

Chapter 2

CBMS2015 Special Projects

Each CBMS survey accepts proposals for special projects from various professional society committees. Special projects chosen for one CBMS survey might, or might not, be continued in the next CBMS survey. This chapter presents data from the special projects of CBMS2015 for two-year and four-year mathematics departments:

- The mathematical education of teachers of pre-college mathematics/statistics (Tables SP.1-SP.7)
- Percentage of departments offering distance learning courses, and practices in distance learning courses (Tables SP.8-SP.11)
- Academic resources and special opportunities available to undergraduates (Tables SP.12-SP.14)
- Interdisciplinary courses in four-year mathematics departments (Tables SP.15)
- Dual enrollment courses in mathematics and statistics (Tables SP.16 and SP.17)
- Requirements in the majors in mathematics and statistics in four-year departments (Tables SP.18 and SP.19)
- Availability of upper level classes in four-year mathematics and statistics departments (Tables SP.20 and SP.21)
- Estimates of post-graduation plans of graduates of four-year mathematics departments and statistics departments (Table SP.22)
- Assessment in four-year mathematics departments and statistics departments (Table SP.23)
- Divisional graduation credit for advanced placement courses in four-year mathematics and statistics departments (Table SP.24)

- Pedagogy and making changes at four-year mathematics and statistics departments (Tables SP.25-27)
- Statistics majors and minors at four-year mathematics departments (Table SP.28)
- Profiles of other full-time faculty at four-year mathematics and statistics departments (Tables SP.29-31)

When there is comparable data in CBMS2010, the appropriate comparison table will be given in the caption, if the 2010 data is not included in the table. Also note that further discussion of selected special project issues at two-year colleges is given in the section “Topics of Special Interest for Mathematics Programs at Two-Year Colleges”, located at the end of Chapter 6.

Terminology: Recall that in CBMS2015, the term “mathematics department” includes departments of mathematics, applied mathematics, mathematical sciences, and departments of mathematics and statistics. These departments may offer a broad spectrum of courses in mathematics education, actuarial science, and operations research, as well as mathematics, applied mathematics and statistics. Computer science courses are sometimes also offered by mathematics departments. The term “statistics department” refers to graduate departments of statistics or biostatistics that offer undergraduate statistics courses. Courses and majors from separate departments of computer science, actuarial science, operations research, etc. are not included in CBMS2015. Departments are classified by the highest degree they offered; for example, “masters-level department” refers to a department that offers a masters degree, but not a doctoral degree.

In the text that follows, the standard error (SE) in many of the estimates is provided along with the estimate (e.g. “estimated 77% (with SE 3.5)”); the standard errors for all CBMS2015 tables can be found in Appendix VIII. The change in an estimate from the estimate in a previous survey is often expressed both as percentage change and as the number of SEs that change represents (e.g. “increased 22% (1.2 SEs)”).

TABLE SP.1 Percentage of mathematics departments whose institutions offer certification programs for some or all grades K-8, and also for secondary teachers, by type of department in fall 2015. (Data for K-8 from fall 2000, 2005, 2010 when available, in parentheses.)

	Percentage whose institutions have a certification program for:			
	K-5	6-8	K-8*	Secondary (9-12)
Mathematics Departments				
Univ (PhD)	52	47	(72,78,62)	75 (79)
Univ (MA)	63	64	(87,92,90)	92 (96)
Coll (BA)	52	50	(85,88,70)	75 (80)
Total Math Depts	53	51	(84,87,72)	77 (82)

*Prior to 2015, CBMS asked about certification for pre-service K-8 teachers, while CBMS 2015 separated K-5 from 6-8. If the results for the two questions on CBMS 2015 are combined, then 63 percent of total mathematics departments responded that they had a program for certification for K-5 and/or 6-8 teachers.

Tables (SP.1-SP.7): The Mathematical Education of Teachers of Pre-College Mathematics and Statistics

Percentages of Four-year Mathematics Departments whose Institutions have Elementary and Secondary Teacher Certification Programs

Table SP.1 shows that, in fall 2015, roughly 63% of all four-year mathematics departments combined reported belonging to an institution that offered a teacher certification program for some or all grades K-8; this compares to an estimated 72% in 2010, 87% in 2005 and 84% in 2000. In 2015, for the first time, departments were asked whether they had a K-5 certification program and/or a 6-8 grades certification program, and there were about equal numbers of departments in each category (an estimated 53% had a K-5 program and 51% had a 6-8 grades program, with SEs of about 3.5 in each case). Table SP.1 breaks these percentages down by the level of department, the masters-level departments having the largest percentage of K-8 teacher certification programs in each of the four CBMS surveys 2000, 2005, 2010, and 2015. Table SP.1 also shows that, in fall 2015, a larger percentage, an estimated 77% (with SE 3.5) of four-year mathematics departments (compared with 82% in fall 2010), belonged to an institution that offered a secondary teacher certification program; again, the percentage was largest for the masters-level departments (92%). It appears that the percentage of four-year mathematics departments whose institutions offer elementary certification, and the percentage offering secondary certification, have declined slightly over 2010.

Teacher Preparation Programs at Two-year Colleges

Table SP.2 updates data regarding public two-year colleges offering programs for pre- and in-service teachers to complete their entire mathematics certification requirements at the two-year college for fall 2015, including historical data for 2010 and 2005. The three types of students mentioned in Table SP.2 are: undergraduates without a bachelors degree ("pre-service teachers"); in-service teachers who already hold certification in some other discipline; and "career switchers" who leave a first career to enter a second career in pre-college teaching. Each category displays decreases from 2010 to 2015 in the percentage of mathematical programs in two-year colleges offering organized teacher preparation programs.

Table SP.2 also shows that two-year institutions were more involved in the preparation of elementary teachers than middle school or secondary teachers. Secondary teachers may take their lower-division mathematical requirements at a two-year institution and those enrollments might not be reflected in this data. In fall 2015, the estimated percentage of public two-year college mathematics programs with a complete certification program at the elementary level was 28% (SE 5), at the middle school level was 14% (SE 3), and at the secondary level was 7% (SE 3). In fall 2010, these estimated percentages were 41% of the colleges having programs at the elementary level, 24% at the middle school level, and 13% at the secondary level.

Table SP.3 presents data on various activities or options related to certification programs at two-year colleges in fall 2015: an estimated 35% (SE 6) of mathematics programs assign a faculty member to coordinate K-8 teacher education in mathematics,

TABLE SP.2 Percentage of mathematical programs at public two-year colleges (TYCs) having organized programs that allow various types of pre- and in-service teachers to complete their entire mathematics course or licensure requirements in fall 2015. (Fall 2005, 2010 data in parentheses.)

	Percentage of TYCs with an organized program in which students can complete their entire mathematics course or licensure requirements	
Pre-service elementary teachers	28	(30,41)
Pre-service middle school teachers	14	(19,24)
Pre-service secondary teachers	7	(3,13)
In-service elementary teachers	12	(12,25)
In-service middle school teachers	6	(15,12)
In-service secondary teachers	4	(2,10)
Career-switchers aiming for elementary teaching	16	(19,30)
Career-switchers aiming for middle school teaching	13	(14,17)
Career-switchers aiming for secondary teaching	5	(6,13)

55% (SE 5) offered a special mathematics course for K-8 teachers, 9% (SE 5) offer a mathematics pedagogy course in their mathematics program, and 6% (SE 2) report that a mathematics pedagogy class is offered outside of the mathematics program. Historical data for 2010 and 2005 are displayed in SP.3.

Further discussion of teacher education programs in two-year colleges is contained at the end of Chapter 6: Topics of Special Interest for Mathematics Programs in Two-Year Colleges. Among the items noted there, in the past ten years, from fall 2000 to fall 2010, the estimated enrollment in the courses in mathematics for elementary school teachers in two-year colleges had doubled (see Tables TYE.3 and TYE.3.2 in Chapter 6), but decreased 45% (5 SEs) from 2010 to 2015.

Four-year Mathematics Departments: Numbers of Mathematics Credits Required for Certification of Pre-service K-8 Teachers

A new question on the 2015 survey inquired about the number of semester hours in four-year

mathematics departments required for certification of pre-service elementary (grades K-5) and middle grade (grades 6-8) mathematics teachers. Table SP.4 contains data, broken down by the level of department, on the number of semester hours in the mathematics department, and the number of semester hours in “fundamental ideas in mathematics appropriate for elementary mathematics teachers” that are required for K-5 teacher certification. Table SP.5 summarizes the analogous data required for grades 6-8 teacher certification.

Previous CBMS surveys asked for slightly different data. In CBMS2010, Table SP.5, p. 51, gave the distribution of the number of mathematics courses (rather than semester hours) required for “early” grade (K-5) certification (if the institution makes a distinction between kinds of K-8 certification, or all K-8 certification if no distinction is made) among the various levels of departments. That table showed that, in fall 2010, most commonly two mathematics courses were required, and the average number of required

TABLE SP.3 Percentage of public two-year colleges (TYCs) that are involved with teacher preparation in various ways in fall 2015. (Data from fall 2005, 2010 in parentheses when available.) This table can be compared to Table SP.4 CBMS 2010.

	Percentage of TYCs
Assign a mathematics faculty member to coordinate K–8 teacher education in mathematics	35 (38,36)
Offer a special mathematics course for preservice K–8 teachers ¹	55 (11,7)
Offer a special mathematics course for preservice secondary teachers ²	19 (na)
Offer mathematics pedagogy courses in the mathematics department	9 (9,5)
Offer mathematics pedagogy courses outside of the mathematics department	6 (10,9)

¹In 2010, this question specifically excluded four courses listed in the detailed course matrix.

²Did not collect in 2010.

mathematics courses, across all levels of mathematics departments combined, was 2.7 courses. In fall 2015, Table SP.4 shows that among departments at institutions with K-5 teacher certification programs, the interval of semester hours chosen by the highest estimated percentage of departments, across all level of departments combined, was “more than 12 required hours” (chosen by an estimated 34% of departments with elementary certification programs, with SE 3.2); in masters-level departments, the 4-6 semester hour interval was chosen most frequently. This data would suggest that, in fall 2015, more semester hours in mathematics generally are required for pre-service elementary teacher certification than in fall 2010. The interval of hours required for K-5 certification in fundamental ideals of mathematics that was chosen by the largest estimated percentage of departments with an elementary education certification program, in fall 2015, was 4-6 hours; the distribution of semester hours required in fundamental ideas in mathematics was relatively uniform for each of the three levels of mathematics departments.

Four-year Mathematics Departments: Courses in Secondary Certification Programs

Table SP.6 gives the estimated percentages, in fall 2015, of four-year mathematics departments that required courses in specified core areas for secondary mathematics certification (grades 9-12), departments where courses in these core areas were not required, but were generally taken by pre-service secondary

teachers, and departments that offered courses specially designed for pre-service secondary teachers in these core areas. In fall 2015, as in fall 2010, the three courses most likely to be required across all levels of departments combined were geometry, statistics, and modern algebra. At all three levels of departments, geometry was required by more than an estimated 85% of departments (with the SE of all departments combined 3). At the bachelors- and masters-level departments, modern algebra was required by at least 80% of departments (with SEs at bachelors-level of 4 and at masters-level of 6). At the doctoral- and masters-level departments, advanced calculus/analysis was required by more than 60% of departments (with SE at doctoral-level of 9 and at masters-level of 6). At masters and bachelors-level departments, statistics was required by more than 80% of departments (with SEs at masters-level of 6 and at bachelors-level of 4). Doctoral-level departments generally were more likely to offer special courses for secondary pre-service teachers than other levels of departments, with special geometry courses offered by 53% (SE 10) of the doctoral-level departments. Table SP.9, p. 54, of the CBMS 2010 report presented comparable data from the 2010 CBMS survey.

Statistics Departments: Courses for Pre-service Teachers

For the first time, in 2015, the statistics questionnaire inquired about pre-service secondary (grades 9-12) teacher education in statistics. Statistics depart-

TABLE SP.4 Among all four-year colleges and universities with a K-5 certification program, the percentage of mathematics departments requiring various numbers of mathematics semester hours for certification, by type of department, in fall 2015. (Table can be compared to Table SP.5 in CBMS2005 and CBMS2010, but the previous surveys asked for the number of courses. Also, the earlier surveys looked at K-8 and at "early" grades, while 2015 asked separately about K-5 and 6-8.)

