of sections (excluding distance-learning sections) of certain introductory-level courses taught by various types of instructors in mathematics departments in fall 2010, by type of department. Also average section sizes and enrollments (not including distance learning enrollments). For comparison, some enrollments in these courses are in Table FY.2, p. 116, in CBMS2005.

Course & Department Type	Percentage of sections taught by												Average Section Size			Enrollment (1000s)					
	Tenured/tenure-eligible/permanent <sup>1</sup> %		Other full-time %		Part-time %		Graduate teaching assistants %		Unknown %												
	PhD	MA	BA	PhD	MA	BA	PhD	MA	BA	PhD	MA	BA	PhD	MA	BA	PhD	MA	BA			
Mathematics for Liberal Arts	16	34	40	24	22	18	17	38	30	20	0	0	23	6	12	43	33	28	43	38	60
Finite Mathematics	10	32	43	47	26	5	14	35	40	22	0	0	7	7	12	59	29	25	27	8	25
Business Math (non-calculus)	10	25	66	40	24	12	17	48	21	25	0	0	9	3	2	52	32	20	22	12	11
Math for Elem Sch Teachers	20	57	53	35	21	7	15	19	27	14	0	0	16	2	12	29	28	24	15	29	36
College Algebra	5	20	34	39	27	18	16	26	32	28	19	0	12	7	16	47	34	30	88	55	99
Trigonometry	8	36	59	27	34	13	15	19	25	42	0	0	7	11	3	45	30	32	16	9	16
College Alg & Trig (combined)	9	33	31	33	27	14	14	23	55	33	0	0	11	17	1	49	38	31	18	7	12
Elem Functions, Precalculus	5	25	48	33	23	17	27	36	30	28	13	0	8	3	5	47	30	25	46	28	39
Intro to Math Modeling	8	62	75	20	0	7	43	38	9	23	0	0	6	0	9	41	40	23	4	1	3
All other intro-level non-Calculus courses	31	23	49	21	26	21	18	45	21	25	6	0	4	0	9	68	28	25	15	18	33
Total All Intro Level Courses	8	27	41	32	26	14	23	33	34	25	9	0	12	5	11	44	31	26	292	206	336

Note: 0 means less than one half of 1%. Inconsistencies in column sums are due to round-off.

<sup>1</sup> Beginning in 2010, the CBMS survey added the word "permanent" to the description "tenured/tenure eligible" that was used previously.



**FIGURE FY.1.1** Percentage of sections (excluding distance-learning sections) in introductory-level mathematics courses taught in mathematics departments by various kinds of instructors in fall 2010, by type of department. (Deficits from 100% represent unknown instructors.)

tenure (estimated at 12% of all four-year mathematics departments in the CBMS2010 survey and 5% in the CBMS2005 survey) were instructed to place permanent faculty in the TTE category. The 2010 survey directors decided to add the label “permanent” to the TTE category, and this change may have added to the TTE category other instructors who have teaching positions that are regarded as permanent, although these faculty do not have tenure and are not eligible for tenure, even if their institution recognizes tenure. The instructions did not define “permanent” beyond the situation where the institution does not recognize tenure, but it seems quite possible that some departments interpreted “permanent faculty” to have this additional meaning, and some of the data suggest that this was the case. Hence, the addition of the word “permanent” may mean that faculty who might be classified as “teaching faculty” who have renewable contracts but are not tenured or tenure-eligible may have been added to the TTE category, even if the institution recognizes tenure. As a consequence of this change, the other full-time category probably consists primarily of postdocs and other temporary academic visitors.

The 2010 CBMS survey followed the practice established in the 2005 survey of presenting findings in terms of percentages of “sections” offered. In analyzing the 2010 survey data, it seems that the notion of “section” varies somewhat among different departments, particularly for lower-level classes

that may be taught with a laboratory component. A further, and possibly related, problem experienced in the 2010 survey was the inconsistent numbers of faculty and sections reported by some departments; this problem had occurred in past surveys and was resolved by creating the category of “unknown” instructors. The 2010 survey produced increased numbers of “unknown” faculty over past surveys, making it difficult to draw conclusions about changes in the percentages of the various ranks of instructors teaching specific courses. When comparing data from CBMS2000 and earlier surveys, one must keep in mind a change made in 2005. In some cases, CBMS2000 and earlier surveys presented data on who taught the course in terms of percentages of enrollments rather than percentages of sections.

