

Teaching Tips

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Preamble: Why we wrote this. Almost all of us do some teaching. All of us could do better. If doing better required a lot of time or a drastic overhaul of our basic personalities, we probably wouldn't bother. Here is a list of *easy* ways to be a better teacher. In the epilogue we talk a bit about teaching styles.

1. Good preparation

- Identify main theme with about three words.
- Relate to bigger theme they've seen or heard of before.
- Select and hone three mathematical punchlines.
- Before each class, look over your notes and make a quick estimate as to how long each topic will take.

2. Good boardwork

- Erase completely.
- Do not erase too quickly.
- Write neatly upper left to lower right, starting with main theme.
- Good headings and pictures.
- End with main theme.

3. Good speaking

- Good start
- Keep talking.
- Repeat yourself (at any given time, 30% of class is not paying attention*).
- Look at audience; read their faces.
- After asking for questions, wait 15 seconds.

4. Learn students' names

*We made up the 30%; true figure probably higher.



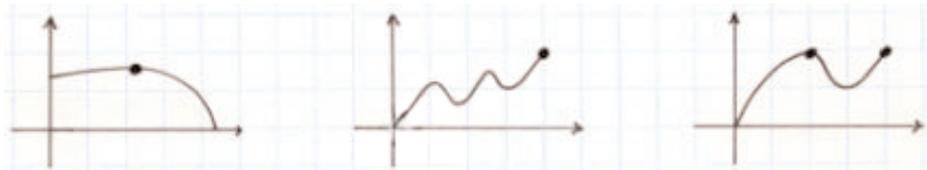
Example of Good Preparation and Good Boardwork

Example taken from the lecture in calculus or real analysis that a continuous function on $[a,b]$ attains a maximum.

State and write on the board that the main theme is:

“Continuous function ATTAINS maximum.”

Draw three graphs of functions on the interval: one with one maximum, one with one absolute maximum and two relative maxima, one with two absolute maxima.



Emphasize that the primary emphasis is on the word “attains.” (Writing merely “Maxima of continuous functions” would be too general and might put students back into the mindframe of finding maxima by setting the derivative equal to 0.) This sets the stage for showing why there is an issue, namely because an *infinite* set of values need not reach a maximum it approaches.

This topic easily can be related to a theme they’ve seen before: the archetypical calculus exercise of finding maxima. But now it addresses the fundamental question of whether you can count on maxima being there to find.

Possible punchlines for the development in a real analysis course are:

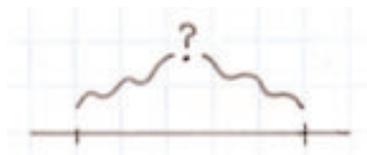
- (1) Result fails if interval is not closed (a,b) or not bounded $[a, \infty)$. (State, examples, restate.)
- (2) Proof is in two steps: continuous image of compact is compact; compact set of reals has a maximum.
- (3) Result is a consequence of the *compactness* of $[a,b]$.

Possible punchlines for the development in a calculus course are:

- (1) Fails for (a,b) , $[a, \infty)$, f not continuous.
- (2) May sound obvious or fussy at first, but exact hypotheses really matter.
- (3) Concern for the proof led to a lot of new mathematics, which the students will see in future math courses.

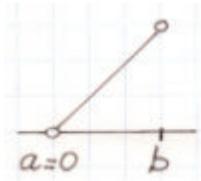
For example, you might write on the board:

“How do you know function *has* a maximum?”



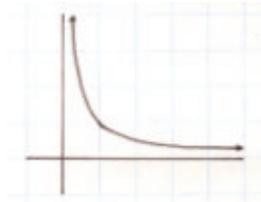
The theorem can fail on an interval (a,b) without endpoints:

$$f(x) = x$$



The theorem can fail on an unbounded interval:

$$f(x) = 1/x$$



The theorem can fail if the function is not continuous:



You really need all those hypotheses (result can fail if interval is missing endpoints or function is not continuous).

Rationale for Good Board Technique

**Erase completely—Do not erase too quickly—Write neatly upper left to lower right—
Good headings and pictures—End with main theme**

What is on the board is what students will use while studying. Since at any given time, a number of people will be daydreaming (think of how you listen to lectures), what is on the board is critical to keep people from losing track. Remember that the material is clearer to you than to the students. They will use their notes, copied from the board, to try to understand the material.

Erasing completely is satisfying to an audience, but it is more important than that. Suppose a stray horizontal line remains. You write a new formula and the ghost of that horizontal line looks, to a daydreaming student in the back of a crowded lecture hall, like a minus sign. Later, while studying, this student is baffled.

Erasing too quickly is the worst. Suppose I am teaching a beginning calculus class. Most students are not yet experts at algebra. If I write on the board

$$f(x) = 4x^3 + 7x^2 - 9 + 3x^3 - 10,$$

immediately erase the $3x^3 - 10$ and alter the previous terms, half the students are lost. They're still two lines behind, and they'll never know what happened. If you leave up everything you do, they have a fighting chance to copy it down and figure it out later.



Good Speaking

It's worth memorizing a good opening sentence, to start with a bang. For the lecture on the existence of a maximum, you might open with a line like:

"If I'm going to invest a lot of time or money in something, I like a guarantee that it will work."

Keep talking. Silence makes the audience nervous. It's OK to repeat yourself while you're carefully writing or drawing pictures on the board. In fact, deliberately repeat yourself. Students are reassured, feel comfortable, maybe comfortable enough to ask a question some day.

Look at the audience; read their faces. Remember, everything you're doing is for them. You're talking to them. Are they getting it?

After asking for questions, wait 15 seconds. Everybody will appreciate the time to catch up, think about the question, and gather the courage to venture a guess or comment.

Learn Students' Names

Learn students' names, not just so that you can talk to the students and call on them, which would be reason enough, but so that they know that they matter, that if they come, someone will notice, that someone cares.

This takes a small amount of effort for the first week or two. Call out their names if you return homework. Take 20 seconds to read the class list aloud every day after class.

A big side bonus is that they learn each other's names and are more likely to help each other or work together on the homework.

Epilogue

Students legitimately wonder why they are taking your math class. You can tell them how your course fits into the curriculum and into mathematics. You bring to a class your own perspective on mathematics, and your own personality. Ideally the two should merge. There are so many different teaching styles, with the potential to succeed or fail gloriously.

Certainly the styles of the two authors are different. Morgan strives for sharp focus: just the right remarks and examples. Gone awry, such an approach could leave the students with too little to gain broad understanding and confidence. But if it works, the students love the basics and the way all else follows.

Garrity on the other hand teaches with an onslaught of intuitions and interconnections. Gone awry, such an approach could leave students with the inability to actually do anything. But if it works, students are caught in a web of excitement and intrigue.

Life is short; don't spend hours agonizing over possible styles. Just give a little time to thinking about your own teaching style, its strengths and weaknesses. Borrow what fits from others. Be moderate. Be happy.

