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Hyperbolic Dynamics, Fluctuations and Large Deviations

Special Semester
Hyperbolic Dynamics, Large Deviations and Fluctuations
January – June 2013
Centre Interfacultaire Bernoulli,
École Polytechnique Fédérale de Lausanne

D. Dolgopyat
Y. Pesin
M. Pollicott
L. Stoyanov
Editors

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Preface

The articles presented in this volume are based on lectures and talks given during the semester-long (January–June, 2013) Special Program on *Hyperbolic Dynamics, Large Deviations and Fluctuations*, which took place at the Centre Interfacultaire Bernoulli, École Polytechnique Fédérale de Lausanne, Switzerland.

The broad theme of the Program was the long term behavior of dynamical systems and their statistical behavior. During the last half century, the statistical properties of dynamical systems of many different types have been the focus of much research interest. Moreover, the results of this study have had a profound effect on many areas across mathematics (including geometry, analysis, and number theory), physics (including both equilibrium and non-equilibrium statistical physics), engineering and biology. The program concentrated on four main research topics.

- large deviations and fluctuations for systems with some hyperbolicity
- limit theorems in hyperbolic dynamics
- fluctuations for systems with zero entropy
- statistical properties of coupled hyperbolic systems

This volume contains all six lectures of the short courses given during the two schools and expanded versions of a number of selected talks given at the two workshops held during the Program. The material presented is primarily directed at researchers and graduate students in the very broad area of dynamical systems and ergodic theory, but will also be of interest to researchers in related areas such as statistical physics, spectral theory and some aspects of number theory and geometry. It may also be profitably read by people who have a keen interest in the applications of statistical methods to other areas of scientific research, and have some background in the physical and mathematical sciences.

The main activities during the Program concentrated on two general topics:

A. *Large deviations and thermodynamical formalism.* Three Mini-courses related to this topic were delivered during the period 11 - 15 March 2013: (i) Y. Kifer (Hebrew University of Jerusalem), *Large deviations in probability and dynamical systems*; (ii) J.Buzzi (Université Paris Sud), *Entropy and measurable classifications of dynamical systems*; (iii) O. Sarig (Weizmann Institute), *Thermodynamic formalism for countable Markov shifts.* The Mini-courses were followed by a Workshop held during the period 18 - 22 March 2013, organised by Jérôme Buzzi (Université Paris Sud), Stefano Luzzatto (International Centre for Theoretical Physics), Yakov Pesin (Pennsylvania State University) and Omri Sarig (The Weizmann Institute for Science).

B. *Limit theorems for dynamical systems.* The following Mini-courses related to this topic were delivered during the period 27 - 31 May 2013: (i) J. Marklof (University of Bristol), *Kinetic limits of dynamical systems*; (ii) G. Forni (University of Maryland), *Limit theorems for classical horocycle flows*; (iii) S. Gouëzel (Universite de Rennes), *Limit theorems in dynamical systems using the spectral method.* These Mini-courses were followed by a Workshop held during the period 3 - 7 June 2013, organised by N. Chernov (University of Alabama at Birmingham), D Kleinbock (Brandeis University), M. Pollicott (University of Warwick) and R. Sharp (University of Warwick).

Another important part of the Program were the *Bernoulli Lectures* delivered by prominent mathematicians and theoretical physicists. During the Program there were three such lectures:

- Yakov Pesin (Pennsylvania State University) *Appearance and genericity of chaos* (7 March 2013)
- Giovanni Gallavotti (Rutgers University) *Hyperbolic systems and fluctuation theorems* (16 May 2013)
- David Ruelle (IHES) *Nonequilibrium statistical mechanics: From heat transport to hydrodynamic turbulence* (30 May 2013).

The Program brought together mathematicians and scientists with a wide variety of different backgrounds and interests, and at different stages of their careers. The schools and workshops proved to be both enlightening and stimulating and we hope that the lecture notes in this volume will be equally beneficial to anyone who wants to understand more about recent developments in aspects of the statistical properties of dynamical systems.

Dmitry Dolgopyat
Yakov Pesin
Mark Pollicott
Luchezar Stoyanov

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