Errata for "Heat Kernel and Analysis on Manifolds"
October 26, 2017

P. 6. Before Remark 1.2 there should be twice “\( t \to 0 \)” instead of “\( t \to \infty \)”.

P. 32. In the top line there should be “After we have proved (2.25)” instead of “After we have proved using (2.25)”.

P. 66. There should be \( v^i = g^{ij}u_j \) instead of \( v^i = g^{ij}u_i \).

P. 167. There should be “To prove (6.47) observe that by (6.45)…” instead of “To prove (6.47) observe that by (6.40)…”

P. 297. There should be \( v^i = g^{ij}u_j \) instead of \( v^i = g^{ij}u_i \).

P. 298. In Remark after Exercise 10.26, there should be: “for any \( \min(\mathcal{M}) \)” instead of “\( \min(\mathcal{M}) \)”.

P. 298. In Exercise 10.27(b) there should be \( u = 1 \mod W_0^1(\mathcal{M}) \)” instead of \( u = 1 \mod W_0^1(\mathcal{M}) \)”.

P. 398. Proof of Corollary 15.6 should be corrected as follows.

Proof. Assume first \( t \leq 2R^2 \). The function \( u(t, \cdot) = P_t f \) satisfies the hypotheses of Theorem 15.1. Since \( \|u(t, \cdot)\|_{L^2} \leq \|f\|_{L^2} \), we obtain

\[
\int_0^t \int_{B(x, R)} u^2_+ d\nu \leq t\|f\|_{L^2}^2,
\]

whence by Theorem 15.1

\[
u^2_+(t, x) \leq \frac{Ca^{-n/2}t}{\min(\sqrt{t}, R)} \|f\|_{L^2}^2 \leq C'a^{-n/2} \left(R^{-n} + t^{-n/2}\right) \|f\|_{L^2}^2.
\]

Applying the same argument to \( u = -P_t f \), we obtain the required estimate for \( |P_t f(x)|^2 \). Next, replacing \( x \) by any point \( x' \in B(x, R/2) \) and applying the above estimate in the ball \( B(x', R/2) \) instead of \( B(x, R) \), we obtain (15.22).

For \( t > 2R^2 \) we apply the already proved case of (15.22) with \( R^2 \) instead of \( t \) and \( P_{t-R^2} f \) instead of \( f \). Since

\[
P_{R^2} P_{t-R^2} f = P_t f,
\]

we obtain

\[
\sup_{B(x, R/2)} |P_t f| \leq C'a^{-n/4} \left(R^{-n/2} + (t - R^2)^{-n/4}\right) \|P_{t-R^2} f\|_{L^2}
\leq C''a^{-n/2} R^{-n/2} \|f\|_{L^2},
\]

which finishes the proof. \( \blacksquare \)

P. 407. Before Corollary 15.17 there should be

\[
p_t(x, y) \sim \frac{c}{t^{n/2}} \left(\frac{\rho^2}{t}\right)^{-\frac{n-1}{2}} \exp \left(-\frac{\rho^2}{4t}\right)
\]

instead of

\[
p_t(x, y) \sim \frac{c}{t^{n/2}} \left(\frac{\rho^2}{t}\right)^{n/2-1} \exp \left(-\frac{\rho^2}{4t}\right).
\]