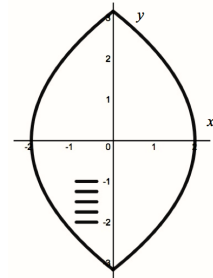


Visualizing Second Order Partial Derivatives on a Football – Class Handout

Draw an xy -coordinate system on a piece of paper, and place a football model on it with the pointed ends of the football along the y -axis as seen in the diagram on the right. As you look down on the football from above, you see the top half of the football, i.e., the part of the football that is farthest above the piece of paper. Make sure that the toothpicks are completely visible from the aerial view in the lower left corner, as seen in the diagram; if they are not, rotate the football until they are visible. Let the height of this surface visible from above be $f(x, y)$. Then complete the problems below.



Football
Orientation

- Pick any point on the football. Hold a toothpick on the surface so that it is parallel with the x -axis.
 - Is f_x positive or negative? How can you tell? Does your answer differ based on the location of the point you picked?
 - Slide the toothpick to the right. As you move the toothpick, what happens to f_x ? Does it increase or decrease? What does this tell you about f_{xx} ?
 - While you moved the toothpick, you traced out a curve along $f(x, y)$. Is that curve concave up or down in the xz -plane? How is concavity related to the sign of f_{xx} ?
- Pick any point on the football. Hold a toothpick on the surface so that it is parallel with the y -axis.
 - Is f_y positive or negative? How can you tell? Does your answer differ based on the location of the point you picked?
 - Slide the toothpick in the positive y -direction. As you move the toothpick, what happens to f_y ? Does it increase or decrease? What does this tell you about f_{yy} ?
 - While you moved the toothpick, you traced out a curve along $f(x, y)$. Is that curve concave up or down in the yz -plane? How is concavity related to the sign of f_{yy} ?
- For this problem, look at the toothpicks already inserted in the plastic covering of the football.
 - What happens to the slopes of the toothpicks, f_x , as you move in the positive y direction along the line drawn on the football? Does the slope increase or decrease? What does this tell you about f_{xy} ?
 - How does your observation of f_{xy} compare to your observations of f_{xx} and f_{yy} ?
 - If the model were a cylinder instead of a football, what would be the sign of f_{xy} ? How can you tell?