Constructing Disjoint Hamiltonian Cycles of K_{2n+1} – Class Handout

Part I: Obtain a set of yarn and pens for your group. Designate one group member as the record-keeper.

Theorem 1. The complete graph K_{2n+1} has a decomposition into n disjoint Hamiltonian cycles.

1. Start by creating a graph with five (human) vertices. How many disjoint Hamiltonian cycles should you be able to find? Create them with different colors of yarn. Draw a picture of your result using the colored pens.

2. Add two human vertices to your graph and create the decomposition as above. See if you can extend your previous configuration instead of starting from scratch (if not, starting from scratch is valid as well!). Draw a picture of this result using the colored pens.

3. Theorem 1 says nothing about the decomposition of a complete graph with an even number of vertices. What can you say about the degree of each vertex in a complete graph? What does this say about the decomposition of an even complete graph K_{2n} into disjoint Hamiltonian cycles?

Theorem 2. The complete graph K_{2n} has a decomposition into n disjoint Hamiltonian paths and a decomposition into $n-1$ disjoint Hamiltonian cycles and a matching.

4. Remove one vertex from your decomposed K_7. Draw a picture of the configuration using the colored pens. Which decomposition does this K_6 demonstrate?

Part II: As the size increases, the complexity quickly becomes unmanageable without employing an algorithm.

Algorithm for decomposing K_{2n+1} into disjoint Hamiltonian cycles:
Number the participants from 1 through $2n+1$. Student $2n+1$ will stand on a chair to hold the final vertex above the others. Connect participants according to the following pattern, closing each cycle:

Cycle 1	$2n+1$	1	$2n$	2	$2n-1$	$\dots$	$n+1$
Cycle 2	$2n+1$	2	1	3	$2n$	$\dots$	$n+2$
Cycle 3	$2n+1$	3	2	4	1	$\dots$	$n+3$
$\vdots$	$2n+1$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Cycle n	$2n+1$	n	$n-2$	$n+1$	$n-3$	$\dots$	$2n$

If your calculations go past $2n$, wrap back around to count from 1 again.

5. Use the algorithm to re-create a decomposed K_7. Is it equivalent to your previous decomposition up to isomorphism?

6. Remove one vertex from your decomposed K_7. *Hint: There may be an obvious candidate here.* Draw a picture of the configuration using the colored pens. Which decomposition does this K_6 demonstrate?

7. Use the algorithm to create a decomposed K_9. Draw a picture of the configuration using the colored pens.

8. Remove one vertex from your decomposed K_9. Which decomposition does this K_8 demonstrate?

9. Using this algorithm to decompose a K_{2n} should always result in the same decomposition. Which one?