Visualizing and Estimating the Mass of a Solid – Class Handout

For this problem we will consider the solid Q that lies under the plane $z = 2 + x - y$ and between the planes $x = 3$, $y = 2 + x$, and $y = 5 - 2x$. We will assume the units for x, y, and z are centimeters. This solid does not have constant density. Instead the density of the solid is given by $\rho(x, y, z) = x + z$ in g/cm^3.

1. Sketch a graph of the region R that lies underneath the solid Q on the 1 cm graph paper at the bottom of this page.

2. Fix a convention such as red cubes have density 1 g/cm^3 and orange cubes have density 2 g/cm^3, etc. Record your color convention below.

Density (g/cm^3)	1	2	3	4	5	6	7	8
Color								

3. Construct a model that approximates the solid Q by stacking blocks over the region R on your 1 cm graph paper. Remember to use your color convention so that your model represents not only the shape of the solid, but also the density.

4. Use your model to estimate the mass of Q. To do this, estimate the mass of each column of blocks. You can record your estimates for each column on the copy of the region R that you have sketched below. Once you have estimated the mass of each column, you can add them together to get an estimate for the mass of the entire solid.

5. In this activity, you have created an approximation for the solid Q by dividing up the solid into cubes. Use the notation Δx, Δy, and Δz for the length, width, and height of each of these cubes, respectively. For fixed values of x and y, what does the expression $\sum\limits_{z=0}^{2+x-y} \rho(x, y, z)\, \Delta z \Delta x \Delta y$ represent?