Number of semester hours in mathematics department required for K-5 certification	Percentage of departments with K-5 certification programs that require various numbers of mathematics courses for certification			
	Univ (PhD) %	Univ (MA) %	Coll (BA) %	All Math %
0 required	8	0	2	2
1-3 required	9	0	6	6
4-6 required	20	37	19	22
7-9 required	22	26	23	23
10-12 required	17	13	11	12
More than 12 required	24	24	38	34
Number of semester hours in fundamental ideas of mathematics required for K-5 certification	Univ (PhD) %	Univ (MA) %	Coll (BA) %	All Math %
0 required	12	5	17	14
1-3 required	6	3	10	8
4-6 required	41	40	46	45
7-9 required	16	21	11	13
10-12 required	11	16	1	5
More than 12 required	14	15	15	15

Some percentages do not total 100% due to round-off.

ments were asked which of a list of statistics courses were required of all students at their institution who were seeking credentials to teach statistics in grades 9-12, which courses were not required at their institution but generally were taken, and for which courses the department offered a special course for per-service secondary teachers. Table SP.7 presents a summary of the responses to those questions. Across all levels of statistics departments combined, an estimated 41% (SE 3.6) required Introductory Statistics, and an estimated 42% (SE 3.6) required Probability and/or Statistics with a calculus prerequisite for certification to teach statistics in grades 9-12. In addition, at another 27% (with SE 4) of institutions, Introductory

Statistics is not required but generally taken, and an estimated 20% (SE 3) of statistics departments offered a special Introductory Statistics course for pre-service secondary teachers.

Statistics departments also were asked for the number of semester hours in statistics that were required by their institution's middle grade (6-8 grade) teacher certification program, and by their institution's elementary grade (K-5) teacher certification program. Table SP.7 shows that an estimated 73% (SE 3.4) of institutions require no statistics for K-5 grade certification; for grades 6-8 certification, 42% (SE 3.9) of institutions require no statistics, while 42% (SE 3.8) require 1-3 semester hours of statistics.

TABLE SP.5 Among all four-year colleges and universities with a 6-8 certification program, the percentage of mathematics departments requiring various numbers of mathematics semester hours for certification, by type of department, in fall 2015. (Table can be compared to Table SP.5 in CBMS2005 and CBMS2010, but the previous surveys asked for the number of courses. Also, the earlier surveys looked at K-8 and at "early" grades, while 2015 asked separately about K-5 and 6-8.)

Number of semester hours in mathematics department required for 6-8 certification	Percentage of departments with grade 6-8 certification programs that require various numbers of mathematics courses for certification			
	Univ (PhD) %	Univ (MA) %	Coll (BA) %	All Math %
0 required	4	0	1	1
1-3 required	0	0	0	0
4-6 required	14	10	4	7
7-9 required	5	3	2	3
10-12 required	6	10	5	6
More than 12 required	71	77	87	83
Number of semester hours in fundamental ideas of mathematics required for 6-8 certification	Univ (PhD) %	Univ (MA) %	Coll (BA) %	All Math %
0 required	15	10	15	14
1-3 required	4		11	8
4-6 required	28	19	26	25
7-9 required	25	16	17	18
10-12 required	15	10	4	7
More than 12 required	13	45	28	29

Some percentages do not total 100% due to round-off.

Tables SP.8-SP.10: Practices in Distance Learning Courses

In the CBMS 2015 survey, a "distance learning course" was defined to be a course offered for credit in which "the majority of the instruction occurs with the instructor and the students separated by time and/or place (e.g. where the majority of the course is taught online, or by computer software, or by other technologies) including MOOC's that are offered for credit. (A MOOC is a 'massive open online course'.)" In Appendix I, enrollments for individual courses both with, and without, distance learning enrollments are given, so that distance learning enrollments can be computed for individual courses taught by four-year mathematics and statistics departments; Chapter 6, Table TYE.12, gives the comparable enrollments at two-year college mathematics programs. In fall 2015,

by the Table E.4 in Chapter 3, total distance learning enrollments in courses in four-year mathematics departments were estimated at 86,197 enrollments (compared to an estimated 36,297 enrollments in fall 2010), and in statistics departments, there were an estimated 4,297 enrollments (about the same as the 2010 estimate of 4,171 enrollments) in distance learning courses; Table TYE.12 shows that in fall 2015 there were an estimated 225,000 enrollments (compared with 188,000 in fall 2010) in distance learning courses at two-year mathematics programs. Enrollments in distance learning courses appear to be growing, and the 2015 survey sought to explore issues regarding their use and pedagogy.

From Table SP.8 we observe that 87% (SE 4.1) of two-year mathematics programs, 64% of statistics departments (SE 3), and 52% (SE 5.2) of four-year mathematics departments (63% at doctoral-level,

TABLE SP.6 Among four-year colleges and universities with secondary pre-service teaching certification programs, for various courses, the percentage of mathematics departments whose program requires the course, or whose students generally take the course, or who offer a special course in the given subject that is designed for secondary teachers, by type of department, in fall 2015. (This table can be compared to Table SP.9, p. 54, in CBMS2010.)

Course	Percentage of departments with secondary certification program where:											
	Course is required				Course is generally taken, but not required				Math dept offers special course in the subject for secondary pre-service teachers			
	Univ (Ph.D) %	Univ (MA) %	Coll (BA) %	All math %	Univ (Ph.D) %	Univ (MA) %	Coll (BA) %	All math %	Univ (Ph.D) %	Univ (MA) %	Coll (BA) %	All math %
Advanced Calculus/ Analysis	69	64	49	54	13	13	16	15	9	3	10	8
Modern Algebra	72	89	81	81	9	12	14	13	23	4	2	6
Number Theory	25	37	11	17	26	24	24	24	7		9	7
Geometry	85	89	90	89	18	7	10	11	53	5	13	18
Discrete Mathematics	56	52	62	60	8	9	16	14	12	5	4	5
Statistics	66	88	85	83	23	7	12	13	4	8	3	4
Probability	62	68	50	55	15	2	18	15	6	9	6	7
History of Math	60	77	39	48	16	7	17	16	39	5	11	15

Some totals are less than 100% due to round-off.

73% at masters-level, and 45% at bachelors-level) offered a distance learning course at least once in the calendar years 2013-2015. These percentages can be compared to those reported in fall 2010 (see CBMS2010, Table SP.10, p. 55), when 88% of two-year mathematics programs, 39% of statistics departments, and 35% of four-year mathematics departments (48% of doctoral-level, 57% of masters-level, and 28% of bachelors-level) reported offering distance learning courses in 2008-10. The survey asked all departments whether, in fall 2015, the department granted credit for a distance learning class that was not taught by faculty in the respondent's institution; an estimated 62% (SE 5.2) of four-year mathematics departments, 50% (SE 3) of statistics departments, and 58% (SE 5.1) of two-year college mathematics programs reported that they did give credit for such courses. Departments were asked if there is a limit on the number of credits in distance learning courses that can be applied toward graduation, and Table SP.8 shows that in fall 2015 an estimated 36% (SE 3.7) of

four-year mathematics departments, 31% (SE 2.9) of statistics departments, and 1% (SE 0.5) of two-year colleges reported that there was such a limit.

Among those departments that offered a distance learning course in 2013-15, Table SP.8 gives the percentages of practices in teaching distance learning courses in four-year mathematics departments, statistics departments, and two-year colleges. Departments were asked to categorize the majority of distance learning courses as completely online, hybrid, or other, and for all three types of departments about two-thirds (66-69%) of the distance learning courses were completely online (with SEs 4-6). Departments were asked to itemize how instructional materials were generally created: by faculty, by commercially produced materials, or by a combination. For the statistics departments combined, an estimated 56% (SE 3.7) indicated faculty created the materials, while at four-year mathematics departments about 36% (SE 4.6) used faculty created materials (and these percentages were about the same across each level of

TABLE SP.7 Among statistics departments at four-year colleges and universities with secondary pre-service teaching certification programs, for various courses, the percentage of statistics departments whose program requires the course, or whose students generally take the course, or who offer a special course in the given subject that is designed for secondary teachers, and the number or semester hours required for certification in grades K-5 and 6-8, by type of department, in fall 2015.

Course	Percentage of departments with secondary certification program where:								
	Course is required			Course is generally taken, but not required			Stat dept offers special course in the subject for secondary pre-service teachers		
	Univ (PhD) %	Univ (MA) %	All stat %	Univ (PhD) %	Univ (MA) %	All stat %	Univ (PhD) %	Univ (MA) %	All stat %
Introductory Statistics	36	57	41	36	0	27	17	29	20
Probability	24	33	26	13	14	13	8	14	9
Probability and/or statistics with calculus prerequisite	36	67	42	4	14	7	12	0	9
Upper level statistics course	12	17	13	9	43	18	8	0	6
Applied statistics course	12	17	13	9	29	14	4	0	3
Other	5	0	4	5	0	4	4	0	4
Number of semester hours required for K-5 grade teachers (%)									
None	85	50	73						
1-3 hours	0	0	0						
4-6 hours	11	50	23						
More than 6 hours	5	0	3						
Number of semester hours required for 6-8 grade teachers									
None	49	25	42						
1-3 hours	33	63	42						
4-6 hours	9	13	10						
More than 6 hours	9	0	6						

Some totals are less than 100% due to round-off.

TABLE SP.8 Percentage of mathematics, statistics, and public two-year college departments offering distance learning¹, and use of various practices with regard to distance learning in fall 2015. This table can be compared to Table SP.10 CBMS 2010 p. 55.

	Mathematics Depts				Statistics Depts			Two-Year Colleges
	Univ (PhD)	Univ (MA)	College (BA)	Total	Univ (PhD)	Univ (MA)	Total	
Give credit for distance learning not taught by faculty in your institution:								
Yes	60	74	60	62	52	42	50	58
No	40	26	40	38	48	58	50	42
Set a limit on the number of credits earned in distance learning classes	33	33	37	36	34	18	31	1
Percentage offering distance learning	63	73	45	52	69	50	64	87
Format of majority of distance learning:								
Complete online	63	60	74	69	70	50	66	69
Hybrid	36	33	21	26	18	50	23	22
Other	1	7	5	5	13		10	8
Instructional materials created by:								
Faculty	37	30	37	36	54	67	56	14
Commercially produced materials	9	6	11	9	3		3	19
Combination of both	55	65	52	55	43	33	41	67
How distance learning students take majority of tests:								
Not at a monitored testing site	15	15	26	22	10	17	11	11
Online, using monitoring technology	10	14	23	19	16	17	16	10
At proctored testing site	49	34	34	37	32	50	35	47
Combination of both	25	37	18	23	41	17	37	32

¹ Distance-learning courses are those courses in which the majority of instruction occurs with the instructor and students separated by time and/or place (e.g. courses in which the majority of the course is taught online, or by computer software, or by other technologies, including MOOCs that are offered for credit).

TABLE SP.9 Percentages of public two-year colleges (TYCs) with various practices in distance-learning courses in fall 2015. (Data from fall 2010 are in parentheses.) This table can be compared to Table SP.11 CBMS 2010 p. 57.

Requirements of faculty whose entire teaching load is distance-learning courses regarding time required to be on campus to meet with students	% of TYCs	
Never	5	(8)
Only for scheduled meeting or student appointment	12	(6)
A specified number of office hours per week	32	(21)
Not applicable or unreported	51	(65)

four-year mathematics department), and at two-year mathematics programs about 14% (SE 4.4) used materials created by faculty. Instructional materials created by a combination of both faculty and commercially produced materials was reported in 41-67% of institutions, across four-year and two-year departments. The administration of tests was addressed in a question about how distance learning students take the majority of their tests: not at a monitored testing site, online using monitoring technology, at a proctored testing site, or using some combination: an estimated 47% (SE 5.1) of two-year college mathematics programs, 35% (SE 3.7) of statistics departments, and 37% (SE 5.9) of four-year mathematics departments (including 49% of the doctoral-level departments) reported using a proctored testing site; these percentages were roughly comparable to those reported in fall 2010.

Table SP.9 examines the time faculty at two-year mathematics programs, whose entire teaching load is distance-learning courses, were required to be on campus. Estimates of percentage of programs with requirements that faculty in two-year college mathematics programs be on campus to meet with students ranged from 5-32% in fall 2015: an estimated 5% (SE 2) of mathematics programs never required faculty to be on campus, 12% (SE 3) required faculty to be on campus only for scheduled meetings or appointments, and 32% (SE 7) required a specific number of on-campus office hours, an 11% increase from 2010 to 2015.

