

**Round Two Qualifying Test**  
**2016 National *Who Wants to Be a Mathematician***

1. An integer between 2 and 7 inclusive is chosen at random. If a pair of fair dice is rolled, which sum (of the top numbers on the two dice) has the same probability of appearing as the randomly chosen integer? \_\_\_\_\_  
**7**
2. How many zeros are at the end (rightmost digits) of  $2015!$ ? \_\_\_\_\_  
**502**
3. In a geometric series  $\sum_{n=1}^{\infty} a_n$ ,  $a_2 = 54$  and  $a_5 = 2$ . What is the sum of the series? \_\_\_\_\_  
**243**
4. A piece of fruit is a perfect sphere of radius  $r$  and has a hard spherical seed at its center of radius 1. If the seed is removed, the volume of the remaining fruit is 7 times the volume of the seed. Find  $r$ .  
\_\_\_\_\_ **2**
5. Which of the following mathematicians was one of the inventors of game theory?  
**a. John von Neumann**   b. Kurt Gödel   c. George Pólya   d. Paul Erdős (circle one)
6. How many even six-digit numbers use every one of the six digits 0,1,2,3,4,5? \_\_\_\_\_  
**312**
7. Which of the following definitions of the binary operation  $*$  on the nonzero rational numbers defines an associative operation? (“max” below denotes the maximum, if  $m = n$ , choose  $m$ )  
a.  $m * n = m - n$    b.  $m * n = 2m + 4n$    c.  $m * n = m^n$    **d.  $m * n = \max\{m, n\}$**  (circle one)
8. Let  $a_0 = 10$  and for each positive integer  $n$ , let  $a_n = 100a_{n-1} + (n + 10)$ . For how many  $n$ ,  $0 \leq n \leq 100$ , is it true that  $a_n$  is a multiple of 3? \_\_\_\_\_  
**67**
9. Suppose  $\sqrt{9 + 4\sqrt{5}} = a + b\sqrt{5}$  where  $a$  and  $b$  are integers. Find  $a + b$ . \_\_\_\_\_  
**3**
10. How many of the elements of the set  $\{11, 111, 1111, \dots, 1111111111\}$  (base 10; starting with eleven, then one hundred eleven, etc. 10 1’s in the last number) are prime? \_\_\_\_\_  
**1**