

2017-18 Who Wants to Be a Mathematician Round Two Test

1. What is the ones (units) digit of the sum $2^{2017} + 3^{2017} + 7^{2017}$? 2

2. Which of the following is largest?
 a. $\sqrt[4]{101}$ b. $5 - \sqrt{3}$ c. $\sqrt{2} + \sqrt{3}$ d. $\sqrt{10}$ b.

3. An *emirp* is a prime number that is also a **different** prime number when its digits are reversed (so for example, 107 is an emirp but 101 is not). How many emirps are there between 1 and 100? 8

4. Evaluate $\left| \frac{(1+\sqrt{3}i)^8}{(1-\sqrt{3}i)^6} \right|$ where $i = \sqrt{-1}$, and $|a + bi| = \sqrt{a^2 + b^2}$. 4

5. Given that $\sum_{k=1}^n \log_a(k^n) = n$, find a in terms of n (where $n > 1$). n!

6. A *googol* is 1 followed by 100 zeroes. A *googolplex* is 1 followed by a googol zeroes. For what exponent n is $\text{googol}^{\text{googol}} = \text{googolplex}^n$? 100

7. What is the sum of the squares of the roots (zeroes) of $x^4 - 8x^3 + 16x^2 - 11x + 5$? 32

8. From each of the three corners of a 5-12-13 right triangle remove a sector of a circle of radius 2 (centered at the corresponding corner, as shown). What is the area of the resulting figure?
30 - 2\pi

9. What is the remainder when 1000^{2018} is divided by 2018? 1090

10. (Tie-breaker—for tie scores, the person closest to the correct answer wins the tie.)
 What is the smallest value of n such that the number of primes less than or equal to n is 25,000?
287,117

Thanks for participating.