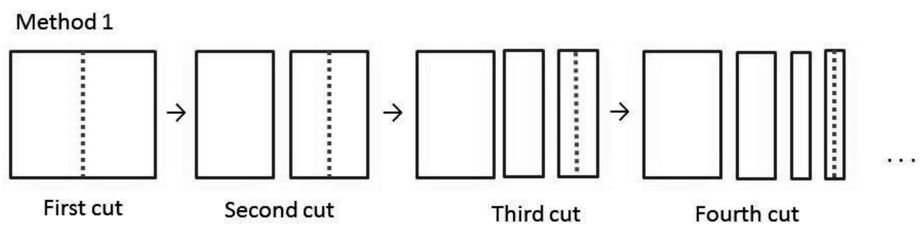
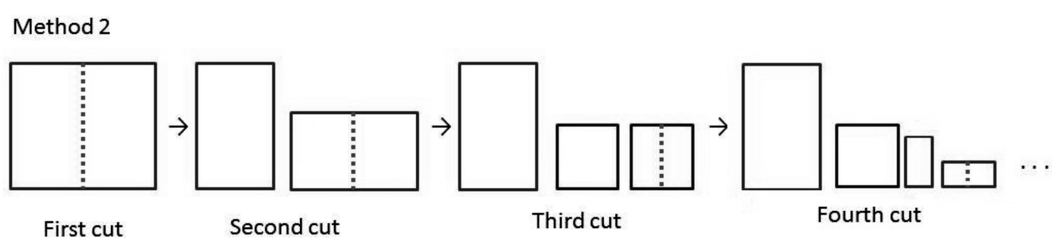
Fun with Paper – Class Handout

1. Begin with two square pieces of paper. (To make a square piece of paper from a rectangular one, fold the $8.5''$ side diagonally down to the $11''$ side, and cut off the part that overlaps.)

2. We are going to cut each square of paper in half repeatedly, using two different methods. The first is to cut the paper longways each time. The second is to rotate the paper 90° between each cut, as shown below.





   We will continue each process indefinitely; that is, make infinitely many cuts. (You, of course, will only be able to make a few cuts with your scissors, since eventually the pieces of paper would be too small.)

3. Answer the following questions about each method, *assuming that you have continued the process ad infinitum*. Note that the answers are not all identical for the two methods!

   (a) How many cuts were made? Method 1: ∞ Method 2: ∞

   (b) How many pieces of paper will there be? Method 1: _______ Method 2: _______

   (c) Assume the original size of the square paper is 1 nice unit by 1 nice unit. How far do the scissors cut in each case? (Another way to ask this question is: How far would the paper extend if you line the pieces up, with all the cut sides along the edge of your table?)

   Method 1: _______ Method 2: _______

   (d) If we don't count time between cuts, how long will the scissors be cutting?

   Method 1: _______ Method 2: _______

   (e) What is the total surface area of all the cut pieces? (Remember that we are assuming "nice units".)

   Method 1: _______ Method 2: _______

   (f) What is the surface area of the nth piece? Method 1: _______ Method 2: _______

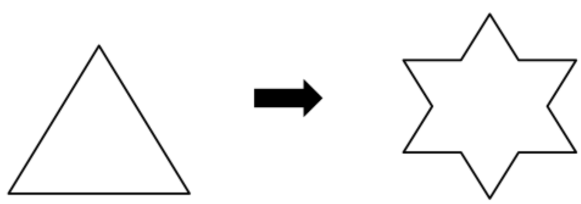
Fun with Fractals – Class Handout

How to make a Koch snowflake:

Step 1. Begin with a drawing of an equilateral triangle.

Step 2. Wherever you see a straight line, draw an equilateral triangle on the middle third of the line segment and erase its base:

The first step will look like:



Step 3. Repeat Step 2 infinitely many times.

Please draw only an approximation on your paper–stop after enough repetitions to give you an idea of the final outcome. You do not have to construct a formal trisection of your segments, or an exact equilateral triangle, but draw carefully.

Complete the following problems.

1. If the original triangle has side length 1 unit, and the area is A square units, fill in the following chart:

Step (n)	Length of one side (s_n)	Number of sides	Perimeter (p_n)	Area of one new triangle (a_n)	Number of new triangles	Area of snowflake (A_n)
0	1	3	3	n/a	0	A
1	$\frac{1}{3}$	12	4	$\frac{1}{9}A$	3	$A + \frac{1}{3}A$
n						

