Errata for *Sets, Groups, and Mappings* (AMSTEXT 39)
March 19, 2020

Chapter 6: Binary Operations

Exercise 6.22, p. 105: Incorrect as stated. Corrected statements are:

(a) Show there are 16 Cayley tables on A. (One approach is to list them all, but that isn't necessary.)

(b) When the elements' names are swapped (i.e., under the bijection $\phi(\circ) = \bullet$ and $\phi(\bullet) = \circ$ in Definition 6.12), each table transforms into an isomorphic table. For instance,

$$\begin{array}{c|cc} & \circ & \bullet \\ \hline \circ & \circ & \circ \\ \bullet & \circ & \circ \end{array} \qquad \longrightarrow \qquad \begin{array}{c|cc} & \bullet & \circ \\ \hline \bullet & \bullet & \bullet \\ \circ & \bullet & \bullet \end{array} \quad = \quad \begin{array}{c|cc} & \circ & \bullet \\ \hline \circ & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{array}$$

Pair up the Cayley tables under isomorphism. (Four of the tables are isomorphic to themselves. The remaining twelve are isomorphic in pairs. Each isomorphism class contains at least one operation having two or fewer products equal to $\bullet$. The remaining parts refer to these representations.)

(c) How many of the ten binary operations are commutative? How many have an identity element? Of these, for how many does each element have an inverse?

(d) Show that precisely five of the ten binary operations on A are associative. That is, for each table, either prove the operation is associative, or exhibit an explicit threefold product whose value depends on grouping. Light's test (Exercise 6.20) is not necessary, but gives a fallback strategy if you can't find a conceptual proof for a specific operation.