# Explorations of the $\epsilon - \delta$ Definition of Continuity – Class Handout

1. A few of your classmates will create an $\epsilon$ region and corresponding $\delta$ region around a point on a continuous function that is made with feather boas on the floor. Complete the following problems concerning the setup.

   (a) Sketch the $\epsilon$ and $\delta$ regions, including the portion of the function bounded by these regions.

   (b) Describe where the function crosses the $\epsilon$ and $\delta$ regions in your diagram from Problem 1a. Does the function cross the yarn representing $\epsilon$ or the yarn representing $\delta$ or both?

   (c) What must be true about the $\delta$ region in order for the definition of continuity to hold? How can you tell that the $\delta$ region is small enough?

2. Two volunteers will decrease the size of the $\epsilon$ region. Two other volunteers then need to decrease the size of the $\delta$ region so that the definition of continuity will still be satisfied. Complete the following problems concerning what you are observing while this happens.

   (a) Sketch the $\epsilon$ and $\delta$ regions, including the portion of the function bounded by these regions.

   (b) Describe where the function crosses the $\epsilon$ and $\delta$ regions in your diagram from Problem 2a. Does the function cross the yarn representing $\epsilon$ or the yarn representing $\delta$ or both? How does your diagram in Problem 1a compare to the region you obtained in Problem 2a?

   (c) Based on the activity so far, would you say that the definition of continuity is more concerned about properties near a point or globally? How can you tell?

3. The feather boas are now used to represent a function with a jump discontinuity. Two volunteers will create a variety of different sized $\epsilon$ regions while two more volunteers attempt to find a $\delta$ region that satisfies the definition of continuity. Complete the following problems concerning what you are observing.

   (a) For very small $\epsilon$ values, is it possible to find a $\delta$ small enough to satisfy the definition of continuity? Should there be a small enough $\delta$ to make the definition work?

   (b) Is it possible to create an $\epsilon$ region that is large enough so that there is a $\delta$ region that meets the criterion of the definition of continuity? If so, does that cause a problem with the definition of continuity? Why or why not?