

**Round Two Qualifying Test**  
**2016 National *Who Wants to Be a Mathematician***

1. An integer between 2 and 7 inclusive is chosen at random. If a pair of fair dice is rolled, which sum (of the top numbers on the two dice) has the same probability of appearing as the randomly chosen integer? \_\_\_\_\_
  
2. How many zeros are at the end (rightmost digits) of  $2015!$  ? \_\_\_\_\_
  
3. In a geometric series  $\sum_{n=1}^{\infty} a_n$ ,  $a_2 = 54$  and  $a_5 = 2$ . What is the sum of the series? \_\_\_\_\_
  
4. A piece of fruit is a perfect sphere of radius  $r$  and has a hard spherical seed at its center of radius 1. If the seed is removed, the volume of the remaining fruit is 7 times the volume of the seed. Find  $r$ .  
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5. Which of the following mathematicians was one of the inventors of game theory?  
a. John von Neumann   b. Kurt Gödel   c. George Pólya   d. Paul Erdős (circle one)
  
6. How many even six-digit numbers use every one of the six digits 0,1,2,3,4,5 ? \_\_\_\_\_
  
7. Which of the following definitions of the binary operation  $*$  on the nonzero rational numbers defines an associative operation? (“max” below denotes the maximum, if  $m = n$ , choose  $m$ )  
a.  $m * n = m - n$    b.  $m * n = 2m + 4n$    c.  $m * n = m^n$    d.  $m * n = \max\{m, n\}$  (circle one)
  
8. Let  $a_0 = 10$  and for each positive integer  $n$ , let  $a_n = 100a_{n-1} + (n + 10)$ . For how many  $n$ ,  $0 \leq n \leq 100$ , is it true that  $a_n$  is a multiple of 3? \_\_\_\_\_
  
9. Suppose  $\sqrt{9 + 4\sqrt{5}} = a + b\sqrt{5}$  where  $a$  and  $b$  are integers. Find  $a + b$ . \_\_\_\_\_
  
10. How many of the elements of the set  $\{11, 111, 1111, \dots, 1111111111\}$  (all numbers are base 10; first number is eleven, then one hundred eleven, etc. 10 1’s in the last number) are prime?  
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*Thank you for participating and congratulations on qualifying for Round Two.*