- Table FY.1 and Figure FY.1.1 present data on who taught introductory-level courses. At doctoral-level mathematics departments, the courses with the lowest percentages of TTE faculty instructors were the cluster of four introductory classes (college algebra, trigonometry, algebra and trigonometry, and precalculus classes); at doctoral-level mathematics departments, over all introductory classes (combined), only 8% of the sections were taught by TTE faculty, 32% by other full-time, 23% by part-time faculty, and 25% by GTAs. At the bachelors-level mathematics departments, 41% of introductory classes were taught by TTE faculty, 14% by OFT faculty, and 34% were taught by

**TABLE FY.2** Percentage of sections of College Algebra in which various practices in teaching are used by mathematics departments at four-year colleges and universities in fall 2010.

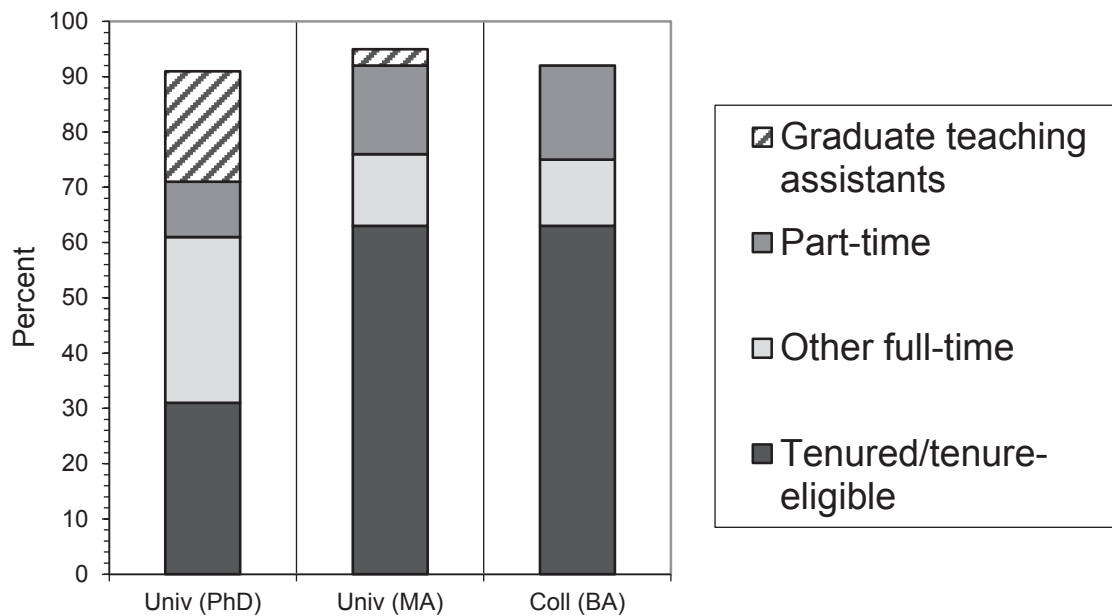
Practices used in teaching College Algebra	Univ (PhD)		Univ (MA)		College (BA)		Total	
	Percentage of all sections, nationally	Mean of department-reported percentages	Percentage of all sections, nationally	Mean of department-reported percentages	Percentage of all sections, nationally	Mean of department-reported percentages	Percentage of all sections, nationally	Mean of department-reported percentages
a. Emphasize problem solving in the modeling sense	38	38	64	60	40	54	44	53
b. Include elementary data analysis	35	24	19	27	25	26	27	26
c. Include writing assignments	11	13	21	15	17	28	16	23
d. Include small group activities	26	24	44	38	39	47	36	42
e. Include small group projects	11	3	32	20	23	27	20	22
f. Include class presentations	4	5	4	4	14	15	9	12
g. Use graphing calculators	46	46	77	78	73	75	66	72
h. Use spreadsheets	1	1	10	0	7	11	5	8
i. Use online homework generating and grading packages	76	71	75	60	58	54	68	58
j. Use classroom response systems (e.g., clickers)	13	10	0	0	10	9	9	8
k. Primarily use a traditional approach	60	64	65	68	69	72	65	70

**TABLE FY.3** Percentage of sections (excluding distance-learning sections) in Mainstream Calculus I and Mainstream Calculus II taught by various types of instructors in four-year mathematics departments in fall 2010, by size of sections and type of department. Also average section sizes and enrollments (not including distance-learning enrollments). This table can be compared to Table FY.3, p. 117 and, for enrollments, to Table FY.4, p.119 of CBMS2005.