Table SP.10 considers courses that four-year and two-year departments offered in both distance learning and regular format, and asked for a comparison of the courses offered in the two formats. Almost all of the departments that offered distance learning courses had some course offered in both formats (estimated at 91% of four-year mathematics departments, and 88% of statistics departments), and almost all believed that the courses had the same course outlines. Tables TYE.3 and TYE.12 in Chapter 6 show that almost every course offered was available in both formats at two-year colleges. An estimated ninety-seven percent (97% with SE of 2.7) of two-year colleges reported that the same course outlines were used for distance-learning courses and face-to-face courses (in four-year mathematics departments the estimated percentage was 94% and in statistics departments it was 88%). Instructors held comparable office hours at an estimated 59% (SE 4.8) of the four-year mathematics departments and 68% (SE 3.7) of the statistics departments. Instructors were evaluated in the same ways at an estimated 87% (SE 4) of the four-year mathematics departments, 91% (SE 2.4) of the statistics departments, and 93% (SE 3) of the two-year college mathematics programs. The courses made the same

use of common exams at an estimated 58% (SE 8) of the four-year mathematics departments, 45% (SE 4) of the statistics departments, and 67% (SE 5) of the two-year college mathematics programs. The classes had the same projects at an estimated 79% (SE 5.4) of the four-year mathematics departments, 68% (SE 3.5) of the statistics departments, and 77% (SE 4.5) of the two-year college mathematics programs. For four-year departments, these numbers are broken down further by the level of department, but the percentages are not very different at the various levels, and are comparable to the data reported in fall 2010.

The 2015 survey asked departments if, during the academic years 2013-15, the department had offered a MOOC (massive open online course) for credit. Out of all the institutions surveyed, one four-year (bachelors-level) mathematics department, one (doctoral-level) statistics department, and two two-year colleges responded "yes". The two-year colleges reported teaching courses in statistics, developmental mathematics, and college-level courses below, and above, calculus-level courses. The four-year mathematics department taught one or more courses that were college-level, but below calculus, and also statistics. The statistics department taught a course that required previous statistical knowledge. Given the few responses, and large SEs, estimates of the percentage of departments offering MOOCs and the enrollments in MOOCs are not included in this report. That is, given the rarity of such MOOCs, a different sample might show a different distribution of courses and different statistics.

Beginning in 2010 the CBMS survey asked four-year departments to check each upper-level course offered in distance learning format. The numbers of departments reporting such courses were small in both 2010 and 2015, and our estimates are likely unreliable, but the data gathered are reported in Tables SP.11A and SP.11.B, and may be compared to the data reported in CBMS2010 Tables 13.A and 13.B, pp. 58-9. There appears to be some growth in upper-level statistics courses offered by statistics departments as distance learning courses. As distance learning courses become more common, these baseline data may be of some interest.

Tables SP.12-SP.14: Academic Resources Available to Undergraduates

Tables SP.12 and SP.13 present a spectrum of academic enrichment activities available in various kinds of mathematics and statistics departments at each level of department. In most cases, the availability of these options in fall 2015 was comparable to what was available in fall 2010; one exception is the reported increase in the estimated percentage

TABLE SP.10 Percentage of four-year mathematics and statistics departments, and public two-year college (TYC) programs, with courses offered in both distance and non-distance-learning formats, and comparison of various practices in the distance learning and the non-distance-learning formats, by type and level of department, in fall 2015. This table can be compared to Table SP.12 CBMS 2010 p. 57.

	Math				Stat			TYC
	Univ (PhD)	Univ (MA)	College (BA)	Total	Univ (PhD)	Univ (MA)	Total	
Some courses in both non-distance and distance-learning formats	91	94	90	91	85	100	88	na ¹
Of those with courses in both formats, the percentage where:								
Instructors hold comparable office hours on campus	71	52	57	59	64	83	68	na
Instructors participate in evaluation in same way	89	81	89	87	89	100	91	93
Same use of common exams as in face-to-face	52	64	58	58	44	50	45	67
Same course outlines as in face-to-face	94	91	95	94	85	100	88	97
Same course projects as in face-to-face	85	73	78	79	62	100	69	77
More course projects than in face-to-face	10	18	14	14	9		7	12

¹See Tables TYE.3 and TYE.12.

of statistics departments that offer participation in statistics contests. Generally, the availability of these options increased as departments offered higher level degrees (e.g. honors sections were available at 69% (SE 5.2) of doctoral-level four-year mathematics departments, but only at 28% (SE 5.7) of the bachelors-level four-year departments). Special programs for women and minorities have increased at almost all levels of four-year mathematics and statistics departments. Two new additions to the CBMS survey questionnaire for four-year mathematics departments and statistics departments in 2015 were the opportunity to tutor, grade papers or TA in the department (offered at 17% (SE 2.9) of all four-year mathematics departments combined, and 75% (SE 2.5) of statistics departments (all levels combined), and the opportunity to participate in a supervised consulting lab with clients (available at 83% (SE 3.2) of four-year mathematics departments and 44% (SE 3.1) of statistics departments).

Another new question, added to the 2015 survey questionnaire, asked four-year mathematics and statistics departments to estimate the number of their majors who had participated in undergraduate research projects in the mathematical sciences, an internship in the mathematical sciences, or mathematical or statistical consulting to clients during September 1, 2014 - August 31, 2015. From these

responses, estimates of the total number of undergraduate majors participating in these activities, broken down by level of department, appears in Table SP.14. The estimated total number for each activity is highest at the bachelors-level mathematics department, with the estimate of majors involved in undergraduate research projects at bachelors-level mathematics departments about four times as large as at doctoral-level mathematics departments (and more than 2 SEs above the doctoral-level department estimate). However, the SEs for the bachelors-level estimates of the numbers of majors involved in undergraduate research were 2,454, and, for internships were 1,726, making these particular estimates for bachelors-level departments unreliable.

As seen in Tables SP.12 and SP.13, fall 2015 saw increases in the percentages of two-year colleges offering various kinds of special mathematics opportunities to students. The largest changes were in the estimated percentage offering outreach in K-12 schools (up to 46% with SE 4 in 2015 from 32% in 2010), the estimated percentage offering special programs for women (up to 15% with SE 3 in 2015 from 6% in 2010) and the estimated percentage offering honors sections of courses for majors (up to 28% with SE 4 in 2015 from 20% in 2010); note that, in fall 2015, the estimated percentage of two-year college programs

TABLE SP.11.A Percentage of four-year mathematics departments offering various upper-level mathematics courses by distance learning, by department type, in fall 2015. This table can be compared to Table SP.13.A in CBMS2010 p. 58.

	Mathematics Departments			
	Univ (PhD)	Univ (MA)	College (BA)	Total
E23. Introduction to Proofs	2		3	2
E24-1. Modern Algebra I	2			0
E24-2. Modern Algebra II				
E25. Number Theory				
E26. Combinatorics				
E27. Actuarial Mathematics				
E28. Logic/Foundations (not E23)				
E29. Discrete Structures	1			0
E30. History of Mathematics	4		1	1
E31. Geometry	2			0
E32-1. Advanced Calculus I and/or Real Analysis I	1			0
E32-2. Advanced Calculus II and/or Real Analysis II				
E33. Advanced Mathematics for Engineering and Physical Sciences				
E34. Advanced Linear Algebra (beyond E17, E19)	2			0
E35. Vector Analysis				
E36. Advanced Differential Equations (beyond E18)				
E37. Partial Differential Equations				
E38. Numerical Analysis I and II		3		0
E39. Applied Mathematics (Modeling)		4		1
E409. Complex Variables		4	1	1
E41. Topology		4		1
E42. Mathematics of Finance (not E26, E38)				
E43. Codes and Cryptology				
E44. Biomathematics				
E45. Operations Research (all courses)			0	0
E46. Senior Seminar/ Independent Study in Mathematics				
E47. Other advanced-level mathematics		7	0	1
E48. Mathematics for Secondary School Teachers		7	1	1

Note: These estimates are based on small numbers and have large standard error. Blank entries represent courses with no responses while zero entries indicate percentages that round to 0%.

TABLE SP.11.B Percentage of four-year mathematics and statistics departments offering upper-level statistics courses by distance learning, by department type, in fall 2015. This table can be compared to Table SP.13.B in CBMS2010 p. 59.

	Mathematics Departments				Statistics Departments		
	Univ (PhD)	Univ (MA)	College (BA)	Total	Univ (PhD)	Univ (MA)	Total
E6. Introductory Probability and/or Statistics for Majors/Minors (no calculus prerequisite)	2	3	5	4	11	15	12
E7. Combined Probability & Statistics (calculus prerequisite)	2	3		1	4	17	7
E8. Probability (calculus prerequisite)	5	7	0	2		8	2
E9. Mathematical Statistics (calculus prerequisite)	3	7	0	2		8	2
E10. Stochastic Processes		3		0			
E11. Applied Statistical Analysis	2	3		1	6	8	7
E12. Data Science/Analytics	2	6		1	3	8	4
E13. Design & Analysis of Experiments	2	3	0	1	7	8	7
E14. Regression (and Correlation)	2	3		1	2		2
F15. Biostatistics		3		0	2		2
E16. Nonparametric Statistics		3		0			
E17. Categorical Data Analysis		3		0			
E18. Sample Survey Design & Analysis		3		0	2	8	3
E19. Statistical Computing and/or Software	2	3		1	4	8	5
E20. Bayesian Statistics	na	na	na	na			
E21. Statistical Consulting	na	na	na	na		8	2
E22. Senior Seminar/ Independent Studies		5		1			
E23. Other upper-level Probability & Statistics	2	5	0	1	2	15	6
E24. Other mathematical science courses	na	na	na	na		8	2

Note: These estimates are based on small numbers and have large standard error. Blank entries represent courses with no responses while zero entries indicate percentages that round to 0%.

TABLE SP.12 Percentage of mathematics and statistics departments in four-year colleges and universities, and of mathematics programs at public two-year colleges, that offer various kinds of special opportunities for undergraduates, by type of department, in fall 2015. (Fall 2010 data in parentheses.) This table can be compared to Table SP.14 in CBMS2010 p. 60.

Percentage with special opportunities for undergraduates	Honors sections of courses for majors %	Math or Stat club %	Special programs for women %	Special programs for minorities %	Math or Stat contests %	Special Math or Stat colloquia for undergrads & %	Outreach in K-12 schools %
Mathematics Departments							
Univ (PhD)	69 (70)	94 (91)	41 (31)	25 (21)	91 (93)	77 (82)	61 (71)
Univ (MA)	39 (40)	91 (96)	37 (21)	31 (21)	78 (82)	87 (88)	77 (75)
Coll (BA)	28 (15)	56 (75)	16 (16)	8 (12)	64 (62)	53 (51)	43 (40)
Total Mathematics Departments	35 (26)	67 (80)	22 (19)	14 (14)	70 (69)	61 (60)	50 (49)
Statistics Departments							
Univ (PhD)	38 (43)	55 (48)	18 (19)	13 (22)	56 (24)	70 (67)	18 (30)
Univ (MA)	50 (55)	18 (45)	0 (0)	8 (0)	45 (36)	42 (82)	42 (18)
Total Statistics Depts	41 (46)	46 (47)	14 (13)	12 (15)	54 (28)	63 (71)	24 (27)
Two-Year College Mathematics Programs	28 (20)	32 (31)	15 (6)	15 (11)	40 (41)	21 (16)	46 (32)

Note: 0 means less than one-half of 1%.

TABLE SP.13 Percentage of mathematics and statistics departments in four-year colleges and universities, and of mathematics programs in public two-year colleges, that offer various additional special opportunities for undergraduates, by type of department, in fall 2015. (Fall 2010 data, where available, in parentheses.) This table can be compared to Table SP.15, p. 61, of CBMS2010.

