

SET[®] in Combinatorics/Discrete Math – Class Handout

- SET is played with a special deck of cards, each of which has symbols characterized by four attributes:
Number: 1, 2, or 3 symbols. **Color:** Red, purple, or green.

Shading: Empty, striped, or solid. **Shape:** Ovals, diamonds, or squiggles.

A *SET* is three cards where, for each attribute, each incidence of that attribute is all the same or all different.

- Two important notes: (1) The number of attributes that are the same can vary.
(2) If you have three cards where there is any aspect where you can say “Two are X but one is not,” then the three cards are not a *SET*. For example, if two are ovals but one is not, it is not a *SET*. If exactly two are striped, it is not a *SET*.
- To play: 12 cards are laid on the table. The first person to spot a *SET* yells “*SET!*”, and takes the cards that make a *SET*, and those cards are replaced. If at some point, everyone agrees that there aren’t any *SETs* on the table, three more cards are added, but if a *SET* gets taken, don’t replace the cards unless there’s no *SET* again. When there are no more cards in the deck and no *SETs* on the table, the winner is the one with the most *SETs*.

1. Quick questions:

- (a) How many cards are there in the whole deck?
- (b) How many *SETs* are there in the whole deck?
- (c) Consider the card with two red solid squiggles. How many *SETs* contain that card? Is the answer unique to that card, or does each card belong to that many *SETs*? Justify your answer.

2. When you lay out the first 12 cards, how many *SETs* do you expect to see on average? *Hint: What is the number of ways of choosing three cards? The probability that three cards form a SET is the fraction (number of SETs)/(number of ways of choosing three cards). Now what do you do?*

3. The questions below ask for the number of *SETs* with various properties. Since you know how many *SETs* are in the deck, also figure out what percentage (or fraction if you prefer) of *SETs* has those properties.

- (a) What is the number of *SETs* with all attributes different?
- (b) What is the number of *SETs* with one attribute the same and three different?
- (c) What is the number of *SETs* with two attributes the same and two different?
- (d) What is the number of *SETs* with three attribute the same and one different?

4. A standard SET deck has cards with four different attributes, but you can make a version with more or less attributes. For example, you can have SET with only three attributes by taking all the solid cards, or you can have SET with five attributes by taking three decks and decorating the backgrounds of two of the decks.

- (a) For 3-attribute SET, what is the number of *SETs* with all attributes different? With two attributes different and one the same? With one attribute different and two the same? What fraction or percentage of the *SETs* has those properties?
- (b) Answer the same questions for 5-attribute SET. (Pay attention: there are more possibilities.)

5. Summarize your results. How might you generalize these results?