Course & Department Type	Percentage of sections taught by												Average Section Size			Enrollment (1000s)						
	Tenured/tenure-eligible/permanent <sup>1</sup> %		Other full-time %		Part-time %		Graduate teaching assistants %		Unknown %		PhD	MA	BA	PhD	MA	BA	PhD	MA	BA			
Mainstream Calculus I																						
Lecture / recitation		33	82	50	29	18	8	12	0	32	19	0	0	7	0	10	71	39	31	70	8	28
Regular section <31		41	56	70	20	22	17	5	12	11	24	0	0	9	11	2	24	25	20	7	7	35
Regular section >30		25	60	63	35	8	2	9	22	13	19	5	0	11	4	22	39	35	35	34	26	18
Total Mainstream Calculus I		31	63	63	30	13	12	10	16	17	20	3	0	9	5	8	52	33	25	110	41	82
Mainstream Calculus II																						
Lecture / recitation		48	97	45	24	3	9	11	0	44	10	0	0	7	0	2	72	39	34	37	3	21
Regular section <31		49	71	83	20	11	6	9	1	5	21	0	0	1	16	6	24	21	18	5	3	14
Regular section >30		39	62	55	31	9	8	9	2	5	12	23	0	9	4	32	40	35	35	19	18	9
Total Mainstream Calculus II		45	67	64	26	9	8	10	2	18	13	16	0	7	6	10	51	32	26	61	23	44
Total Mainstream Calculus I & II		36	64	64	29	11	10	10	11	18	17	8	0	8	5	9	52	33	26	171	65	126

Note: 0 means less than one half of 1%. Inconsistencies in column and row sums are due to round-off.

<sup>1</sup> Beginning in 2010, the CBMS survey added the word "permanent" to the description "tenured/tenure eligible" that was used previously.



**FIGURE FY.3.1** Percentage of sections (excluding distance learning) in Mainstream Calculus I in four-year mathematics departments by type of instructor and type of department in fall 2010. (Deficits from 100% represent unknown instructors.)

**TABLE FY.4** Percentage of four-year mathematics departments with various practices in teaching Honors Calculus in fall 2010, by type of department.

	Mathematics Departments			
	Univ (PhD)	Univ (MA)	College (BA)	All Depts. Combined
Percentage that offer an Honors Calculus course	65	26	10	20
Of those that offer Honors Calculus, the percentage of depts that offer it for:				
Calculus I	71	73	66	69
Calculus II	88	85	97	91
Calculus III	74	32	17	48
Of those that offer Honors Calculus, compared to Mainstream Calculus, the percentage of departments where Honors Calculus:				
Contains more theory	95	84	84	89
Contains more applications	57	59	88	69
Is aimed at mathematics majors	32	56	43	40
Requires a test or placement mechanism as a prerequisite	75	95	59	72
Can be selected by any interested student	18	5	17	15



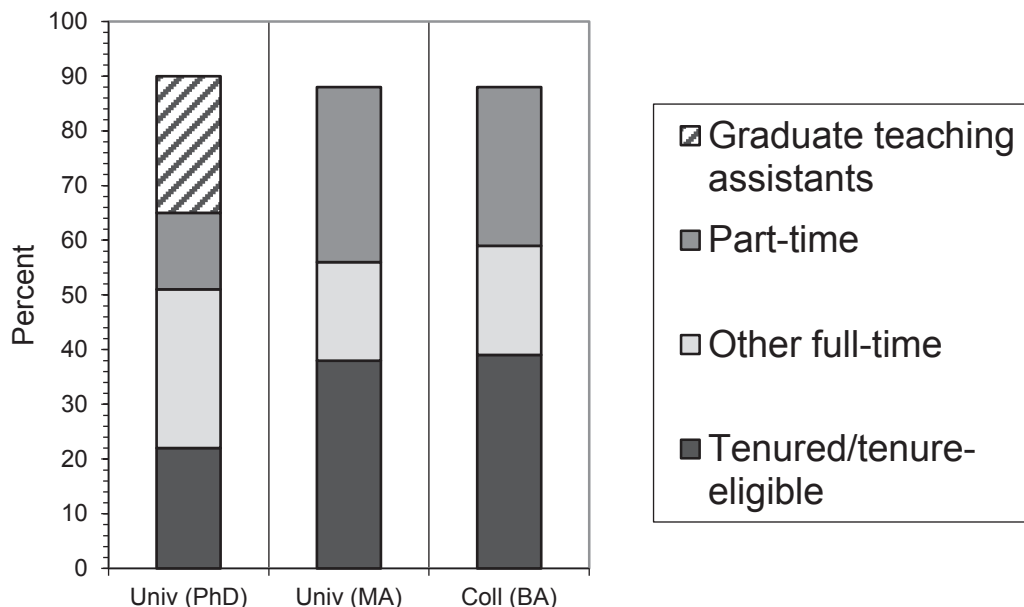
**TABLE FY.5** Percentage of sections (excluding distance-learning sections) in Non-Mainstream Calculus I and in Non-Mainstream Calculus II, III, etc.<sup>2</sup> taught by various types of instructors in mathematics departments in fall 2010, by size of sections and type of department. Also average section size and enrollments (not including distance-learning enrollments). This table can be compared to Table FY.5, p. 121 and, for enrollments, to Table FY.6, p. 123 in CBMS2005.