Percentage with additional opportunities for undergraduates	Undergrad. Research opportunity %	Indep. Studies opportunity %	Assigned advisors in dept. %	Senior thesis opportunity %	Math career day %	Graduate school advising %	Internship opportunity %	Senior seminar opportunity %	Consulting lab with clients %	Tutor, grade papers, or TA %
Mathematics Departments										
Univ (PhD)	94 (96)	90 (96)	88 (90)	73 (63)	46 (40)	67 (67)	69 (50)	50 (47)	89	21
Univ (MA)	89 (91)	93 (100)	93 (100)	59 (56)	23 (46)	58 (70)	69 (67)	71 (66)	82	19
Coll (BA)	72 (83)	85 (94)	85 (90)	52 (58)	21 (17)	51 (46)	61 (55)	61 (59)	82	15
Total mathematics depts	77 (86)	87 (95)	86 (91)	56 (59)	25 (24)	55 (52)	63 (56)	60 (58)	83	17
Statistics Departments										
Univ (PhD)	91 (85)	95 (90)	73 (89)	60 (54)	50 (30)	90 (66)	72 (69)	46 (30)	41 (32)	80
Univ (MA)	69 (82)	92 (100)	83 (73)	42 (27)	27 (45)	50 (64)	69 (91)	27 (27)	54 (55)	62
Total statistics depts	86 (84)	94 (93)	76 (84)	56 (46)	45 (35)	80 (66)	71 (75)	42 (29)	44 (39)	75
Two-Year College Mathematics Programs	17 (14)	41 (36)	49 (42)	na	na	na	na	na	na	na

offering honors sections of courses is the same as that for bachelors-level mathematics departments.

Table SP.15: Interdisciplinary Courses in Four-Year Mathematics Departments

CBMS2015 was also interested in the existence of interdisciplinary courses. Table SP.15 gives the estimated percentages of mathematics departments at four-year colleges and universities that offered various interdisciplinary courses in fall 2015, broken down by the level of the department. Across all levels of four-year mathematics departments combined, the most likely interdisciplinary courses to be taught were in mathematics and education (41%, with SE 4.3), mathematics and business or finance (35% with SE 3.9), and mathematics and computer science (31% with SE 4.7). Some interdisciplinary courses were more likely to be taught at doctoral-level departments (e.g. mathematics and biology was offered at an estimated 47% (SE 7.8) of doctoral-level departments, 36% (SE 7.7) of masters-level departments and 3% (SE 2.6) of bachelors-level departments). A different question regarding interdisciplinary courses was asked on the 2010 survey; in fall 2010, departments were asked about new interdisciplinary courses offered in the last five years (that data is in CBMS2010, Table SP.17, p. 53).

Tables SP.16 and SP.17: Dual Enrollments – College Credit for High School Courses

Dual enrollment courses were defined to be “courses conducted on a high school campus and taught by high school teachers, for which high school students may obtain high school credit and, simultaneously, college credit through your institution”. This arrangement is not the same as obtaining college credit based on an AP or IB exam, or high school students enrolling in a course at a college. Dual enrollment is encouraged by many state governments as a way of utilizing state-wide educational resources efficiently.

Table SP.16 gives the estimated number of dual enrollments in the courses College Algebra,

Precalculus, Calculus I (Mainstream I and Non-Mainstream I, combined), Statistics and “Other” courses that were offered by four-year mathematics departments, two-year mathematics programs, and statistics departments in spring 2015 and fall 2015. In past CBMS surveys (see e.g. CBMS2010, Table SP.18, p. 65), these courses were offered predominately by mathematics programs at two-year colleges; in fall 2010, an estimated 61% of mathematics programs at two-year colleges, 17% of mathematics departments at four-year colleges and universities, and 8% of statistics departments offered dual enrollment courses; in fall 2015, the estimated percentage of four-year mathematics departments offering dual enrollment courses rose to 26% (SE 4.1) (the percentages of two-year colleges and statistics departments offering dual enrollment courses in 2015 were about comparable to percentages in 2010). However, the estimated enrollments in dual enrollment courses offered in 2015 by four-year mathematics departments increased dramatically over the number of dual enrollments estimated in 2010. The estimated enrollment in dual enrollment courses offered by mathematics departments in four-year colleges and universities in spring and fall (combined) 2010 was 42,862, with slightly more than half of the enrollments in the fall 2010; in 2015, the estimated number of enrollments had risen to 117,399, and, again, slightly more than half of the enrollments were in fall 2015. Mathematics programs in two-year colleges had an estimated total of 170,970 enrollments in spring and fall (combined) 2015 (compared to 158,097 enrollments in spring and fall (combined) 2010). In 2010, mathematics programs at two-year colleges had almost four times the estimated dual enrollments of mathematics departments at four-year colleges and universities, while in 2015, the estimated enrollments in four-year college dual enrollment courses were about 2/3 of the estimated enrollments in dual enrollment courses offered by two-year colleges. Statistics departments had a much smaller estimated number of dual enrollments, 1,478 in 2015, compared with 1,573 dual enrollments in 2010.

TABLE SP.14 Total number of majors (best estimate) who participated in various activities over Sept. 1, 2014, through Aug. 31, 2015.

Activity	All Math Depts	PhD Math	MA Math	BA Math	All Stat Depts	PhD Stat	MA Stat
Undergraduate research project in the mathematical sciences	12168	2091	1733	8344	575	534	42
Internship in mathematical sciences	6031	1198	766	4068	714	680	34
Mathematical or statistical consulting to client	975	243	170	562	317	300	17

TABLE SP.15 Percentage of all four-year mathematics departments offering interdisciplinary courses, by type of department, in fall 2015.

	Univ (PhD)	Univ (MA)	Coll (BA)	All departments
Offered course in:	Offered course %	Offered course %	Offered course %	Offered course %
Mathematics and finance or business	46	44	31	35
Mathematics and biology	47	36	14	22
Mathematics and the study of the environment	16	8	3	6
Mathematics and engineering or the physical sciences	29	23	13	17
Mathematics and economics	15	11	9	10
Mathematics and social sciences other than economics	5	16	7	8
Mathematics and education	33	59	40	41
Mathematics and the humanities	8	9	14	13
Mathematics and computer science	27	41	30	31
Other	10	6	10	10

By Table SP.16, the percentage of two-year mathematics programs entering into dual enrollment agreements increased to 63% (SE 6.4) in 2015 from 61% in 2010. Large increases were reported in College Algebra and Statistics dual enrollments, with decreases in Precalculus, Calculus, and Other categories. Estimated dual enrollments in College Algebra for spring and fall combined increased to 90,460 in 2015 from 52,828 in 2010 (71% increase). Elementary Statistics dual enrollments for spring and fall combined increased to 18,983 in 2015 from 11,768 (61% increase). Precalculus dual enrollments in spring and fall combined decreased to 32,047 in 2015 from 43,778 in 2010 (21% decrease). Calculus I dual enrollments for spring and fall combined decreased to 10,954 in 2015 from 20,531 in 2010 (47% decrease). The “Other” course category dual enrollments for spring and fall combined decreased to 18,524 in 2015 from 29,192 in 2010 (37% decrease). In 2015, two-year mathematics programs estimated

fall dual enrollments represented 16% of estimated College Algebra enrollments, 13% of Precalculus enrollments, 6% of Calculus I enrollments, and 3% of Elementary Statistics enrollments.

Table SP.16 gives the dual enrollments, broken down by course. The largest course estimated dual enrollments in both four and two-year mathematics departments in fall and spring 2015 (combined) occurred in College Algebra. Estimated enrollments in dual enrollment courses in four-year mathematics departments showed large gains across all courses: estimated dual enrollments in College Algebra rose from about 17,000 in 2010 (fall and spring combined) to almost 46,000 in 2015, estimated dual enrollments in Precalculus rose from about 5,000 in 2010 to over 30,000 in 2015, estimated dual enrollments in Calculus I rose from about 10,000 in 2010 to about 20,000 in 2015, estimated dual enrollments in Statistics rose from about 6,000 in 2010 to about 7,000 in 2015, and estimated dual enrollments in “Other” rose from

about 4,900 in 2010 to about 13,000 in 2015. Dual enrollments represent a growing percentage of total enrollments in four-year mathematics departments; for example, dual enrollments in College Algebra were about 18% of other College Algebra enrollments at four-year mathematics departments in 2015, and about 7% in 2010. It also should be noted that the SEs on the individual dual enrollments are large; for example, the SE on the number of dual enrollments in College Algebra at four-year mathematics departments in fall 2015 is about 8,400 enrollments. However, it seems clear from the data that four-year colleges' dual enrollments have increased over previous CBMS surveys, and that dual enrollment courses are no longer confined primarily to two-year colleges.

There has been some concern about the degree of quality control exercised by the department through which college-level credit for the courses is awarded. The lower portion of Table SP.16 gives the estimated percentages of departments offering dual enrollment courses that require teaching evaluations. That percentage increased at two-year colleges from 48% in 2010 to 72% in 2015. Only an estimated 34% (SE 7.2) of four-year mathematics departments offering dual enrollment courses in 2015 required teaching evaluations for the instructors, compared to an estimated 40% in 2010. In earlier CBMS surveys other questions related to the control of the quality of dual enrollment courses by the credit granting department were asked; these questions were not repeated in 2015.

The increase in required teaching evaluations at mathematics programs in two-year colleges mentioned in the preceding paragraph may be a response to a concern at two-year colleges regarding dual enrollment courses as reported in Tables TYF.24 and TYF.25. Among all survey respondents (including respondents from two-year colleges that do not have dual enrollment arrangements), in fall 2015, an estimated 7% (SE 3) of mathematics program heads in two-year colleges saw dual enrollment courses as a "major problem" in 2015 (11% in 2010). Another 36% (SE 5) found dual enrollment arrangements "somewhat of a problem" in 2015, up 20 points from 2010.

Table SP.17 examines the practice of colleges and universities sending their own faculty members into high schools to teach courses that grant both high school and college credit; this differs from dual enrollment courses where the instructor is a high school teacher. The number of students involved in these courses has been smaller than the enrollment in dual enrollment courses. However, these programs have grown from 2005 to 2015 at two-year colleges, but, in fall 2015, involved only a small number of four-year departments. In fall 2010, an estimated 22% of two-year and 4% of four-year mathematics departments assigned and paid their own faculty to teach courses in a high school that awarded both high

school and college credit. In fall 2015, this estimated percentage was 6% (SE 1.8) at four-year mathematics departments and had doubled to 44% (SE 6.5) at two-year mathematics programs. A two-year college faculty member teaching a dual enrollment course usually was classified as a part-time faculty member at the two-year college that awarded college credit for the course, even though the salary was paid completely by a third party, e.g., the local school district. The 2015 estimate of the number of students enrolled in courses where the two-year college assigned their own faculty members to teach the courses is not displayed in Table SP.17, since it cannot be reliably estimated from the 2015 data because there was one large outlier that increased the SE (and the estimate) significantly. These direct-pay faculty members at two-year colleges were reported in 2010 to have taught 6,358 students, and the 2015 data indicates this number is much larger (perhaps about 30,000) in 2015. The estimated enrollment in four-year mathematics departments, in fall 2015, was 4,014 (about the same as in 2010), with the large SE of 1,649, and no four-year statistics departments reported being involved in this practice.

Table SP.18 and SP.19: Curricular Requirements of Mathematics and Statistics Majors in Four-Year Departments

Requirements for a major in mathematics have become more flexible, as can be seen, for example, in the MAA's Committee on Undergraduate Programs in Mathematics (CUPM) recommendations on requirements for the mathematics major. Departments seem to have more tracks (sets of graduation requirements) and more flexible requirements for mathematics majors. The CBMS 2005 and 2010 surveys asked about these requirements, and some of these questions were repeated in the 2015 survey. Table SP.18 summarizes data from four-year mathematics departments on whether each course option was required in all their majors, required in some but not all of their majors, or required in none of their majors; these numbers are broken down by the level of the department. Table SP.18 can be compared to CBMS2010 Table SP.20, p. 67.

Table SP.18 shows that in fall 2015 (as in fall 2010) the requirement selected most frequently by four-year mathematics departments as being required for all mathematics majors was "at least one computer science course" (required for all majors by more than an estimated 60% of departments at all levels (with SEs of 6-7)); the estimated percentage of four-year mathematics departments requiring a statistics course for all majors decreased at the doctoral and the masters-levels of mathematics departments from fall 2010 to fall 2015 (at the bachelors-level departments, it increased from 32% in 2005, to 55% in 2010, to 59% (SE 5.4) in 2015). The requirement that all

TABLE SP.16 Percentage of departments offering dual-enrollment courses taught in high school by high school (HS) teachers, enrollments in various dual-enrollment courses in spring 2015 and fall 2015 compared to total of all other enrollments in fall 2015, and (among departments with dual-enrollment programs) percentage of departments requiring teacher evaluations, by type of department. (Fall 2010 data in parentheses.) The comparable data in the CBMS2010 report is in Table SP.18 p. 65.

