A Bridge to Advanced Mathematics

From Natural to Complex Numbers

– Corrections and Improvements –

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Typos/Corrections

This section contains a list of typos from our book.

- On **page 10**, in Exercise 1.1.12, the conclusion should be

  .. there must be at least two among them whose sum is $2n+1$

  instead of

  .. there must be at least two among them whose sum is $2n$.
- The formula stated in Exercise 1.3.9 on **page 34** is not correct. It should be
$$1 + 2x + \cdots + n\,x^{n-1} = \frac{nx^{n+1} - (n+1)x^n + 1}{(1-x)^2}.$$
- In Exercise 1.5.1 on **page 51** the first four expressions do not make sense. Recall that in an expansion with respect to a base $b \geq 2$ occur only digits belonging to $\{0, \ldots b-1\}$. Those who want to practice with correct expressions may replace the first four numbers by 11_2, 102_3, 42_5 and 343_5, respectively.
- On **page 77** in Exercise 1.7.6 the formula to prove is not correct. It should be
$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} = \binom{n}{0} - \binom{n}{1} + \cdots + (-1)^n \binom{n}{n} = 0\,.$$
  For example, if $n = 1$ the formula stated in the book leads to
$$\binom{1}{0} + (-1)^{1-1}\binom{1}{1} = 1 + 1 = 2 \neq 0\,.$$
- On **page 129** in Exercise 2.5.10 we refer to Exercise 2.2.10. It should be Exercise 2.3.8.
- On **page 155**, Exercise 2.7.9 should ask for constructing Costas arrays of order 10 (not 11 as stated, but using the integers mod 11) and 12 (not 13 as stated, but using the integers mod 13).
- On **page 172**, in Example 3.2.3, the equation should be
$$(x^2 - 2)^2 - 12x^2 = 0$$
  instead of the equation
$$(x^2 + 2)^2 - 12x^2 = 0.$$
- On **page 172**, there is a mistake $\cos(30°)$ does not equal $1/2$, but is $\sqrt{3}/2$. The proof showing that $\cos(10°)$ should be corrected to something like the following:

  Assume that $\cos(10°)$ is rational. Therefore, $\cos(30°) = 4\cos^3(10°) - 3\cos(10°)$ must be rational. However, $\cos(30°) = \sqrt{3}/2$ which is an irrational number. This contradiction implies that $\cos(10°)$ is irrational.
- On **page 181** the estimate after (3.3.3) is incorrect. It should be
$$\frac{-1}{b^n} \leq \alpha - 0.a_1 \cdots a_{m-1}(a_m + 1) < 0\,.$$
  This cannot be true for all $n \geq 1$. Note that $1/b^n$ becomes arbitrarily small as n tends to infinity.

- On **page 220**, in Exercise 3.5.2, the conclusion should be $f = \alpha b + \beta d$ instead of $f = \alpha c + \beta d$.
- On **page 220**, in Exercise 3.5.8, the pentagon should be *convex*.
- On **page 233**, in Exercise 3.7.5, the statement should end with .. *with at most two terms* instead of ... *with two terms*.
- On **page 235**, in Exercise 3.8.19, the part

  If a and b are natural numbers such that $\frac{3}{p} = \frac{1}{a} + \frac{1}{b}$

  should be

  If a and b are distinct natural numbers such that $\frac{3}{p} = \frac{1}{a} + \frac{1}{b}$
- On **page 243**, in Exercise 4.1.9, the last inequalities should be reversed, the exercise should state that $r_{2n} < \sqrt{2} < r_{2n+1}$ instead of the current version $r_{2n+1} < \sqrt{2} < r_{2n}$.
- On **page 243**, in Exercise 4.1.10, the last inequalities should be reversed, the exercise should state that $s_{2n} < \sqrt{5} < s_{2n+1}$ instead of the current version $s_{2n+1} < \sqrt{5} < s_{2n}$.
- The formula in Exercise 4.2.10 on **page 246** contains an error. The correct version is

  $$(x_1 - x_2)(x_4 - x_3) + (x_1 - x_3)(x_2 - x_4) + (x_1 - x_4)(x_3 - x_2) = 0.$$
- On **page 271**, the equation 4.6.6. (ii) should be

  $$3^{x-2} - 3^{\frac{2x-1}{3}} + 3^{\frac{x+1}{3}} = 1.$$
- On **page 271**, the inequality in Exercise 4.6.10 (ii) should be

  $$x^x \cdot y^y \geq x^y \cdot y^x$$

  instead of $x^x \cdot y^y > x^y \cdot y^x$.
- On **page 289**, Exercise 4.8.7 has a typo and it should state that