Course & Department Type	Percentage of sections taught by										Average Section Size		Enrollment (1000s)								
	Tenured/ tenure-eligible/ permanent <sup>1</sup> %	PhD	MA	BA	Other full-time %	Part-time %	Graduate teaching assistants %	Unknown %	PhD	MA	BA	PhD	MA	BA	PhD	MA	BA				
Non-Mainstream Calculus I																					
Lecture / recitation	31	60	29	28	20	39	17	20	26	15	0	0	9	0	6	74	33	29	27	3	5
Regular section <31	16	43	41	21	23	15	11	20	32	45	2	0	7	13	12	27	25	22	6	3	7
Regular section >30	18	31	44	33	16	13	13	38	25	24	0	0	13	15	18	52	39	36	27	15	5
Total Non-Mainstream Calculus I	22	38	39	29	18	20	14	32	29	25	0	0	10	12	12	54	35	27	60	22	17
Total Non-Mainstream Calculus II, III, etc. <sup>2</sup>	18	22	60	21	32	0	12	44	10	25	0	0	24	3	31	35	33	19	12	5	5
Total Non-Mainstream Calculus I, II, III, etc.	21	35	45	27	21	14	13	34	23	25	0	0	13	11	18	50	35	25	72	27	23

Note: 0 means less than one half of 1% in columns 1 through 18. Inconsistencies in row and column sums are due to round-off.

<sup>1</sup> Beginning in 2010, the CBMS survey added the word "permanent" to the description "tenured/tenure eligible" that was used previously.

<sup>2</sup> The 2010 survey mistakenly asked for *Non-mainstream Calculus I, II, and III, etc.*; the data here are our best estimate for *Non-mainstream Calculus II, III, etc.*



**FIGURE FY.5.1** Percentage of sections (excluding distance-learning sections) in Non-mainstream Calculus I in four-year mathematics departments taught by various kinds of instructors, by type of department in fall 2010. (Deficits from 100% represent unknown instructors.)

part-time faculty. The percentages for masters-level departments were generally in between the doctoral- and the bachelors-level departments. See Table FY.1 and Figure FY.1.1.

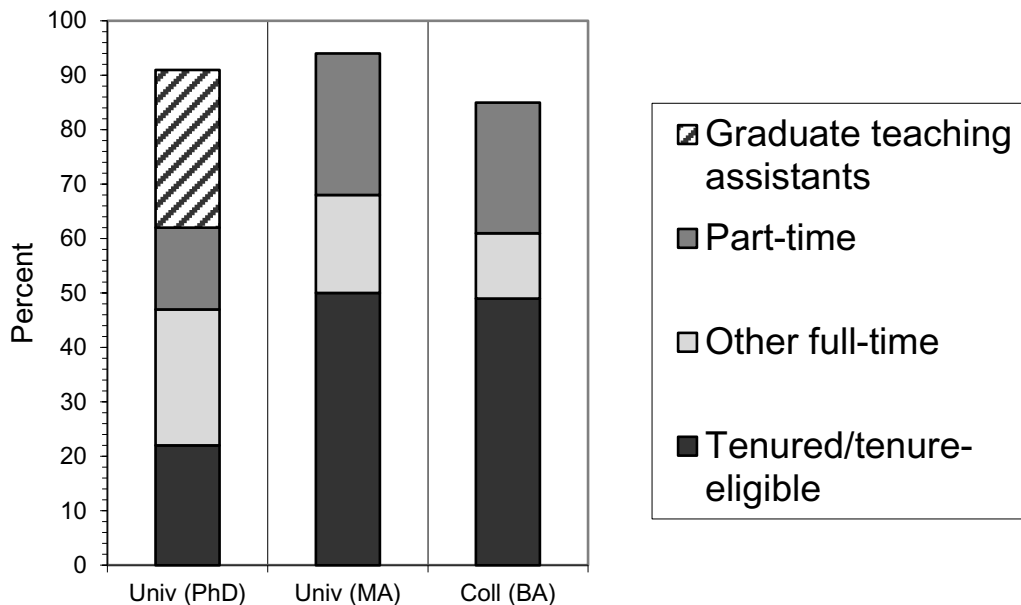
- Table FY.3 and Figure FY.3.1 present data on who taught Mainstream Calculus I and II. For Mainstream Calculus I, at doctoral-level mathematics departments, over all types of sections, 31% of the sections were taught by TTE faculty, while at the bachelors- and masters-level mathematics departments, over all types of sections, 63% of Mainstream Calculus I sections were taught by TTE faculty. In 2005, these percentages were 36% for doctoral-level departments, 73% for masters-level departments, and 79% for bachelors-level departments. The average section size for the total Mainstream Calculus I at the doctoral-level departments was double that of the bachelors-level departments, and the average section sizes in 2010 were close to those in 2005. Across all types of faculty in fall 2010, the percentages of faculty teaching Mainstream Calculus II and its average section size were relatively close to those for Mainstream Calculus I. A notable change from 2005 was the percentage of TTE faculty who taught Mainstream Calculus II at bachelors-level departments: down to 64% in 2010 from 94% in 2005, though there is a large standard error (13%) in the 2010 estimate. See Table FY.3 and Figure FY.3.1.
- Table FY.5 and Figure FY.5.1 present data on who taught Non-Mainstream Calculus. At the doctoral level, for Non-Mainstream Calculus I in fall 2010, slightly over 20% of the sections were taught by TTE faculty, while at the bachelors- and masters-level, this percentage was slightly under 40%. This is a notable decrease from 2005, when these percentages were 43% at doctoral-level departments, 45% at masters-level departments, and 68% at bachelors-level departments (but there are large standard errors for masters- and bachelors-level estimates in 2010). The average section sizes of Mainstream and Non-Mainstream Calculus I in 2010 are approximately the same size, and the average section size across all sections of Non-Mainstream Calculus I was up by 2 students in 2010 over 2005 at each of the three levels of departments.
- Table FY.6 and Figure FY.6.1 present data on who taught three elementary probability and statistics courses that do not have a calculus prerequisite in mathematics departments of four-year colleges and universities. At the doctoral-level mathematics departments, almost 25% of the total sections of the three courses were taught by TTE faculty, while at the bachelors- and masters-level departments, the percentage was roughly 50%. This percentage was about the same at the doctoral- and masters-level departments and was slightly down from the percentages in 2005 at the bachelors-level depart-

