Proceedings of Symposia in
PURE MATHEMATICS

Volume 92

Lie Algebras, Lie Superalgebras, Vertex Algebras and Related Topics

2012–2014 Southeastern Lie Theory Workshop Series

Categorification of Quantum Groups and Representation Theory
April 21–22, 2012, North Carolina State University

Lie Algebras, Vertex Algebras, Integrable Systems and Applications
December 16–18, 2012, College of Charleston

Noncommutative Algebraic Geometry and Representation Theory
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2010 *Mathematics Subject Classification.* Primary 17B37, 17B55, 17B56, 17B65, 17B67, 17B69, 20C08, 20C11, 20G05, 20G42.

Library of Congress Cataloging-in-Publication Data

Names: Misra, Kailash C., 1954- editor. — Nakano, Daniel K. (Daniel Ken), 1964- editor. — Parshall, Brian, 1945- editor. Title: Lie algebras, lie superalgebras, vertex algebras, and related topics : Southeastern Lie Theory Workshop Series 2012-2014 : Categorification of Quantum Groups and Representation Theory, April 21-22, 2012, North Carolina State University : Lie Algebras, Vertex Algebras, Integrable Systems and Applications, December 16-18, 2012, College of Charleston : Noncommutative Algebraic Geometry and Representation Theory, May 10-12, 2013, Louisiana State University : Representation Theory of Lie Algebras and Lie Superalgebras, May 16-17, 2014, University of Georgia / Kailash C. Misra, Daniel K. Nakano, Brian J. Parshall, editors. Description: Providence, Rhode Island : American Mathematical Society, [2016] — Series: Proceedings of symposia in pure mathematics ; volume 92 — Includes bibliographical references. Identifiers: LCCN 2015043322 — ISBN 9781470418441 (alk. paper) Subjects: LCSH: Lie algebras–Congresses. — Lie superalgebras–Congresses. — Vertex operator algebras–Congresses. — AMS: Nonassociative rings and algebras – Lie algebras and Lie superalgebras – Quantum groups (quantized enveloping algebras) and related deformations. msc — Nonassociative rings and algebras – Lie algebras and Lie superalgebras – Homological methods in Lie (super)algebras. msc — Nonassociative rings and algebras – Lie algebras and Lie superalgebras – Cohomology of Lie (super)algebras. msc — Nonassociative rings and algebras – Lie algebras and Lie superalgebras – Infinite-dimensional Lie (super)algebras. msc — Nonassociative rings and algebras – Lie algebras and Lie superalgebras – Kac-Moody (super)algebras; extended affine Lie algebras; toroidal Lie algebras. msc — Nonassociative rings and algebras – Lie algebras and Lie superalgebras – Vertex operators; vertex operator algebras and related structures. msc — Group theory and generalizations – Representation theory of groups – Hecke algebras and their representations. msc — Group theory and generalizations – Representation theory of groups – p-adic representations of finite groups. msc — Group theory and generalizations – Linear algebraic groups and related topics – Representation theory. msc — Group theory and generalizations – Linear algebraic groups and related topics – Quantum groups (quantized function algebras) and their representations. msc Classification: LCC QA252.3 .L5534 2016 — DDC 512/.482–dc23 LC record available at http://lccn.loc.gov/2015043322

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Preface

Algebraic, analytic and geometric representations of Lie algebras, quantum groups and related algebraic structures have become a comprehensive and mainstream research area in mathematics with numerous applications in mathematics and theoretical physics. Research in representation theory includes quantized enveloping algebras and their connections with representations of algebraic and finite groups, quantum function algebras, Kac-Moody Lie algebras, Hecke algebras, vertex (operator) algebras, Hall algebras, A-infinity algebras, quivers, cluster algebras, Hopf algebras, and Khovanov-Lauda-Rouquier algebras. In particular, representation theory of quantized Kac-Moody Lie algebras and their categorical and geometric constructions have taken the lead not only within Lie theory but also in other areas of mathematics and physics such as combinatorics, group theory, number theory, integrable systems, partial differential equations, topology and conformal field theory.

The notion of categorification has been prevalent in the works of I. Frenkel, Ariki, Grojnowski, Brundan and Kleshchev since Chuang and Rouquier systematically developed the theory and showed that it could be a powerful tool in proving significant results. The ideas and methods soon brought about the discovery of Khovanov-Lauda-Rouquier algebras to categorify quantum groups and their representations. Moreover, this categorical approach was used in proving the Kazhdan-Lusztig positivity conjecture using Soergel bimodules. It is anticipated that these methods will lead to substantial progress in the field. The workshop at North Carolina State University in April 2012 brought together specialists in these areas to explore these topics in more depth.