  $$\sqrt[3]{4} - \sqrt[3]{2} = -1 + \sqrt[3]{9\sqrt[3]{2} - 9}$$

  or

  $$\sqrt[3]{4} - \sqrt[3]{2} + 1 = \sqrt[3]{9\sqrt[3]{2} - 9}.$$

  instead of

  $$\sqrt[3]{4} - \sqrt[3]{2} = 1 + \sqrt[3]{9\sqrt[3]{2} - 9}$$
- In Exercises 4.8.17 and 4.8.18 on **page 290**, the inequality signs go in the wrong direction. The correct inequalities are

  $$x_1y_1 + x_2y_2 + x_3y_3 \geq x_1y_2 + x_2y_3 + x_3y_1 \geq x_1y_3 + x_2y_2 + x_3y_1$$

  and

  $$x_1y_1 + \ldots + x_ny_n \geq x_1y_{\sigma(1)} + \ldots + x_ny_{\sigma(n)} \geq x_1y_n + \ldots + x_ny_1, ,$$

  respectively.
- On **page 297** the second sentence of the last paragraph *We can take as many often the digit* 2 *as we wish,* should be replaced by *We can repeat the digit* 2 *as many times as we wish,*.

- On **page 302**, in Proposition 5.2.4, the equation $\sup_{n\geq x_n} = \infty$ should be replaced by $\sup_{n\geq 1} x_n = \infty$.
- On **page 303**, in Example 5.2.3, change

  Hence, let us choose N as some fixed number satisfying $N > K/4$.

  to

  Hence, let us choose N as some fixed number satisfying $N > 4K$.
- On **page 303**, in Example 5.2.3, change

  Then $n \geq N$ implies $4n > K$ ...

  to

  Then $n \geq N$ implies $n > 4K$ ...
- On **page 311**, in the first sentence above Corollary 5.3.2, it should $\sup_{n\geq 1} x_n = \infty$ instead of $\sup_{n\geq 1} = \infty$.
- On **page 363** in Exercise 5.8.9 an assumption about the sequence $(x_n)_{n\geq 1}$ is missing. One has to suppose that the limit of this sequence is different of zero. Otherwise the quotient x_{n+k}/x_n needs not converge to 1 as, for example, can be seen by taking $x_n = q^n$ for a certain $0 < q < 1$.
- On **page 367**, in the first footnote, it should be written *Mirzakhani* instead of *Mirzhakhani*.
- On **page 374** it is more natural to solve the equations in Exercise 6.1.2 not in $\mathbb{R}$, but in $\mathbb{C}$. So one should better ask for complex solutions x and y of the four equations.
- On **page 400**, Exercise 5.6.9 should be

  Let A, B, and C be three points in the Cartesian plane such that the triangle ABC is equilateral. Prove that $z_A = z_B + \gamma(z_C - z_B)$ or $z_A = z_B + \overline{\gamma}(z_C - z_B)$, where $\gamma = e^{\pi \mathrm{i}/3}$.
- On **page 400**, Exercises 5.6.10 should be

  Let A, B, and C be three points in the Cartesian plane. Show that the triangle ABC is equilateral if and only if

  $$z_A + \omega z_B + \omega^2 z_C = 0 \text{ or } z_A + \omega z_C + \omega^2 z_B = 0.$$

  where $\omega = e^{\frac{2\pi \mathrm{i}}{3}} = \cos\left(\frac{2\pi}{3}\right) + \mathrm{i}\sin\left(\frac{2\pi}{3}\right)$ is a third root of unity.
- On **page 416** in Exercise 6.8.6 the expression for the cosine should be

  $$\cos(n\alpha) = \frac{z^{2n}+1}{2z^n} \quad \text{instead of} \quad \cos(n\alpha) = \frac{z^{2n}+1}{z^n}.$$
- On **page 416** in Exercise 6.8.9 some absolute values are missing. The correct formula to prove is

  $$(n-2)\sum_{j=1}^{n} |z_j|^2 + \left|\sum_{j=1}^{n} z_j\right|^2 = \sum_{1\leq k<\ell\leq n} |z_k + z_\ell|^2.$$
- In Exercise A.3.5 on **page 453** the second formula should be

  $$f(f^{-1}(Y)) = Y \cap f(A) \quad \text{not} \quad f(f^{-1}(Y)) = Y \cap f(X).$$

- On **page 512** problem (2) in Exercise A.8.15 is badly formulated. The letter "n" occurs twice in different meaning. Correctly it should be: Check whether ϕ from $(\mathbb{Z},+)$ to $(\mathbb{Z}_n,+)$ with

  $$\phi(m) = r \quad \text{if} \quad m \equiv r \pmod{n}$$

  is a homomorphism.