*A First Course in Sobolev Spaces*, First edition
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For the original text I use the color Red , for corrections the color Green, and for improvements and additions the color Blue. Names in brackets refer to the persons who called the error to my attention (to the best of my recollection) or suggested improvements and additions.[1]

## CHAPTER 1:

p. 5 In Exercise 1.5 one should assume that $u$ is bounded from below, since otherwise the series could diverge to $\infty$. Alternatively, one should choose a point $x_0 \in I^\circ$ at which $u$ is continuous and then take

$$u_{\mathrm{J}}(x) := \begin{cases} +\sum_{y\in I,\, x_0\le y<x} (u_+(y) - u_-(y)) + u(x) - u_-(x) & \text{if } x \ge x_0, \\ -\sum_{y\in I,\, x<y\le x_0} (u_+(y) - u_-(y)) + u(x) - u_+(x), & \text{if } x \le x_0. \end{cases} \quad (\boxplus)$$

p. 32 In Exercise 1.44 Consider the Banach space

$$X := \{u : [0,1] \to \mathbb{R} : u \text{ is continuous, } u(0) = 0, \text{ and } u(1) = 1\},$$

where we take the supremum norm

$$\|u\|_\infty := \max_{x\in[0,1]} |u(x)|.$$

should be replaced by Consider the complete metric space

$$X := \{u : [0,1] \to \mathbb{R} : u \text{ is continuous, } u(0) = 0, \text{ and } u(1) = 1\},$$

where we take the metric induced by supremum norm

$$\|u\|_\infty := \max_{x\in[0,1]} |u(x)|.$$

[Rinat Kashaev]

## CHAPTER 3:

p. 109 In Corollary 3.74 one should either assume that $u$ is bounded from below or replace (3.33) with ($\boxplus$) (above in this file).

## CHAPTER 10:

p. 305 In Exercise 10.51, $p$ should be $\infty$ . Also the fact that $W^{1,\infty}(\Omega)$ is a dual space is not obvious. See the file "Lipschitz functions as a dual space". [M.G. Mora]

[1] The style of this file is inspired by http://www.hss.caltech.edu/ kcb/IDA-Errata.pdf

## CHAPTER 11:

p. 305 In line 8, by (1.15) (with $u$ replaced by $u_n$) should be replaced by reasoning somewhat as in (1.15). [M.G. Mora]

p. 316 In Step 4

$$v_n(x) := \begin{cases} |u(x)| - \frac{1}{n} & \text{if } \frac{1}{n} \le |u(x)| \le n, \\ 0 & \text{if } |u(x)| < \frac{1}{n}, \\ n - \frac{1}{n} & \text{if } |u(x)| > \frac{1}{n}. \end{cases}$$

should be replaced by

$$v_n(x) := \begin{cases} |u(x)| - \frac{1}{n} & \text{if } \frac{1}{n} \le |u(x)| \le n, \\ 0 & \text{if } |u(x)| < \frac{1}{n}, \\ n - \frac{1}{n} & \text{if } |u(x)| > n. \end{cases}$$

[Robert Simione]

## CHAPTER 12:

p. 353 In Exercises 12.6 and 12.7, $d_{\text{reg}}$ be its regularized distance should be replaced by $d_{\text{reg}}$ be the regularized distance corresponding to $\mathbb{R}^N \setminus \overline{\Omega}$.

p. 354 In Line 2, $td_{\text{reg}}(x)$ should be replaced by $Ctd_{\text{reg}}(x)$, where $C$ is the constant given in Exercises 12.6. See also the file "Extension domains for higher order Sobolev spaces" for a complete proof.

p. 356 In formula (12.7), $\frac{1}{\varepsilon}$ should be replaced by $\frac{M}{\varepsilon}$ .

p. 359 In lines 2-6 and in Remark 12.16, $\frac{1}{\varepsilon}$ should be replaced by $\frac{M}{\varepsilon}$ .

p. 370 In Remark 12.35, $\frac{1}{4}$ should be replaced by $\frac{1}{2}$ . [Marco Barchiesi]

## CHAPTER 13:

p. 402 Thanks to Paolo Piovano for pointing out that the proof of Theorem 13.30 is too fast. It should be divided in two steps. Assuming first that $u \in L^{1^*}(\mathbb{R}^N)$, by mollification it follows that

$$\left( \int_{\mathbb{R}^N} |u(x)|^{1^*} \, dx \right)^{\frac{1}{1^*}} \le C \, |Du| \left( \mathbb{R}^N \right).$$

To remove the additional hypothesis that $u \in L^{1^*}(\mathbb{R}^N)$, one should truncate $u$ as in Step 4 of the proof of Theorem 11.2. The problem here is the fact the mollification of a function vanishing at infinity needs not vanish at infinity. See also the file "An extension of the Sobolev–Gagliardo–Nirenberg theorem".

**CHAPTER 14:**

p. 422 In Exercise 14.13 Prove that if $g$ is decreasing, then there exists

$$\lim_{s\to 0^+} \|g\|_{s,\infty} = \lim_{h\to\infty} g(h).$$

What happens if we remove the hypothesis that $g$ is decreasing?should be replaced by Prove that if $g$ is increasing, then there exists

$$\lim_{s\to 0^+} \|g\|_{s,\infty} = \lim_{h\to\infty} g(h).$$

What happens if we remove the hypothesis that $g$ is increasing?[Xiang Xu]

p. 422 In Exercise 14.13 Prove that if $\frac{g(h)}{h}$ is increasing, then there exists

$$\lim_{s\to 1^-} \|g\|_{s,\infty} = \lim_{h\to 0^+} \frac{g(h)}{h}.$$

What happens if we remove the hypothesis that $\frac{g(h)}{h}$ is increasing?should be replaced by Prove that if $\frac{g(h)}{h}$ is decreasing, then there exists

$$\lim_{s\to 1^-} \|g\|_{s,\infty} = \lim_{h\to 0^+} \frac{g(h)}{h}.$$

What happens if we remove the hypothesis that $\frac{g(h)}{h}$ is decreasing?[Xiang Xu]

**More to come for sure... :(**