**TABLE FY.6** Percentage of sections (excluding distance-learning sections) in Elementary Statistics (non-Calculus) and Probability and Statistics (non-Calculus) taught by various types of instructors in mathematics departments in fall 2010, by size of sections and type of department. Also average section size and enrollments (not including distance learning enrollments). Comparable 2005 data is in CBMS2005, Table FY.7, p. 125 and for enrollments, in Table FY.8, p. 127.

	Percentage of sections taught by										Average Section Size			Enrollment (1000s)							
	Tenured/ tenure- eligible/ permanent <sup>1</sup> %	PhD	MA	BA	Other full-time %	PhD	MA	BA	Part-time %	PhD	MA	BA	PhD	MA	BA	PhD	MA	BA			
Course & Mathematics Department Type	PhD	MA	BA	PhD	MA	BA	PhD	MA	BA	PhD	MA	BA	PhD	MA	BA	PhD	MA	BA			
Elementary Statistics (F1) (non-calculus)																					
Lecture / recitation	36	66	43	22	18	3	10	3	32	21	0	0	11	13	21	48	38	30	6	6	34
Regular section <31	6	39	50	28	22	16	6	35	27	29	1	0	31	3	8	27	20	22	4	4	46
Regular section >30	23	50	56	25	15	16	20	30	8	31	0	0	1	5	21	65	38	37	28	16	30
Total Elementary Statistics	22	50	49	25	18	12	15	26	24	29	0	0	9	6	14	55	33	27	38	27	110
Probability & Statistics (non-Calculus) (F3 + F4)	30	52	47	17	10	7	15	24	21	20	5	7	18	9	18	57	32	25	4	7	9
Total, all non-calculus elementary probability & statistics courses	23	51	49	24	16	12	15	25	24	28	1	1	10	7	14	55	33	27	42	34	119

Note: 0 means less than one half of 1%. Some row and column sums appear inconsistent due to round-off.

<sup>1</sup> Beginning in 2010, the CBMS survey added the word "permanent" to the description "tenured/tenure eligible" that was used previously.



**FIGURE FY.6.1** Percentage of sections (excluding distance-learning sections) in Elementary Statistics (non-Calculus) in four-year mathematics departments, by type of instructor and type of department in 2010. (Deficits from 100% represent unknown instructors.)

ments. At doctoral-level departments, about 28% of the sections of the combined courses were taught by GTAs (compared to 22% in 2005). The average section size at doctoral-level mathematics departments was up from 47 students in 2005 to 55 students in 2010 (but with a standard error of about 7 students).

