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Modern Aspects of Random Matrix Theory

AMS Short Course
Random Matrices
January 6–7, 2013
San Diego, California

Van H. Vu
Editor

American Mathematical Society

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Preface

The theory of random matrices is an amazingly rich topic in mathematics. Beside being interesting in its own right, random matrices play a fundamental role in various areas such as statistics, mathematical physics, combinatorics, theoretical computer science, number theory and numerical analysis, to mention a few. A famous example is the work of the physicist Eugene Wigner, who used the spectrum of random matrices to model energy levels of atoms, and consequently discovered the fundamental semi-circle law which describes the limiting distribution of the eigenvalues of a random hermitian matrix.

Special random matrices models where the entries are iid complex or real gaussian random variables (GUE, GOE or Wishart) have been studied in detail. However, much less was known about general models, as the above mentioned study relies very heavily on properties of the gaussian distribution. In the last ten years or so, we have witnessed considerable progresses on several fundamental problems concerning general models, such as the Circular law conjecture or Universality conjectures. More importantly, these new results are proved using novel and robust approaches which seem to be applicable to many other problems. Surprising connections to the emerging field of free probability have also been made and fortified. Equally surprising is the discovery that many practical tricks for numerical problems (to make the computation of eigenvalues faster or more reliable, say) can also be used as powerful theoretical tools to study spectral limits.

Another area where we see rapid progressions is the theory of computing and applications (which includes numerical analysis, theoretical computer science, machine learning and data analysis). Here properties of random matrices have been used for the purpose of designing and analyzing practical algorithms. As already realized by von Neumann and Goldstine at the dawn of the computer era, bounds on the condition number of large random matrices would play a central role in a vast number of numerical problems. Their questions were posed 70 years ago, but effective ways to estimate this number have only been found in recent years. As a model for random noise/error, random matrices enter all problems concerning large data, perhaps one of the most talked about subjects in applied science in recent years. Today, random matrices are studied not only for their own mathematical beauty, but also for a very real purpose of making digital images sharper or computer networks more reliable. These new goals have motivated new lines of research, such as non-asymptotic or large deviation theory for random matrices.

This volume contains surveys by leading researchers in the field, written in introductory style to quickly provide a broad picture about this fascinating and rapidly developing topic. We aim to touch most of the key points mentioned above (and many more) without putting too much technical burden on the readers. Most of the surveys are accessible with basic knowledge in probability and linear algebra.

We also made an attempt to discuss a considerable amount of open problems. Some of these are classical but many are new, motivated by current developments. These problems may serve as a guideline for future research, especially for young researchers who would like to study this wonderful subject.

Van H. Vu

New Haven, Fall 2013.

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