	Four-year Mathematics				Two-year Mathematics				Four-year Statistics			
	Dual Enrollments		Other enrollments		Dual enrollments		Other enrollments		Dual enrollments		Other enrollments	
	spring 2015	fall 2015	fall 2015	fall 2015	spring 2015	fall 2015	fall 2015	fall 2015	spring 2015	fall 2015	fall 2015	fall 2015
Percentage of departments with dual-enrollment courses	26% (17%)				63% (61%)				12% (8%)			
Number of dual enrollments in:												
College algebra	15534	30310	255416	255416	32937	57523	292138	292138	na	na	na	na
Precalculus	15090	15702	122302	122302	18869	13178	87014	87014	na	na	na	na
Calculus I ¹	6329	14480	344988	344988	4596	6358	91993	91993	na	na	na	na
Statistics	3866	3292	226441	226441	11919	7064	251279	251279	299	1179	89756	89756
Other courses	8016	4780	na	na	8478	10046	na	na	na	na	na	na
Departmental teaching evaluations required in dual-enrollment courses	34% (40%)				72% (48%)				26% (0%)			

¹The question on dual enrollments did not differentiate between mainstream and non-mainstream calculus. To provide comparable data, the column for "Other enrollments" also combines mainstream and non-mainstream calculus even though separate statistics are shown elsewhere in this report.

TABLE SP.17 Percentage of departments in four-year colleges and universities and in public two-year colleges that assign their own full-time or part-time faculty members to teach, in high school, courses that award both high school and college credit, and number of students enrolled, in fall 2015. (Fall 2010 data in parentheses.) This table was Table SP.19 in CBMS2010.

	Four-year Mathematics Departments	Two-year Mathematics Departments	Statistics Departments
Assign their own members to teach dual-enrollment courses (90)	6 (4)	44 (22)	(0)
Number of students enrolled	4014 (3,932)	* (6,358)	0

*The estimate of 36,368 from the data shows very large standard errors. The only clear finding is that there has been a large increase in this practice, but not necessarily as large as the estimate indicates.

majors take at least one applied mathematics course (beyond calculus) increased at all levels of mathematics departments from 2010 to 2015. Comparable data from 2010 is in CBMS2010 Table SP.20, p. 67, and for 2005 is in CBMS2005 Table SP.20 p. 67.

Historically, Modern Algebra and Real Analysis were considered required courses for all mathematics majors; for example, in the 1990 CBMS survey report, Table D.2 p. 62, showed that Modern Algebra was required for the major at 56% of doctoral-level departments, 70% of masters-level departments, and 78% of bachelors-level departments (in 2015 Table SP. 18 shows that the corresponding percentages were 34%, 34%, and 54%), while in 1990, Real Analysis/Advanced Calculus was required at 70% of doctoral-level departments, 66% of masters-level departments, and 65% of bachelors-level departments (in 2015 Table SP.18 shows that the corresponding percentages were 31%, 49%, and 36%), Table SP.18 shows that at all levels of departments, the estimated percentage of departments requiring Modern Algebra, and the estimated percentage requiring Real Analysis, in all majors, were about the same, or decreased, from 2010 to 2015, while the estimated percentage of departments requiring of all majors either Modern Algebra or Real Analysis (major can choose either) increased at all levels of departments. Of these two courses, Modern Algebra I was a more popular required course at bachelors-level departments (required for all majors at an estimated 54% (SE 8.5) of bachelors-level departments in 2015 (down from 62% in 2010). At the bachelors-level departments, an estimated 41% (SE 6.3) of departments did not require Real Analysis in any major in 2015 (up from 36% in 2010).

Some departments found ways to create more depth in their mathematics major, without requiring particular mathematics courses. In doctoral-level

departments, beyond the required computer science course, the requirement most often cited for all majors was the requirement that the major take a one-year sequence (required for all majors by an estimated 48% (SE 8) of all doctoral-level departments); at the masters (respectively, bachelors) level departments, a capstone experience (e.g. a senior project, thesis, seminar, internship) was required for all majors by an estimated 68% (SE 8) (respectively, 76% (SE 4.5)) of all departments.

Table SP.19.A and Table SP.19.B examine the estimated percentages of departments that had various options that were required in all majors, required in some majors, and not required in any major for an undergraduate statistics majors; Table SP.19.A summarizes these percentages for the degrees in statistics awarded by mathematics departments, and Table SP.19.B examines the requirements for the degrees awarded by statistics departments. Table SP.19.A appears for the first time in a CBMS survey, and Table SP.19.B can be compared to CBMS2010, Table SP.21, p. 68.

According to Tables SP.19A and B, the requirements for undergraduate statistics degrees awarded by mathematics and statistics departments in fall 2015 were relatively similar. As might be expected, in mathematics departments it was slightly more likely that mathematics courses (Multivariable Calculus, Linear Algebra, an applied mathematics course, Mathematical Statistics) and also a Probability course were required of all statistics majors than in statistics departments, while statistics departments were more likely to require a course in Linear Models and Computer Science of all majors than were mathematics departments. In fall 2015, a larger estimated percentage of mathematics departments required an applied statistics course for all majors (74% (SE 9.8) of

TABLE SP.18 Percentage of four-year mathematics departments requiring certain courses (or exit exam) in all, some, or none of their majors, by type of department, in fall 2015. These percentages can be compared to Table SP.20 in CBMS2010 p. 67.

Mathematics Department Requirements	Required in all majors			Required in some but not all majors			Not required in any major		
	Univ (PhD) %	Univ (MA) %	College (BA) %	Univ (PhD) %	Univ (MA) %	College (BA) %	Univ (PhD) %	Univ (MA) %	College (BA) %
Modern Algebra I	34	34	54	40	62	27	26	4	19
Real Analysis I	31	49	36	49	45	23	20	6	41
Modern Algebra I or Real Analysis I (major may choose either to fulfill this requirement)	21	33	24	23	27	14	56	40	62
A one-year upper-level sequence	48	26	28	19	43	6	33	31	66
At least one computer science course	55	67	69	19	13	6	26	20	25
At least one statistics course	31	46	59	37	47	8	32	8	34
At least one applied mathematics course beyond course E21	32	36	43	47	40	16	21	24	41
A capstone experience (senior project, thesis, seminar, internship)	32	68	76	27	17	5	41	15	19
An exit exam (written or oral)	3	10	31	3	15	2	94	75	67

TABLE SP.19A Percentage of mathematics departments that offer a major in statistics requiring certain courses (or exit exam) in all, some, or none of their majors, by type of department, in fall 2015.

Percentage of statistics departments that require:	Required in all majors			Required in some but not all majors			Not required in any major		
	Univ (PhD) %	Univ (MA) %	College (BA) %	Univ (PhD) %	Univ (MA) %	College (BA) %	Univ (PhD) %	Univ (MA) %	College (BA) %
(a) Calculus I	100	100	91			9			
(b) Calculus II	100	100	83			17			
(c) Multivariable Calculus	100	100	67			17			16
(d) Linear algebra/Matrix theory	92	100	83	6		17	2		
(e) At least one Computer Science course	60	85	67	8	7	33	32	7	
(f) At least one applied mathematics course, not incl. (a), (b), (c), (d)	42	47		8		16	49	53	84
(g) A capstone experience (e.g., a senior thesis or project, seminar, or internship)	16	100	83	18			66		17
(h) An exit exam (oral or written)			9	8			92	100	91
(i) One Probability Course	100	75	83		7	9		18	9
(j) One Mathematical Statistics Course	100	85	50		15	17			33
(k) One applied statistics course	74	85	75	8	15	25	18		
(l) One Linear Models Course	29	43	67	8	57	9	62		25
(m) One Bayesian Inference Course	7	19		8	8	25	84	73	75

TABLE SP.19.B Percentage of statistics departments requiring certain courses (or exit exam) in all, some, or none of their majors, by type of department, in fall 2015. This table can be compared to Table SP.21 in CBMS2010 p. 68.

Percentage of statistics departments that require:	Required in all majors		Required in some but not all majors		Not required in any major	
	Univ (PhD) %	Univ (MA) %	Univ (PhD) %	Univ (MA) %	Univ (PhD) %	Univ (MA) %
(a) Calculus I	97	83	3	17		
(b) Calculus II	97	83	3	17		
(c) Multivariable Calculus	88	50	5	33	8	17
(d) Linear algebra/Matrix theory	86	50	11	33	3	17
(e) At least one Computer Science course	86	67	6	17	7	17
(f) At least one applied mathematics course, not incl. (a), (b), (c), (d)	23	33	28		49	67
(g) A capstone experience (e.g., a senior thesis or project, seminar, or internship)	35	17	14	17	51	67
(h) An exit exam (oral or written)	2		6	17	92	83
(i) One Probability Course	75	50	11	17	13	33
(j) One Mathematical Statistics Course	89	33	8	33	3	33
(k) One applied statistics course	79	50	19	50	2	
(l) One Linear Models Course	60	17	9		31	83
(m) One Bayesian Inference Course	11	17	15		74	83

TABLE SP.20 Percentage of mathematics departments offering various upper-division mathematics courses at least once in the two-academic years 2014-2016 and 2015-2016, plus historical data on the two year period 2009-2011, by type of department. The table can be compared to Table SP.23 in CBMS2010 p. 70.

Upper-level mathematics courses	All Math Depts 2009-2011 %	Academic Years 2014-2015 & 2015-2016			
		All Math Depts 2014-2016 %	PhD Math %	MA Math %	BA Math %
Modern Algebra I	80	78	81	89	75
Modern Algebra II	27	27	57	48	17
Number Theory	51	37	59	65	27
Combinatorics	27	22	39	45	15
Actuarial Mathematics	13	21	38	40	14
Foundations/Logic	11	12	15	19	10
Discrete Structures	30	21	20	27	20
History of Mathematics	49	47	58	66	41
Geometry	74	71	79	77	68
Math for Secondary Teachers	35	33	45	59	26
Adv Calculus/ Real Analysis I	79	72	84	95	65
Adv Calculus/Real Analysis II	31	31	78	49	17
Adv Mathematics for Engineering/Physics	12	12	36	16	5
Advanced Linear Algebra	23	22	56	54	8
Introduction to Proofs	57	56	65	76	50

TABLE SP.20 (continued) Percentage of mathematics departments offering various upper-division mathematics courses at least once in the two academic years 2014-2015 and 2015-2016, plus historical data on the two-year period 2009-2011, by type of department. The table can be compared to Table SP.23 in CBMS2010 p. 71.

Upper-level math courses, continued	All Math Depts 2009-2011 %	Academic Years 2013-2014 & 2015-2016			
		All Math Depts 2014-2016 %	PhD Math %	MA Math %	BA Math %
Vector Analysis	11	11	32	9	7
Advanced Differential Equations	16	16	58	23	5
Partial Differential Equations	26	29	71	61	14
Numerical Analysis I and II	42	43	66	74	33
Applied Math/Modeling	37	36	45	53	31
Complex Variables	44	43	64	55	36
Topology	25	28	51	53	18
Mathematics of Finance	12	13	35	23	7
Codes & Cryptology	11	11	19	18	8
Biomathematics	12	8	26	10	4
Operations Research	17	18	15	35	16
Math senior seminar/Ind study	65	66	63	81	65
All other advanced-level mathematics	25	25	34	47	19

doctoral-level, 85% (SE 11.5) of masters-level, and 75% (SE 19) of bachelors-level mathematics departments) than did the masters-level statistics departments (50% (SE 10.9)). A larger estimated percentage of doctoral-level statistics departments (35% (SE 4)) required a capstone experience of all majors than did doctoral-level mathematics departments (16% (SE 8)), but an estimated 100% (respectively, 83% (SE 12)) of masters (respectively, bachelors)-level mathematics departments required a capstone experience of all statistics majors.

Comparing Table SP.21 from 2010 to Table SP.19.B from 2015, we see that among doctoral-level statistics departments, a larger estimated percentage of departments required Multivariable Calculus, Linear Algebra, Computer Science, and Mathematical Statistics of all majors in 2015 than in 2010. The estimated percentage of doctoral-level statistics departments requiring a Bayesian Inference course, while still small, increased slightly in 2015 over 2010. The option of a course in applied statistics as a requirement in all majors was a new option in the 2015 CBMS survey, and, in fall 2015, an applied statistics course was required of all majors in an estimated 79% (SE 2.7) of doctoral-level statistics departments and 50% (SE 10.9) of masters-level statistics departments.