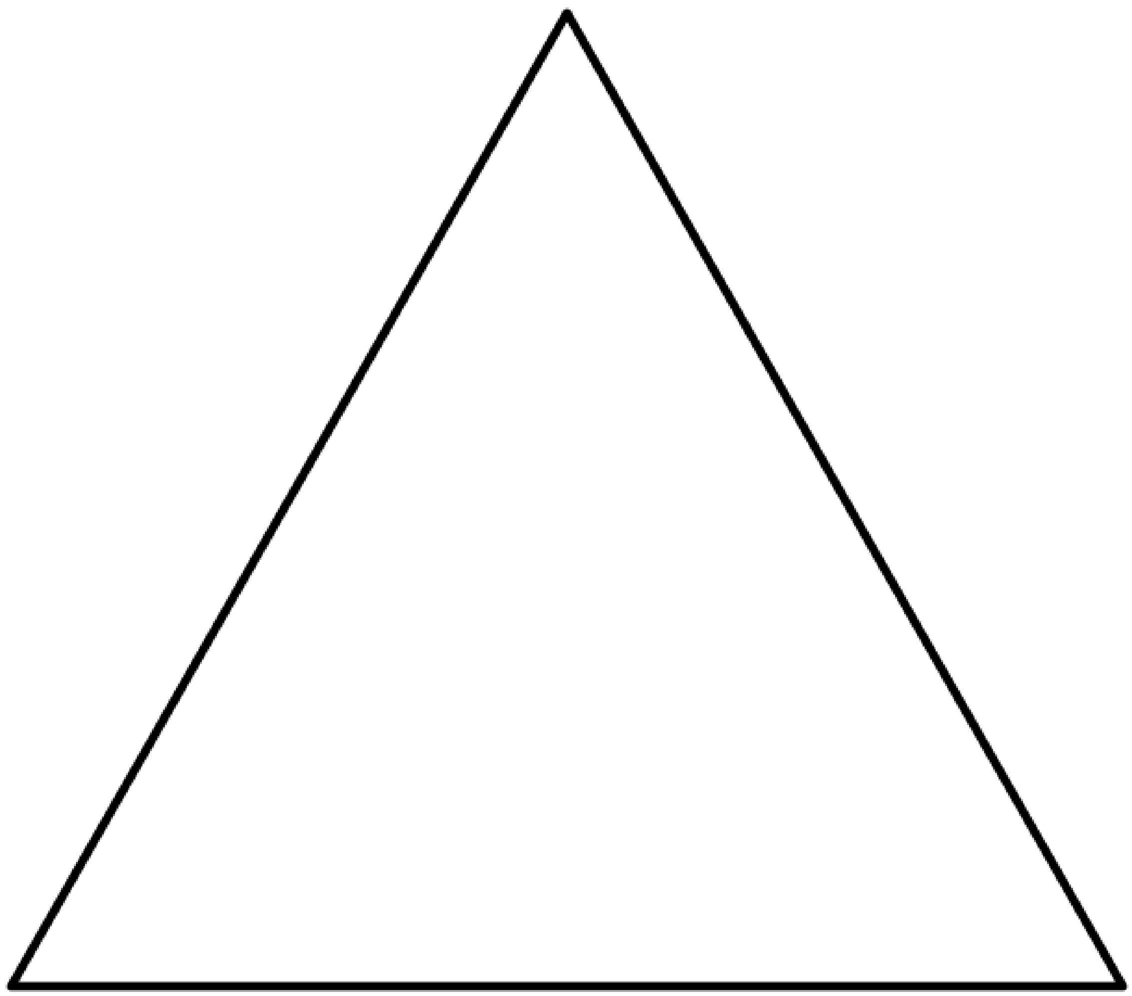
2. What happens to s_n, the length of each side, as n (the number of iterations) goes to infinity?

3. What happens to p_n, the perimeter, as n goes to infinity?

4. What happens to a_n, the area of one added triangle, as n goes to infinity?

5. What happens to A_n, the total area of the snowflake, as n goes to infinity?

Triangle for Fun with Fractals



Fun with Gravity – Class Handout

1. Obtain a meter stick and a bouncy ball.
2. Drop the ball *once* from a height of 1 meter, let it bounce three times, and record the height of each of these three bounces.

   This may take some practice and teamwork! Assign each team member a "bounce" to measure – that is, one person watches to see how high the ball goes after the *first* bounce, a second person watches to see how high the ball goes after the *second* bounce, etc.
3. Figure out to what percent of its height the ball rebounds each time.
4. We are going to pretend we live in a perfect world as we model this phenomenon – this is called "making an assumption." Make an assumption about what height the ball would reach on the rebound if dropping from a height of h meters.

   We do still want meaningful results – so this assumption should be based on the data you collected and calculated in steps 2 and 3.
5. Suppose the ball bounced at this rebound percentage infinitely many times. Find the total distance travelled by the ball, if it is dropped from a height of 10 meters. (Hint: Write an infinite series.)
6. From physics, the height of the ball at time t if it is dropped from a height of h meters with an initial velocity of 0 meters per second is $s(t) = h - 4.9t^2$. How long does each of the first three up-and-down bounces take the ball, if it is dropped from a height of 10 meters?
7. Find the total amount of time that the ball would bounce. (Hint: Write an infinite series. Look for a pattern in your answers in the previous step, or do some algebra.)