Exploring the $\epsilon - N$ Definition of Sequence Convergence – Class Handout

1. Let $a_n = 3$. As a class, we will represent this sequence by having one student stand at each of the points $(1, 3)$, $(2, 3)$, $(3, 3)$, ... Two people (referred to as the *epsilon pair*) will use string to represent a large ϵ region. Since this sequence converges to 3, the epsilon pair needs to stand on the edges of the coordinate system along the line $y = 3$. They should start with their arms wide open, representing a large ϵ region.

   (a) Are all the students from the sequence inside the ϵ region?

   (b) What happens as the students representing the ϵ region make the region smaller? Are all the students from the sequence still in the ϵ region?

   (c) Is it possible to find an ϵ region small enough so that someone from the sequence will not be in the ϵ region? Why or why not?

2. Let $b_n = \frac{3}{n}$. As a class, we will represent this sequence by having one student stand at each of the points $(1, \frac{3}{1}), (2, \frac{3}{2}), (3, \frac{3}{3}), (4, \frac{3}{4})$, ... Two people will use string to represent a large ϵ region. Since this sequence converges to 0, the epsilon pair needs to stand on the edges of the coordinate system along the line $y = 0$. They should start with their arms wide open, representing a large ϵ region.

   (a) Are all the students from the sequence inside the ϵ region?

   (b) What happens as the students representing the ϵ region make the region smaller? Are all the students from the sequence still in the ϵ region?

   (c) For smaller ϵ values, which students remain inside the ϵ region? What does this have to do with N?

3. Let $c_n(x) = (-1)^n$. As a class, we will represent this sequence by having one student stand at each of the points $(1, -1)$, $(2, 1)$, $(3, -1)$, $(4, 1)$, ... Two people will use string to represent a large ϵ region. You may want to try 0, 1, or -1 as possible values for the limit.

   (a) Are all the students from the sequence inside the ϵ region?

   (b) Make ϵ smaller. What happens to the number of students inside the ϵ region as ϵ decreases?

   (c) Does this situation satisfy the definition of convergence? Why or why not? Does c_n converge to zero? Does c_n converge to 1? Does c_n converge to -1?