Tables SP.20 and SP.21: Availability of Upper-level Courses in Mathematics and Statistics

Concerns about the availability of upper-level courses in mathematics and statistics led to questions on the CBMS surveys. Generally, the availability of upper-level mathematics courses was slightly less in 2014-16 than in 2009-11, and the availability of upper-level statistics courses in statistics departments was greater than in 2014-16 than in 2009-11. As noted in Chapter 1 Table S.2 (and will be seen in more detail in Chapter 3 Table E.3), estimated enrollments in upper-level courses were up (particularly in statistics courses) in fall 2015 over fall 2010.

Table SP.20 examines the availability of many upper-division mathematics courses offered in four-year mathematics departments at least once during the two academic years 2014-2015 and 2015-2016 (and the comparison to 2009-11), and Table SP.21 examines the same question for upper-division statistics courses offered in four-year mathematics and statistics departments; both tables are broken down by level of department. These tables can be compared to the CBMS2010 Tables SP.23 and SP.24, pages 70-72. For mathematics courses, Table SP.20 shows that over all mathematics departments combined, the percentages of departments offering specific upper-division courses in 2014-2016 were less, but only slightly less, than the percentages in 2009-11 for almost every course; two noticeable exceptions

were Number Theory, which was available at an estimated 51% of mathematics departments in 2009-2011 and at only an estimated 37% (SE 4.2) of departments in 2014-16, and Actuarial Mathematics, which was available at an estimated 13% of mathematics departments in 2009-11 and at an estimated 21% (SE 2.6) of departments in 2014-16 (and the estimated percentage of mathematics departments that offered Actuarial Mathematics increased at each level of department from 2009-11 to 2014-16). While there were differences in individual course percentages, the trends in 2014-16 over 2009-11 were about the same over all levels of mathematics departments. With the exception of Mathematics for Secondary Teachers and Mathematics for Engineering/Physics, all the estimated percentages of mathematics departments that offered a given course in 2014-16 were above the corresponding estimated percentages ten years ago (2004-6), and these changes are most notable at the bachelors-level departments; for example, in the 2005 survey report (CBMS2005, Table SP.22, p. 70) an estimated 52% of bachelors-level departments offered Modern Algebra I in 2004-6, while an estimated 75% (SE 4.6) of bachelors-level departments offered it in 2014-16. Similarly, an estimated 57% of bachelors-level departments offered Real Analysis I in 2004-6, while an estimated 65% (SE 4.8) offered it in 2014-16. However, both Modern Algebra II (offered at an estimated 15% of bachelors-level departments in 2004-6 and 17% of bachelors-level departments in 2014-16) and Real Analysis II (offered at 17% of bachelors-level departments in both 2004-6 and 2014-16) were offered at roughly the same low percentages in 2004-6 and in 2014-16 (for comparison, at doctoral-level departments, in 2014-16, Modern Algebra II was offered at an estimated 84% (SE 6.4) of departments, and Real Analysis II was offered at an estimated 78% (SE 6.2) of departments).

It is interesting to compare the availability of upper-level mathematics classes in 2014-16 to the reported availability in much earlier CBMS surveys. For example, Table SE 5 p. 10, of the CBMS1995 report presents the reported availability of a smaller list of upper-level mathematics courses in 1984-86, 1989-91, and 1995-96 (the latter only a one-year window). The percentages for the courses listed are roughly comparable to those reported in 2014-16, with the exception of Topology, offered by 35% of all departments (combined) in 1989-91 and 50% of all departments in 1995-96 (compared to 25% in 2014-16), and Foundations of Mathematics, offered by 22% of all departments in 1998-91 and 24% of all departments in 1995-96 (compared to 11% in 2014-16).

Table SP.21 examines the analogous question for statistics courses offered in mathematics departments and in statistics departments, providing data for the academic years 2009-11 and 2014-16. The list

TABLE SP.21 Percentage of mathematics and statistics departments offering various undergraduate statistics courses at least once in two academic years 2009-2010 and 2010-2011 and at least once in the two academic years 2014-2015 and 2015-2016, by type of department. This table can be compared to Table SP.24 in CBMS2010 p. 72.

Upper-level statistics courses	All Math Depts 2009-2011 %	AY 2014-15 & 2015-16				All Stat Depts 2009-2011 %	AY 2014-15 & 2015-16		
		All Math Depts %	PhD Math %	MA Math %	BA Math %		All Stat Depts %	PhD Stat %	MA Stat %
Introductory Probability and/or Statistics	na	18	14	28	16	na	48	54	31
Mathematical Statistics	42	34	47	42	30	78	73	82	46
Probability	37	37	53	41	32	63	70	77	46
Combined Probability and Statistics	26	32	33	45	30	37	48	48	46
Stochastic Processes	9	12	26	25	6	37	49	55	31
Applied Statistical Analysis	13	12	19	29	7	50	46	46	46
Experimental Design	10	9	13	26	5	51	59	58	62
Regression & Correlation	11	15	19	38	10	71	78	84	62
Biostatistics	4	7	11	9	6	27	36	40	23
Nonparametric Statistics	5	6	9	14	4	30	44	46	38
Categorical Data Analysis	1	4	8	11	2	31	30	35	15
Sample Survey Design	2	4	6	13	2	41	50	56	31
Stat Software & Computing	5	11	17	23	8	35/41*	62	64	54
Data Science	na	7	11	17	5	na	36	38	31
Bayesian Statistics	na	na	na	na	na	36	47	55	23
Statistical Consulting	na	na	na	na	na	29	34	38	23
Senior Seminar/ Independent Study	12	9	13	20	6	44	56	59	46

Note: 0 means less than one-half of one percent.

*In 2010, this appeared as two separate items in the statistics questionnaire, with 41 percent reporting courses in statistical computing and 35 percent reporting courses in statistical software.

TABLE SP.22 Departmental estimates of the percentage of graduating mathematics or statistics majors from academic year 2014-2015 who had various post-graduation plans, by type of department, in fall 2015. (Data from fall 2010 in parentheses.)

Departmental estimates of post-college plans	Mathematics Departments			Statistics Departments	
	Univ (PhD) %	Univ (MA) %	College (BA) %	Univ (PhD) %	Univ (MA) %
Students who went into pre-college teaching	12 (13)	25 (48)	26 (27)	1 (1)	1 (1)
Students who went to graduate school in the mathematical or statistical sciences	11 (15)	13 (12)	12 (17)	17 (23)	10 (29)
Students who went to graduate or professional school outside of mathematics/statistics	8 (10)	4 (4)	7 (8)	10 (5)	1 (5)
Students who took jobs in business, government, etc.	27 (27)	19 (19)	34 (30)	34 (41)	20 (45)
Students who had other plans known to the department	3 (5)	3 (3)	4 (4)	3 (2)	0 (3)
Students whose plans are not known to the department	40 (30)	36 (14)	16 (13)	36 (29)	68 (18)

TABLE SP.23 Percentage of four-year mathematics and statistics departments undertaking various assessment activities during the last six years, by type of department, in fall 2015. (Data from fall 2010 when known in parentheses.)

Percentage using various assessment tools	Four-year Mathematics Departments			Statistics Departments	
	Univ (PhD) %	Univ (MA) %	College (BA) %	Univ (PhD) %	Univ (MA) %
Consult outside reviewers	36 (53)	57 (48)	40 (31)	44 (42)	42 (80)
Survey program graduates	67 (71)	83 (80)	59 (71)	70 (63)	67 (70)
Consult other departments	44 (54)	42 (45)	38 (26)	46 (47)	17 (60)
Study data on students' progress in later courses	63 (62)	77 (65)	62 (55)	21 (41)	33 (40)
Assessed teaching objectives	78	81	85	98	67
Evaluate placement system	72 (72)	52 (51)	57 (60)	18 (12)	25 (30)
Change undergraduate program due to assessment	80 (78)	76 (76)	70 (69)	76 (61)	75 (80)

of statistics courses was revised in 2010, increasing the number of upper-divisional statistics offerings for undergraduates that could be reported in statistics departments, and a few changes were made to the list of statistics course options in mathematics and statistics departments in the 2015 survey. Generally, the estimated percentages of statistics departments offering each upper-level course was up slightly in 2014-16 from 2009-11; for example, in 2009-11, an estimated 30% of statistics departments offered a course in nonparametric statistics, while this percentage increased to 44% (SE 3.1) in 2014-16. However, many of the percentages were larger in 2000-2001 than in 2014-16; for example, by CBMS2000 Table SP.23, p. 72, in 2000-1 (a one-year period) Applied Statistical Analysis was offered at 70% of statistics departments, while in 2014-16 (a two-year period) it was offered at 50% (SE 3.2) of statistics departments. Estimated percentages of mathematics departments offering various upper-level statistics courses in 2014-16 were roughly comparable to the estimated percentages in 2009-11, and these percentages were smaller than in statistics departments; for example, an estimated 6% (SE 1.2) of mathematics departments offered a course in nonparametric statistics in 2014-16 (the estimated percentage was 5% for 2009-2011). Over the past fifteen years, the offering of Mathematical Statistics has decreased: in the 2000 survey it was offered by an estimated 52% of mathematics departments and an estimated 90% of statistics departments in the one-year period (2000-1), but, in 2014-16 (a two-year period), it was offered by an estimated 34% (SE 4.3) of mathematics departments and 73% (SE 2.6) of statistics departments (both estimated percentages slightly less than in 2009-11).

Table SP.22: Estimates of Post-Graduation Plans of Graduates of Four-Year Mathematics Departments and Statistics Departments

Table SP.22 presents estimates from four-year mathematics departments and statistics departments of the post-graduation plans of their 2014-2015 graduating undergraduate majors, broken down by the level of department. Departments do not know the post-graduation plans of many of their majors, and, in fact, the estimated percentages of students with unknown post-graduation plans rose among all levels of four-year mathematics and statistics departments from 2009-10 graduates to 2014-15 graduates. The estimated percentage of 2014-15 graduates with post-graduation plans unknown to the department was estimated at 40% (SE 4) among doctoral-level mathematics departments, 36% (SE 9.7) among masters-level mathematics departments, and 18% (SE 2) among bachelors-level mathematics departments; among statistics departments, these

estimated percentages were 38% (SE 2.8) among doctoral-level statistics departments (up from 29% in 2009-10 graduates) and 68% (SE 11.3) among masters-level statistics departments (up from 18% of 2009-10 graduates). Given the large percentages of students whose plans were unknown, the plans of the 2014-15 graduates known to the department were roughly comparable to the plans of the 2009-10 graduates, and the plans of the 2014-15 mathematics graduates were roughly similar to the plans of the 2014-15 statistics graduates, except for the small percentage of statistics graduates entering pre-college teaching. Among students whose plans were known to the department, at doctoral (respectively, bachelors) level mathematics departments, the largest estimated percentage 27% (SE 2.7) (respectively, 34% (SE 3)) of 2014-15 graduates took jobs in business, government, etc., and among masters-level mathematics departments, the largest estimated percentage of students (25% (SE 4.7) of 2014-15 graduates, down from 48% of 2009-10 graduates), accepted jobs in pre-college teaching. Among statistics departments, the largest estimated percentage of students whose plans were known took jobs in business, government, etc. (34% (SE 2) at doctoral-level statistics departments and 20% (SE 7.4) at masters-level departments). The estimated percentage of 2014-15 graduates of statistics departments known to go on to graduate study in the statistical sciences was down from the estimated percentage of 2009-10 graduates at both the doctoral and the masters-level statistics departments, but was comparable to the percentages of graduates from mathematics departments that went on to graduate study in the mathematical sciences (these estimates were about the same as the estimates made for 2009-10 graduates). The estimated percentages of 2014-15 graduates of mathematics departments who went into pre-college teaching was slightly down for graduates of all three levels of mathematics departments, and remained estimated at 1% of statistics department graduates.

Table SP.23: Assessment Activities in Four-Year Mathematics Departments and Statistics Departments

State governments, national accrediting agencies, and professional organizations such as the Mathematical Association of America have placed great emphasis on department assessment activities. Beginning with the 2005 CBMS survey, four-year mathematics and statistics departments were asked to identify which of a list of assessment activities they had performed over the last six years. This question was repeated in the 2010 and 2015 CBMS surveys; a summary of the responses to the 2010 and 2015 surveys can be found in Table SP.23. The results obtained in fall 2015 were roughly compa-

table to those reported in fall 2010. The estimated percentage of doctoral-level mathematics departments that had consulted with outside reviewers dropped from an estimated 53% in 2010 to 36% (SE 6.7) in 2015. The percentage of bachelors-level mathematics departments that had surveyed program graduates dropped from an estimated 71% in 2010 to 59% (SE 5.4) in 2015. The percentage of doctoral-level statistics departments that had studied data on students' progress in later courses dropped from 41% in 2010 to 21% (SE 2.7) in 2015 (compared to 63% (SE 6.4) of doctoral-level mathematics departments). An additional option, added to the 2015 CBMS survey questionnaire, asked about assessment of teaching objectives, which, according to Table SP.23, was reportedly performed at more than an estimated 78% (SEs 3-8) of all the mathematics departments, 98% (SE 0.5) of the doctoral-level statistics departments, and 67% (SE 7) of the masters-level statistics departments. For all levels of mathematics and statistics departments, over 70% (SEs 3-7) said that their assessment activities had resulted in changes to their undergraduate programs.