- Table FY.9 and Figure FY.9.1 present data on who taught introductory probability and statistics courses for non-majors/minors in statistics departments. The percentage of TTE faculty who taught the course (labeled E2 on the statistics questionnaire) with a calculus prerequisite was 36% at doctoral-level departments and 59% at masters-level departments, while the course without the calculus prerequisite (course E1) had TTE faculty teaching 19% of the sections in doctoral-level departments and 44% of masters-level departments (smaller percentages than for the no-calculus-prerequisite course taught in mathematics departments). At doctoral-level departments, the percentage of sections taught by GTAs was 24% for course E1 (about the same as in 2005) and half that percentage for course E2. The average section sizes for the no-calculus-prerequisite statistics course taught in mathematics departments (course F1) and statistics departments (course E1) were about the same.

**Teaching Methods** (Tables FY.2, FY.4, FY.7, FY.8)

**College Algebra** (Table FY.2):

The questions on the teaching of College Algebra were constructed with the help of the MAA's CRAFTY (Curriculum Renewal Across the First Two Years) committee that had written a report [CRAFTY] on the teaching of College Algebra. The precise wording of the questions can be found by consulting the Four-Year Mathematics Questionnaire, question H1, located in Appendix IV. The survey instrument instructed each department to give the number of sections of the course College Algebra to which each of 11 aspects of College Algebra pedagogy applied. Table FY.2 presents two different averages: first, the overall average number of sections where each aspect is present (i.e., the total number of sections in the U.S. where the aspect was present, divided by the number of all sections of College Algebra in the U.S.), and second, the average of the departmental average numbers of sections where the aspect is present (i.e. for each department, the number of sections where the aspect was present was divided by the number of sections of College Algebra at that department, then the average of these averages was computed); the table is broken down by the level of the department. About two-thirds of each level of department described their College Algebra course as "primarily

**TABLE FY.7** Percentage of mathematics departments using various practices in the teaching of Elementary Statistics (no calculus prerequisite) in fall 2010 by type of department.

	Mathematics Departments			
	Univ (PhD)	Univ (MA)	College (BA)	All Depts. Combined
Percentage of departments that offer elementary statistics course with no calculus prerequisite	58	90	87	84
Of those that offer the course, the percentage of departments in which the majority of sections use real data for the following percentages of class sessions:				
0-20%	33	29	15	18
21-40%	18	15	30	27
41-60%	26	14	20	19
61-80%	5	12	18	16
81-100%	18	30	18	20
Percentage of departments where the majority of sections use in-class demonstrations for the following percentages of class sessions:				
0-20%	36	23	10	14
21-40%	21	9	33	29
41-60%	20	16	11	13
61-80%	6	16	29	25
81-100%	16	35	17	19
Percentage of departments using the following kinds of technology in the majority of sections:				
Graphing calculators	52	79	72	71
Statistical packages	49	63	54	55
Educational software	26	16	18	19
Applets	20	15	17	17
Spreadsheets	57	55	50	51
Web-based resources	61	53	54	54
Classroom response systems	11	9	10	10
Percentage of departments where the majority of sections require assessments beyond homework, exams, and quizzes	24	51	46	45

using a traditional approach (i.e., sections that were basically the same College Algebra course that was taught in 1990)". The "modeling approach: model => data => interpretation" was used most heavily at the masters-level departments. Graphing calculators were used in about three-quarters of the masters- and bachelors-level departments sections, and less than half of the doctoral-level sections. Online homework was used in about three-fourths of the sections at the doctoral- and masters-level departments, and a little over half of the bachelors-level departments. Of the less traditional methods, small group activities seemed to be used the most frequently—overall at 26% of the doctoral-level departments, 44% of the masters-level departments, and 39% of the bachelors-level departments.

**Calculus** (Table FY.4):

Since there was another major national study of calculus instruction ("Characteristics of Successful Programs in College Calculus") (<http://www.maa.org/cspcc/>) conducted parallel to the CBMS2010 survey, the CBMS survey restricted its questions about calculus pedagogy to a topic not covered in the other survey, namely "honors calculus" courses. Table FY.4 shows that 65% of doctoral-level, 26% of masters-level, and 10% of bachelors-level departments offered some kind of honors calculus course in fall 2010. Of departments that offered such a course, of the three levels of calculus at which such a course might be offered, Calculus II had the largest percentage of departments offering it. A third question asked about how honors calculus differed from Mainstream Calculus, and typically it covered more theory than Mainstream Calculus (at 95% of doctoral-level departments and 84% of both masters- and bachelors-level departments), though at bachelors-level departments it was even slightly more likely to cover more applications than Mainstream Calculus. According to Table FY.4, such "honors" courses typically required some sort of selection procedure, though at 17% of all levels of departments the course could be selected by any student.

