Modeling of Fishing and Restocking with Pennies – Class Handout

A lake in northern Montana is dominated by arctic grayling (henceforth called "species A") but the Department of Fish, Wildlife, and Parks is planning to slowly introduce bull trout ("species B"). The lake is popular with sport fishermen who remove both species of fish from the lake regularly.

The Department of Fish, Wildlife, and Parks has carefully estimated the number of fish taken by sport fishing each week, and they have decided to keep the fish population as constant as possible by replacing the fish lost by an equal number of arctic grayling and bull trout. Consider a lake with $N = 50$ fish in the lake at the beginning of the week. Fishermen remove $M = 10$ fish during that week, then the fish and wildlife people will restock the lake with 5 arctic grayling and 5 bull trout. Hence the population of the lake will remain $N = 50$ fish at the end of each week, assuming no new fish are born. Both fish species swim freely throughout the lake and both are targeted by similar bait used by sport fisherman.

1. **Conjecture**

   (a) What do you think will happen to the populations of species A and B over a long period of time?

   (b) Is it possible that species A will be eliminated from the lake with the restocking plan? Explain.

2. **Simulate**

   (a) Use pennies to represent your N fish and decide with your partner(s) which coin face represents which species (e.g., Species A will be heads). Start your lake with 100% species A.

   (b) Decide with your partner(s) how to simulate the swimming of fish, the fishermen, and the Department of Fish, Wildlife, and Parks' restocking plan. Simulate roughly 15 weeks of the fish population representing species A and B with coins. Be sure to let the fish swim thoroughly around the lake and keep track of the proportions of species A and B. Keep track of your data in a table with the following headers:

Week #	Number in population		Proportion of population	
	species A	species B	species A	species B
0	50	0	1	0
⋮	⋮	⋮	⋮	⋮

3. **Model**

   (a) Propose a verbal model for the rate of change of species B in the lake.

   rate at which species B changes = _____ + _____

   (b) Explicitly state any assumptions that you are using in your verbal model.

   (c) Introduce mathematical notation for your proposed model and write your verbal model mathematically. Be sure to include any necessary condition(s).

   model: _____

   condition(s): _____

4. **Analyze**

   (a) According to your model, what is the long term effect on the fish population in the lake? Use your model to justify your answer algebraically and graphically.

   (b) Solve your mathematical model (either numerically or analytically) and compare with your data. How well does the model interpolate and extrapolate the data?