Table SP.24: Institutional or Divisional Graduation Requirements Satisfied by Advanced Placement Courses in Four-Year Mathematics and Statistics Departments

In 2015 the CBMS survey asked four-year mathematics and statistics departments whether advanced placement courses (taken when in high school) could be used to meet their institution's mathematical sciences divisional graduation requirements. Across all levels of mathematics and statistics departments, in fall 2015, the estimated percentage of departments that reported that these courses did meet divisional graduation requirements was at least 83% (with SEs of 2-3, except at masters-level mathematics departments, where the SE was 7.8).

Tables SP.25-SP.27: Pedagogical Methods and Making Changes at Four-Year Mathematics and Statistics Departments

The 2015 CBMS survey included several new questions asking about pedagogical methods used in mathematics and statistics departments. In asking department chairs to comment on pedagogical methods used in their department, it is useful to determine what information was available to them. Table SP.25 summarizes the information on teaching that was collected in four-year mathematics and statistics departments in fall 2015. The data show that almost all four-year mathematics and statistics departments collected course syllabi, few (an estimated 16% (SE 2.4)) of all mathematics departments combined collected teaching portfolios, but an esti-

mated 36% (SE 2.9) of all statistics departments reported collecting teaching portfolios. Peer evaluation of teaching was done at an estimated 64% of all levels of mathematics departments combined, and all levels of statistics departments combined (the SE for mathematics departments was 3.5, and the SE for statistics departments was 3). Self-evaluation was available less often, and primarily at masters-level mathematics and statistics departments, and at bachelors-level mathematics departments. Departmental discussions of teaching methods were held at about 2/3 of mathematics and statistics departments, across all levels of departments (SE was 5 for all levels of mathematics departments combined, and SE was 2.8 for all levels of statistics departments combined).

Four-year mathematics and statistics departments were asked if each in a list of teaching strategies was used by some member of their department; Table SP.26 presents a summary of the responses, broken down by level of department. No definitions of these strategies were given in the instrument, allowing for broad interpretation of what constitutes "inquiry based learning" (generally regarded as a strategy aimed at promoting active learning that starts by posing questions, rather than presenting established facts) or "flipped classrooms" (typically where the instructional content is delivered outside of the classroom, and class sessions are devoted to activities that might otherwise be done as homework). At least 50% of most levels of mathematics and statistics departments reported that a member of their faculty was using inquiry based learning in a class. In mathematics, across all levels of departments, flipped classrooms were used by someone at more than 50% of the departments, at each level of mathematics departments; flipped classrooms were used less frequently (estimated at 39% (SE 2.9)) across both levels of statistics departments combined. At least one faculty member taught a class largely online in almost 50% of mathematics and statistics departments, except at bachelors-level mathematics departments, where the percentage was estimated at 33% (SE 7.2). Activity based learning was used at an estimated 66% (SE 5.3) of all mathematics departments combined, and 77% (SE 2.7) of all statistics departments combined. Technology was used to develop conceptual understanding at an estimated 86% (SE 3) of all mathematics departments and 84% (SE 3) of all statistics departments. The survey questions did not address how many individual faculty members were using each of these methods (a more difficult question for a chair to answer) but this data shows that these pedagogical methods are represented by at least one faculty member at most mathematics and statistics departments.

Four-year mathematics and statistics departments were asked if the department had experienced major change in the types of pedagogy used in the depart-

TABLE SP.24 Percentage of mathematics and statistics departments that allow a student to meet an institutional or divisional graduation requirement using an advanced placement course.

Meets requirement	All Math Depts	PhD Math	MA Math	BA Math	All Stat Depts	PhD Stat	MA Stat
Yes (%)	88	97	83	87	86	84	92
No (%)	12	3	17	13	14	16	8

TABLE SP.25 Percentage of four-year mathematics and statistics departments reporting that various items are significant sources of information to the department about the types of pedagogy used.

Activity	All Math Depts	PhD Math	MA Math	BA Math	All Stat Depts	PhD Stat	MA Stat
Syllabi for classes	87	95	96	84	98	98	100
Teaching portfolios	16	23	28	12	36	35	42
Peer evaluation of instructors	64	78	74	60	64	60	75
Self-evaluation of instructors	51	28	47	57	29	22	50
Department discussions of teaching practices	69	66	64	71	73	68	92
None of these are available	2	2	3	1			

ment during last 10 years, and an estimated 60% of mathematics departments and 80% of statistics departments reported that it had (see Table SP.27). Of those departments experiencing change, respondents were asked to attribute the change to any of a list of factors (they could check all that applied), and Table SP.27 summarizes the responses. The overwhelming factor, cited by 91% (SE 3.2) of mathematics departments combined and 88% (SE 2.4) of the statistics departments combined, was the advocacy of some member of their faculty. Educational research was the next most cited factor, noted by an estimated 61% (SE 5.7) of the mathematics departments combined and 49% (SE 3.6) of the statistics departments combined. Advocacy by the institution's administration was cited by an estimated 47% (SE 3.5) of the statistics departments combined and 37% (SE 4.7) of the mathematics departments combined, and advocacy by a professional organization was cited by 39% (SE 4.5) of the mathematics departments combined and 38% (SE 3.5) of the statistics departments combined. Advocacy

by another department was cited by 16% of both the mathematics departments combined (where the SE was 4.5) and the statistics departments combined (where the SE was 2.5).

Table SP.28: Statistics Minors and Majors in Four-Year Mathematics Departments

A new set of questions in the 2015 CBMS survey dealt with statistics minors and majors in mathematics departments; the responses to these questions are summarized in Table SP.28. By Table SP.28, in fall 2015, the estimated percentage of mathematics departments offering a major in statistics is 10% (SE 1.8) across all levels of mathematics departments combined; it is 25% (SE 5.7) at doctoral-level departments, 26% (SE 8.2) at masters-level departments, and 4% (SE 1.6) at bachelors-level departments. The estimated percentage of departments offering a minor in statistics is 16% (SE 2.1) across all levels of mathematics departments combined, but 52% (SE 7.5) at

TABLE SP.26 Percentage of four-year mathematics and statistics departments reporting that various pedagogical strategies are used by some member of the department faculty.

Activity	All Math Depts	PhD Math	MA Math	BA Math	All Stat Depts	PhD Stat	MA Stat
Inquiry based class	58	56	71	57	54	56	45
Flipped classroom	58	61	52	59	39	35	55
Class conducted largely online	38	49	53	33	48	49	45
Activity based learning	66	64	71	65	77	70	100
Technology used to develop conceptual understanding	86	82	91	86	84	84	82

TABLE SP.27 Percentage of mathematics and statistics departments reporting major changes in the kinds of pedagogy used in their departments, and the percentage citing various reasons for those changes.

Activity	All Math Depts	PhD Math	MA Math	BA Math	All Stat Depts	PhD Stat	MA Stat
Department experienced major changes over the last 10 years	60	62	65	58	80	78	85
Of those experiencing change, the percent attributing the change to:							
Educational research	61	67	77	56	49	53	36
Advocacy of some faculty member in the department	91	99	90	90	88	88	91
Advocacy by another department	16	23	14	15	16	21	0
Advocacy by institution's administrators	37	47	30	35	47	48	45
Advocacy by a professional organization	39	31	33	43	38	36	45

masters-level departments. Between July 1, 2014-June 30, 2015, an estimated 1,012 students (SE 213) graduated with a minor in statistics that was obtained in a mathematics department.

Tables SP.29-SP.31 Profiles of other full-time faculty in four-year mathematics and statistics departments

Concern has been voiced about the early career profiles of individuals with Ph.D.s in the mathematical sciences. There are increasing numbers of postdocs and decreasing numbers of tenure-eligible positions, and there seems to be a growing number of non-tenure-eligible positions (see, e.g. Amy Cohen, “Disruptions of the Academic Math Employment Market”, *Notices of the American Mathematical Society*, October 2016, pp. 1057-1060). Data on numbers of faculty obtained from the CBMS survey in fall 2015 are contained in Table S.15, and in the Chapter 4 tables. As a part of the CBMS 2015 survey, and the Annual Survey administered by the American Mathematical Society that is a part of the CBMS survey, a separate instrument (see Appendix V) was sent to mathematics and statistics departments to gain more information about postdocs and other full-time faculty who are not tenure-eligible. This survey consisted of three sets of questions related to the profiles of research postdocs, non-tenure-eligible faculty with renewable appointments, and non-tenure-eligible faculty with fixed-term (nonrenewable) appointments.

The first set of questions was intended to study the career profile of (research) postdoctoral faculty; it inquired about positions postdocs accept after leaving

a postdoc position. The question asked departments, first, for the number of individuals in their department in 2014-2015 who were postdoctoral faculty (defined as: “those in a temporary position primarily intended to provide an opportunity to extend graduate training or to further research experience”), and, next, for the number of those individuals who were postdocs in 2014-15, but were not classified as postdoctoral research faculty in fall 2015-16 (including postdocs who remained in the department in a different position), i.e. the number of individuals who were postdocs in 2014-15, and left the position of postdoc at that institution after the 2014-15 academic year. For those individuals who were no longer postdocs, responders were given six choices (and “unknown”) for the current positions of these postdocs; these options were intended to illuminate the career path of postdocs. The responses from this set of questions are summarized in Table SP.29, which is broken down by the level of the responding mathematics department, and by doctoral-level statistics department.

Table SP.29 shows that in the masters and bachelors-level mathematics departments, a large percentage of postdocs left the postdoc position after 2014-15 (an estimated 71% (SE 1) at masters-level departments, and 89% (SE 5.1) at bachelors-level departments), while an estimated 39% (SE 1.4) of the postdocs at doctoral-level mathematics departments, and 30% (SE 5.8) of postdocs who were at doctoral-level statistics departments, left a postdoc position after 2014-15 (hence about 1/3 of postdocs in 2014-15 ended their appointment as a postdoc at the same doctoral-level department, which would be expected with postdocs usually serving a 3-year appointment). These data

TABLE SP.28 Percentage of four-year mathematics departments offering a minor in statistics, the number of students graduating with such a minor between July 1, 2014, and June 30, 2015, and the percentage of four-year mathematics departments offering a major in statistics.

Number of tracks	Mathematics Departments			Total
	Univ (PhD)	Univ (MA)	College (BA)	
Offer a minor in statistics (%)	13	52	10	16
Number of graduates	305	323	384	1012
Offer a major in statistics (%)	25	26	4	10

Some totals are less than 100% due to round-off.

TABLE SP.29 Profile of 2014-2015 Postdocs who left the position at the end of the 2014-2015 academic year.