**Elementary Statistics** (Tables FY.7 and FY.8):

As already noted, probability and statistics course enrollments have expanded, and there has been considerable interest in how these courses are taught, particularly since they are often taught outside of statistics departments (see e.g. [CAUSE], [GAISE], [Moore]). The CBMS2010 pedagogy questions about statistics courses focused on the course "Introductory Statistics (no calculus prerequisite)" in mathematics departments (course F1 in the Four-Year Mathematics Questionnaire) and "Introductory Statistics (no Calculus prerequisite) for non-majors/minors" in statistics departments (course E1 in the Four-Year Statistics Questionnaire). The questions

for four-year mathematics departments were the same as the questions in Section G of the statistics questionnaire, and they begin with question H5 in the mathematics questionnaire. The same questions were used in both instruments so that the results (Table FY.7 for mathematics departments and Table FY.8 for statistics departments) can be compared; each of these tables is broken down by level of department.

Generally, the results of the CBMS survey indicated that in teaching elementary statistics, in fall 2010, statistics departments made more use of real data, modern technology, and in-class activities that encourage student involvement than mathematics departments did. However, mathematics departments held a small edge in assigning projects beyond routine assignments. All of these aspects have been cited as important elements in teaching elementary statistics courses.

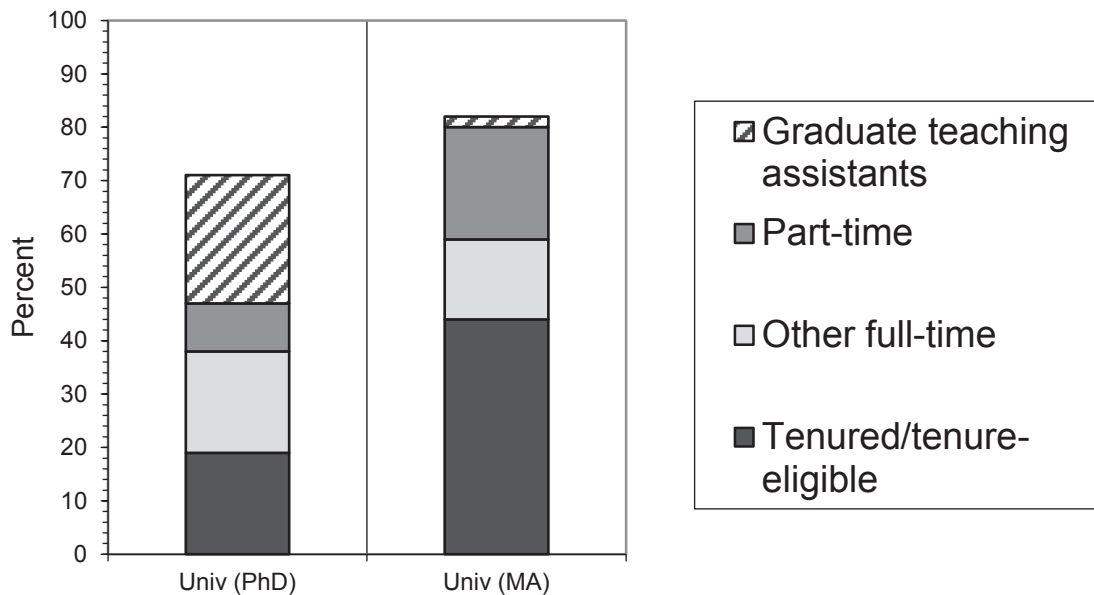
Table FY.7 shows that an elementary statistics course, with no calculus prerequisite, was offered at over half of the doctoral-level mathematics departments and at about 90% of the masters-level and bachelors-level mathematics departments. Table FY.8 shows that an elementary statistics course for non-majors/minors, with no calculus prerequisite, was offered at 90% of the doctoral-level statistics departments and at 85% of the masters-level statistics departments. The remaining table entries contain percentages of sections from departments that offered these courses. The distribution of class sessions in which real data was used shows that this distribution is more skewed to lower use of real data at mathematics departments than at statistics departments (see Tables FY.7 and FY.8), and among mathematics departments, the doctoral departments typically reported fewer sessions spent using real data than the bachelors-level departments (with the masters-level departments generally between the doctoral-level and bachelors-level; see Table FY.7). Both tendencies were also present regarding class sessions spent using "in-class demonstrations and/or in-class problem solving activities/discussions". Among mathematics departments, graphing calculators were used at about three-quarters of the bachelors-level and masters-level departments, at a little over half of the doctoral-level mathematics departments (Table FY.7), and at under 50% of statistics departments (Table FY.8). Statistical packages were used in 87% of statistics departments but only in 55% of mathematics departments (66% at masters-level departments), so statistics departments were generally using the more sophisticated technology. Similarly, educational software was used in 40% of the statistics department sections but only in 19% of all mathematics department sections (26% of doctoral-level mathematics department sections). Applets were used in 34% of statistics department sections and in 17% of mathematics department

