

Round Two Qualifying Test
2016 National *Who Wants to Be a Mathematician*

1. An integer between 2 and 7 inclusive is chosen at random. If a pair of fair dice is rolled, which sum (of the top numbers on the two dice) has the same probability of appearing as the randomly chosen integer? _____
7
2. How many zeros are at the end (rightmost digits) of $2015!$? _____
502
3. In a geometric series $\sum_{n=1}^{\infty} a_n$, $a_2 = 54$ and $a_5 = 2$. What is the sum of the series? _____
243
4. A piece of fruit is a perfect sphere of radius r and has a hard spherical seed at its center of radius 1. If the seed is removed, the volume of the remaining fruit is 7 times the volume of the seed. Find r .
_____ **2**
5. Which of the following mathematicians was one of the inventors of game theory?
a. John von Neumann b. Kurt Gödel c. George Pólya d. Paul Erdős (circle one)
6. How many even six-digit numbers use every one of the six digits 0,1,2,3,4,5? _____
312
7. Which of the following definitions of the binary operation $*$ on the nonzero rational numbers defines an associative operation? (“max” below denotes the maximum, if $m = n$, choose m)
a. $m * n = m - n$ b. $m * n = 2m + 4n$ c. $m * n = m^n$ **d. $m * n = \max\{m, n\}$** (circle one)
8. Let $a_0 = 10$ and for each positive integer n , let $a_n = 100a_{n-1} + (n + 10)$. For how many n , $0 \leq n \leq 100$, is it true that a_n is a multiple of 3? _____
67
9. Suppose $\sqrt{9 + 4\sqrt{5}} = a + b\sqrt{5}$ where a and b are integers. Find $a + b$. _____
3
10. How many of the elements of the set $\{11, 111, 1111, \dots, 1111111111\}$ (base 10; starting with eleven, then one hundred eleven, etc. 10 1’s in the last number) are prime? _____
1