Properties of Functions on Finite Sets Using Candy – Class Handout

$S =$ the set of different colored Skittles® (one of each of the five colors)

M= the set of different colored M&M's® (one of each of the six colors)

1. Draw a diagram of the function $f : S \to M$ that you created. What are the *orders* (number of distinct elements) of the domain and the codomain? Use $|S|$ to denote the order of the set S and $|M|$ for the order of M. Put a box around the range of the function.

2. Draw a diagram of a relation from S to M that is not a function and write down how it violates the definition of a function. Draw a second diagram of a relation from S to M that is not a function for a different reason.

3. A function f is said to be *onto* or *surjective* if for each element y in the codomain, there exists an element x in the domain such that $f(x) = y$. In other words, the range is equal to the codomain. Let Y be the set consisting of the red, blue, green and yellow M&M's. Create an onto function from S to Y. Create a second function that is not onto from S to Y. Draw the diagrams of both.

4. A function f is said to be *one-to-one* or *injective* if for all elements x_1 and x_2 in the domain, if $f(x_1) = f(x_2)$ then $x_1 = x_2$. In other words, every element in the domain maps to a unique element in the codomain. Create a function from S to M that is one-to-one. Create a second function from S to M that is not one-to-one. Draw the diagrams of both.

5. A function is said to be a *bijection* or *bijective* if it is both one-to-one and onto. Define a domain A and a codomain B consisting of Skittles and M&M's respectively, and then create a function that is a bijection from A to B. Draw the diagram.

6. Is it possible to define a function from M to S that is a bijection? Why or why not?

7. What can you say about the orders of the domain and codomain when you know that a function is onto, one-to-one, or a bijection? Does one have to be larger than the other? Can they be equal? Or is there not enough information to decide? Draw some examples below, and when you have a conjecture, think about how you might prove it is true.

   If a function $f : A \to B$ is onto, then $|A|$? $|B|$. That is, how is the size of A related to B?
   If a function $f : A \to B$ is one-to-one, then $|A|$? $|B|$.
   If a function $f : A \to B$ is bijective, then $|A|$? $|B|$.