**TABLE FY.8** Percentage of statistics departments using various practices in the teaching of Introductory Statistics for non-majors/minors (no calculus prerequisite) in fall 2010 by type of department.

	Statistics Departments		
	Univ (PhD)	Univ (MA)	All Depts. Combined
Percentage of departments that offer Introductory Statistics for non-majors/minors with no calculus prerequisite	90	85	88
Of those that offer the course, the percentage of departments in which the majority of sections use real data the following percentages of the time:			
0-20%	6	20	9
21-40%	16	20	17
41-60%	21	0	16
61-80%	24	10	20
81-100%	34	50	38
Percentage of departments where the majority of sections use in-class demonstrations in the following percentages of class sessions:			
0-20%	22	10	19
21-40%	16	40	22
41-60%	21	0	16
61-80%	16	20	17
81-100%	24	30	26
Percentage of departments using following kinds of technology in the majority of sections			
Graphing calculators	45	33	43
Statistical packages	89	80	87
Educational software	38	44	40
Applets	31	44	34
Spreadsheets	45	56	48
Web-based resources	79	60	74
Classroom response systems	26	40	29
Percentage of departments where the majority of sections require assessments beyond homework, exams, and quizzes	31	50	36

sections, while spreadsheets were used at roughly half of all surveyed departments. Web-based resources were used in 74% of statistics department sections and in 54% of mathematics department sections (61% at doctoral-level mathematics department sections). Classroom response systems (e.g. clickers) were used in 29% of statistics department sections and in 10% of mathematics department sections. One aspect of

reform pedagogy in which mathematics departments held a slim advantage was in the use of non-routine assignments. A slightly higher percentage of mathematics department sections (45%, but only 24% of doctoral-level department sections) than statistics department sections (36%) had assessments beyond homework, exams, and quizzes (e.g. projects, oral presentations, or written reports).



**FIGURE FY.9.1** Percentage of sections (excluding distance-learning sections) in Elementary Statistics (non-Calculus) taught in statistics departments in fall 2010, by type of instructor and type of department. (Deficits from 100% represent unknown instructors).



**TABLE FY.9** Percentage of sections (excluding distance-learning sections) in Introductory Statistics (non-Calculus for non-majors/minors) and Introductory Statistics (Calculus prerequisite for non-majors/minors) taught by various types of instructors in statistics departments in fall 2010, by size of sections and type of department. Also average section size and total (non-distance-learning) enrollments. Enrollments in 2005 can be found in Table FY.10, p.131 of CBMS2005.

Course & Statistics Department Type	Percentage of sections taught by										Average Section Size		Enrollment (1000s)		
	Tenured/tenure-eligible permanent <sup>1</sup> %	Other full-time (with PhD) %	Other full-time (without PhD) %	Part-time %	Graduate teaching assistants %	Unknown %	PhD	MA	PhD	MA	PhD	MA	PhD	MA	
Introductory Statistics (non-Calculus for non-majors/minors )															
Lecture / recitation	19	11	13	11	18	27	11	17	5	27	41	65	54	29	9
Regular section <31	32	17	0	13	15	24	13	23	0	0	0	16	26	1	4
Regular section >30	17	5	4	4	39	31	4	24	0	4	4	47	29 <sup>2</sup>	10	4
Total Introductory Statistics (non-Calculus)	19	10	9	9	24	28	9	21	2	28	17	55	37	40	17
Introductory Statistics (calculus prerequisite for non-majors/minors )															
Lecture / recitation	36	14	4	11	13	23	11	5	0	23	32	50	34	6	1
Regular section <31	32	10	1	3	6	47	3	3	11	8	8	15	44 <sup>2</sup>	1	3
Regular section >30	39	13	1	17	17	13	6	6	6	6	6	36	42	4	1
Total Introductory Statistics (Calculus)	36	13	2	11	12	26	11	4	7	15	15	36	40	11	5

Note: 0 means less than one half of 1%. Row and column sums may appear inconsistent due to round-off.

<sup>1</sup> Beginning in 2010, the CBMS survey added the word "permanent" to the description "tenured/tenure eligible" that was used previously.

