

2016-17 Round Two Qualifying Test for *Who Wants to Be a Mathematician*

1. What is the sum of the roots of the polynomial $x^2 - 12x + 4$? _____
2. What is $\tan^{-1}(\cos(\sin(\cos(\pi/2))))$? _____
3. What is the last name (family name) of the famous British mathematician who worked with Ramanujan and wrote *A Mathematician's Apology*? _____
4. A machine makes parts, but 1% of the parts are defective. If an inspector selects 100 parts at random, which of the following intervals below contains the probability that none of the 100 parts are defective? (Choose one.)
a. $[0, 0.2)$ b. $[0.2, 0.4)$ c. $[0.4, 0.6)$ d. $[0.6, 0.8)$ e. $[0.8, 1.0]$
5. Find the remainder when $18^6 + 87^6$ is divided by 7. _____
6. How many divisors (factors) of $10!$ (including 1) are perfect squares? _____
7. Spheres of equal radius are stacked inside a cylinder of the same radius so that the spheres extend from the bottom of the cylinder to the top. The volume of the space inside the cylinder that is not taken up by the spheres is equal to the volume of three of the spheres. How many spheres are inside the cylinder? _____
8. Suppose that $a, b, c,$ and d are positive integers such that $a^3 = b^2, c^3 = d^2$ and $c - a = 9$. Find $a + b + c + d$. _____
9. How many distinct real solutions are there to $(x + 1)^7 + (x + 1)^6(x - 1) + (x + 1)^5(x - 1)^2 + \dots + (x + 1)(x - 1)^6 + (x - 1)^7 = 0$? _____
10. (Tie-breaking question. In the event of a tie for high score in a region, the person with an answer closest to the correct answer without exceeding the correct answer will be chosen for the semifinals in Atlanta.)
What is the smallest (minimum) value of n such that $(n + 1)!$ has four more digits than $n!$? _____