In the late 1970s, Lepowsky-Wilson and I. Frenkel-Kac gave constructions of representations of affine Kac-Moody Lie algebras in terms of certain differential operators on the space of polynomials in infinitely many variables, that were recognized as the so-called vertex operators appearing in string theory. These constructions spurred the development of the field in various directions. One such application is related to combinatorial identities, including a representation-theoretic proof of the Rogers-Ramanujan type identities. Another is a beautiful connection to integrable systems such as the KP hierarchy, developed by the Kyoto school. The mysterious connections to the largest sporadic simple finite group (known as the "Monster") and to modular forms led R. Borcherds to introduce the notion of a vertex algebra. It turns out that this mathematical notion is essentially equivalent to the physical notion of a chiral algebra in conformal field theory. At the same time, vertex algebras provide a natural framework for the representation theory of infinite-dimensional Lie algebras and generalizations. The q-analog of the Frenkel-Kac construction given by Frenkel and Jing motivated the consideration of "quantum" vertex algebras, which are expected to play the same role with respect to quantum affine algebras. The workshop at the College of Charleston in December 2012 focused on these topics.

In the past few decades, the interactions between noncommutative geometry and representation theory have led to new and interesting developments in these areas. In his celebrated ICM talk Drinfeld laid the foundation for the interplay between the geometry of Poisson structure on Lie groups and their homogeneous spaces, and the representation theory of the related quantum function algebras. Representation theory also plays a fundamental role in related approaches to noncommutative algebraic geometry via Calabi–Yau algebras, superpotential algebras and quiver algebras. Calabi–Yau algebras and categories are nowadays an important ingredient in the categorification theory of cluster algebras. These subjects also have connections to geometric representation theory, whose main theme is the use of (derived) categories of sheaves on various varieties to describe categories of representations. An early success was the use of perverse sheaves in the proof of the Kazhdan–Lusztig conjectures. Since then, perverse sheaves have led to important results in the representation theory of finite reductive groups, Hecke algebras, affine Lie algebras, and quantum groups. More recently, derived categories of coherent sheaves on varieties such as the nilpotent cone and the Springer resolution have begun to play a role, especially in the context of the geometric Langlands program, and this has led to connections between noncommutative geometry and algebra. The workshop at Louisiana State University in May 2013 focused on research in this direction.

The workshop at the University of Georgia in May 2014 featured new developments in the representation theory of Lie superalgebras and their connections with geometry and Lie theory. In recent work, Boe, Kujawa and Nakano have utilized methods from tensor triangular geometry to reveal the Balmer spectrum for classical Lie superalgebras. These underlying spaces are prime candidates for applying methods in derived algebraic geometry. Progress in this direction, along with open conjectures, was presented by Kujawa in a series of lectures. Another major development has been the formulation of character formulas for the Lie superalgebra in type A by Brundan and the recent verification by Cheng, Lam and Wang via canonical basis and super duality. In another series of lectures, Wang outlined the general program and presented new work on this question for other classical Lie superalgebras.

Six years ago the three editors established a consortium called the "Southeastern Lie Theory Network" to enhance regional research collaboration and provide a stronger educational environment for graduate students and junior researchers. The editors also initiated an annual (three to four day) workshop series in Lie theory. The aim of these workshops was to bring together senior and junior researchers as well as graduate students to build and foster cohesive research groups in the region. Each workshop included expository talks by senior researchers and discussion sessions for junior researchers and graduate students with the purpose of communicating new ideas and results in the field. These workshops have been supported by the National Science Foundation and the host universities in the region. Since the inception of the consortium, eight workshops have been held. Each of these workshops have been well attended. The Proceedings of the first three workshops were published in this book series (volume 86) in 2012. The plenary speakers in the following four workshops during 2012–2014 were invited to contribute to this proceedings. Most of the articles presented in this book are self-contained. The survey articles, by Martin Schlichenmaier and Oleg K. Sheinman are accessible to a wide audience of readers.

The editors would like to take this opportunity to thank the conference participants, the contributors, and the editorial offices of the American Mathematical Society for making this volume possible.

Kailash C. Misra
Daniel K. Nakano
Brian J. Parshall

North Carolina State University, April 2012

College of Charleston, December 2012

Louisiana State University, May 2013

University of Georgia, May 2014

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ISBN 978-1-4704-1844-1

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