	Doctoral Math	Masters Math	Bachelors Math	All Math	Doctoral Stat	Masters Stat	All Stat
Postdocs during 2014-2015 academic year	1297	46	119	1463	100	0	100
Number who left the position for fall 2015	501.3	32.8	106.1	640.2	30.0	0.0	30
Percent who left the position for fall 2015	38.6%	70.5%	88.8%	43.7%	30%		30%
Of those who left the position for fall 2015:							
Number who took tenure-track position	179.5	8.3	72.5	260.4	7.2	0.0	7.2
Percent who took tenure-track position	36%	25%	68%	41%	24%		24%
Number who took another postdoc position	111.0	5.8	0.0	116.9	3.8	0.0	3.8
Percent who took another postdoc position	22%	18%	0%	18%	13%		13%
Number who took renewable appointment for fall 2015	66.7	13.3	28.9	108.8	15.4	0.0	15.4
Percent who took renewable appointment for fall 2015	13%	41%	27%	17%	51%		51%
Number who took non-renewable appointment for fall 2015	30.1	0.0	0.0	30.1	1.8	0.0	1.8
Percent who took non-renewable appointment for fall 2015	6%	0%	0%	5%	6%		6%
Number who took non-academic appointment for fall 2015	28.8	2.9	4.7	36.5	1.8	0.0	1.8
Percent who took non-academic appointment for fall 2015	6%	9%	4%	6%	6%		6%
Number unemployed for fall 2015	1.9	0.0	0.0	1.9	0.0	0.0	0.0
Percent unemployed for fall 2015	0%	0%	0%	0%	0%		0%
Number whose status is unknown for fall 2015	83.3	2.3	0.0	85.7	0.0	0.0	0.0
Percent whose status is unknown for fall 2015	17%	7%	0%	13%	0%		0%

suggest that typically a postdoc position at a doctoral-level department is a different experience than at a masters-level or bachelors-level department. The responding departments reported that there were no postdocs that they would classify as unemployed in fall 2015, but the precise status of their former postdocs was not always known (e.g. an estimated 17% (SE 1.5) of postdocs leaving positions at doctoral-level mathematics departments after 2014-15 had “unknown” status in fall 2015, and possibly many of these former postdocs were unemployed). Of those postdocs who left a postdoc position after 2014-15, an estimated 68% (SE 7.6) of the postdocs at bachelors-level departments, 25% (SE 11.4) of the postdocs at masters-level departments, 36% (SE 2.1) of the postdocs at doctoral-level mathematics departments, and 24% (SE 11.1) of postdocs at doctoral-level statistics departments, were employed in a tenure-eligible position in fall 2015. The percentages of postdocs who left a postdoc position after 2014-15, and who were known to be in another postdoc position in fall 2015, was an estimated 22% (SE 1.8) of the postdocs leaving doctoral-level mathematics departments, 18% (SE 7.2) of postdocs leaving masters-level mathematics departments, and 13% (SE 6.2) of postdocs leaving doctoral-level statistics departments. The percentages of postdocs who left a postdoc position after 2014-15 and were in a renewable (but not postdoc or tenure-eligible) position in fall 2015 was estimated at 13% (SE 1.4) of the postdocs leaving doctoral-level mathematics departments, 41% (SE 11) of postdocs who left masters-level mathematics departments, 27% (SE 7) of postdocs who left bachelors-level departments, and 51% (SE 10.8) of postdocs who left doctoral-level statistics departments. The percentages of postdocs who left postdoc positions after 2014-15 and took nonacademic or non-renewable academic positions were small. The data in Table SP.29 provides some light on the career path of postdocs at various kinds of institutions, and, if confirmed by further studies, suggests that the career path of a postdoc varies according to the level of institution where the postdoc was completed. For example, it appears that about half of postdocs at doctoral-level statistics departments took a subsequent renewable appointment, and about a quarter took tenure-track positions after completing a postdoc, that postdocs at bachelors-level departments generally did not take another postdoc, but were likely to find a tenure-eligible job or a renewable position after completing the postdoc, that postdocs at doctoral-level mathematics departments tended to accept tenure-track or renewable positions or another postdoc, etc.

The second set of questions related to the profile of faculty with renewable, but not tenure-eligible (and not postdoc), appointments; these were faculty with

positions such as Lecturer, Teaching Professional, Professor of the Practice, Instructor, etc. Data was collected on the number of such positions, the number leaving these positions after 2014-15, and the typical responsibilities of faculty in these positions.

The first question in this second set of questions asked for the number of faculty in renewable positions in 2014-15, and, of those, how many of these faculty were no longer in that position in fall 2015. The survey also asked for the number of faculty who were in such a renewable position in 2015-16. Finally, department chairs were asked, of those faculty who were in such a position in 2015-16, for the number of renewable-term faculty who typically were engaged in each of a list of nine different activities. The responses from this set of questions are contained in Table SP.30, which is broken down by level of mathematics and statistics department.

Table SP.30 shows that, in fall 2015, essentially all faculty with renewable appointments taught, and that in both doctoral and masters-level mathematics departments an estimated 14% (SE 1) (21% (SE 2) in bachelors-level departments) and 8% (SE 2) across both levels of statistics departments left the renewable position after 2014-15 for new position in fall 2015. Across all levels of mathematics departments combined, an estimated 16% (SE 0.8) were active in research; in doctoral-level statistics departments an estimated 33% (SE 2.8) were active in research. Support for attending conferences would appear not to be a standard benefit of renewable positions in fall 2015, as less than an estimated 20% (with SEs around 1 in each level of mathematics department and 2.3 in both levels of statistics departments combined) of faculty with renewable positions would be supported to attend a research conference (even at the doctoral-level statistics departments), and, support to attend a teaching conference was available to only an estimated 29% (SE 1) of faculty with renewable positions across all levels of mathematics departments combined (to an estimated 37% (SE 2.1) at bachelors-level mathematics departments), and to an estimated 13% (SE 2.1) of faculty with renewable positions across all levels of statistics departments combined. Across all levels of departments, more than half of the faculty with renewable positions typically would serve on departmental committees, and less than 1/3 would serve as a course coordinator (except at masters-level statistics departments, where 54% (SE 10.5) of faculty with renewable positions would serve as a course coordinator). Except at bachelors-level mathematics departments and masters-level statistics departments, less than an estimated 20% (SEs 1-2) of faculty with renewable positions would serve on college/university committees. Across all levels of mathematics departments combined an estimated

TABLE SP.30 Profile of Non-tenure-track faculty with renewable appointments.

	Doctoral Math	Masters Math	Bachelors Math	All Math	Doctoral Stat	Masters Stat	All Stat
Renewable positions filled for 2014-2015	1641.1	850.2	1778.1	4269.4	214.4	50.7	265.1
Number that left renewable position for 2015	228.9	121.6	375.3	725.8	15.0	5.3	20.3
Percent that left renewable position for 2015	14%	14%	21%	17%	7%	11%	8%
Renewable positions filled for 2015-2016	1645.2	865.2	1808.5	4318.8	253.2	34.7	287.9
Number active in teaching	1625.1	865.2	1794.3	4284.6	243.6	34.7	278.3
Percent active in teaching	99%	100%	99%	99%	96%	100%	97%
Number active in research	276.4	92.3	310.7	679.4	91.6	2.7	94.3
Percent active in research	17%	11%	17%	16%	36%	8%	33%
Number that attend research conf. with support	174.6	79.7	341.0	595.4	39.0	2.7	41.7
Percent that attend research conf. with support	11%	9%	19%	14%	15%	8%	14%
Number that attend teaching conf. with support	377.5	218.9	665.6	1261.9	37.2	0.0	37.2
Percent that attend teaching conf. with support	23%	25%	37%	29%	15%	0%	13%
Number that serve on dept. committees	866.4	512.0	1145.2	2523.6	137.2	21.3	158.5
Percent that serve on dept. committees	53%	59%	63%	58%	54%	62%	55%
Number that advise undergrad. research projects	200.1	89.6	363.0	652.8	39.8	10.7	50.5
Percent that advise undergrad. research projects	12%	10%	20%	15%	16%	31%	18%
Number that serve as academic advisor	336.9	208.4	725.3	1270.6	77.2	10.7	87.9
Percent that serve as academic advisor	20%	24%	40%	29%	30%	31%	31%
Number that serve on univ. committees	234.0	176.0	711.3	1121.3	30.6	13.3	43.9
Percent that serve on univ. committees	14%	20%	39%	26%	12%	38%	15%
Number that serve as course coordinator	540.4	179.5	503.9	1223.8	50.6	18.7	69.3
Percent that serve as course coordinator	33%	21%	28%	28%	20%	54%	24%

29% (SE 1) of faculty with renewable positions typically would serve as an academic advisor (40% (SE 2.1) at bachelors-level departments), and across all levels of statistics departments an estimated 31% (SE 3) of faculty with renewable positions would serve as an academic advisor. Across all levels of mathematics departments, the percentage of faculty with renewable positions who typically would supervise undergraduate research projects was about the same as the percentage who were active in research. In doctoral-level statistics departments, an estimated 36% (SE 3) of faculty with renewable positions were active in research while an estimated 16% (SE 2.5) would supervise undergraduate research projects; in masters-level statistics departments (which reported an estimated total of only 51 such faculty), an estimated 8% (SE 6.1) of faculty with renewable positions were research-active, but an estimated 31% (SE 10.5) typically would supervise undergraduate research projects.

The final set of questions dealt with the profile of faculty in fixed-term (non-renewable) appointments, and the same questions were asked about this group of faculty that were asked about faculty with renewable appointments. The responses to these questions are summarized in Table SP.31, which is broken down by level of mathematics and statistics department.

From Table SP.31 we see that, in fall 2015, there were estimated to be fewer fixed-term (non-renewable) faculty appointments than renewable-term faculty appointments (an estimated total of 4,269 (SE 187) renewable positions, and 1,503 (SE 127) fixed-term positions, across all levels of mathematics departments combined; for statistics departments,

the estimates were 265 (SE 29) renewable and 53 (SE 11) fixed-term appointments). Across all levels of mathematics, about 1/3 of those faculty who were in a fixed-term appointment in 2014-15 were not in the department in fall 2015 (the estimates for statistics departments are small, and the numbers very variable). Across all levels of mathematics departments, a larger percentage of faculty with fixed-term appointments were active in research than the percentage of faculty with renewable appointments, and, except for masters-level mathematics departments, the faculty with fixed-term appointments were more likely to be supported to attend a research conference (e.g. at doctoral level mathematics an estimated 11% (SE 0.7) of renewable-term faculty typically would receive support to attend a research conference, while an estimated 27% (SE 2) of fixed-term faculty would typically receive such financial support). There was a smaller estimated percentage of fixed-term appointment faculty who would typically be supported to attend a teaching conference than the estimated percentage for faculty with renewable appointments. There was a small estimated percentage of fixed-term faculty who typically were involved in the other activities listed (serving on a departmental committee, serving on a university committee, serving as an academic advisor, supervising an undergraduate research project, or serving as a course coordinator); one exception was, at bachelors-level mathematics departments, the estimated percentage of fixed-term appointment faculty typically supervising an undergraduate research project was 27%, (SE 3) while the percentage of renewable-term appointment faculty typically supervising such a project was estimated at 20% (SE 1.7).

TABLE SP.31 Profile of Non-tenure-track faculty with fixed-term (non-renewable) appointments.

	Doctoral Math	Masters Math	Bachelors Math	All Math	Doctoral Stat	Masters Stat	All Stat
Number of Fixed-term positions filled for 2014-2015	511.4	311.4	680.5	1503.3	47.8	5.3	53.1
Number that left fixed-term position for 2015	159.1	81.0	212.5	452.6	25.6	5.3	30.9
Percent that left fixed-term position for 2015	31.1%	26.0%	31.2%	30.1%	54%	100.0%	58%
Number of Fixed-term positions filled for 2015-2016	574.1	382.5	658.5	1615.1	54.6	13.3	67.9
Number active in teaching	567.4	382.5	655.9	1605.9	48.6	13.3	61.9
Percent active in teaching	99%	100%	100%	99%	89%	100.0%	91%
Number active in research	213.7	44.8	267.5	526.0	28.4	2.7	31.1
Percent active in research	37%	12%	41%	33%	52%	20.0%	46%
Number that attend research conf. with support	153.0	27.2	241.6	421.8	9.6	2.7	12.3
Percent that attend research conf. with support	27%	7%	37%	26%	18%	20.0%	18%
Number that attend teaching conf. with support	60.8	40.9	158.7	260.4	0.0	0.0	0.0
Percent that attend teaching conf. with support	11%	11%	24%	16%	0%	0.0%	0%
Number that serve on dept. committees	73.4	117.4	246.1	436.9	9.6	2.7	12.3
Percent that serve on dept. committees	13%	31%	37%	27%	18%	20.0%	18%
Number that advise undergrad. research projects	19.5	32.2	175.6	227.3	4.0	0.0	4.0
Percent that advise undergrad. research projects	3%	8%	27%	14%	7%	0.0%	6%
Number that serve as academic advisor	17.8	14.2	112.8	144.8	4.0	0.0	4.0
Percent that serve as academic advisor	3%	4%	17%	9%	7%	0.0%	6%
Number that serve on university committees	7.2	27.2	78.5	112.8	0.0	0.0	0.0
Percent that serve on university committees	1%	7%	12%	7%	0%	0.0%	0%
Number that serve as course coordinator	44.1	26.0	99.8	170.0	0.0	0.0	0.0
Percent that serve as course coordinator	8%	7%	15%	11%	0%	0.0%	0%