Finding the GCD – Instructions for Initial Setup

In your team of three, each of you should select a role: Student A, Student B, or Student C. Then work together to complete the following steps to get ready for Round 1.

- Use station markers to designate ten stations that are 5-10 feet apart.
- Place a pen at each of the first nine stations.
- Place a sheet of 6 mailing labels at each of the first nine stations.
- Student A: Take a calculator, two flying discs, and an extra sheet of six mailing labels to Station 1. Once at Station 1, place a mailing label on each of the flying discs and write the number 14304 on one of the labels and 3356 on the other one.
- Student B: Take a calculator and one flying disc to Station 2. Once at Station 2, place one mailing label on the flying disc, and then use the pen to write the number 3356 on that mailing label.
- Student C: Take a calculator and go to Station 3.

Now you are ready to begin Round 1.

Finding the GCD – Class Handout

1. Run the algorithm with $a = 21,748$ and $b = 16,612$. Record a, b, r, and q at each step.

2. Run the algorithm with $a = 233$ and $b = 144$. Repeat for 144 and 89. Both pairs of numbers are adjacent Fibonacci numbers. Record a, b, r, and q at each step. What do you notice? Make a conjecture about what happens when you run the algorithm with any two adjacent Fibonacci numbers.

3. We define the sequences $\{a_i\}$, $\{b_i\}$, $\{q_i\}$ and $\{r_i\}$ to be the values for a, b, q, and r that the person doing the ith step has. Explain what the following three equations mean in terms of the activity.

$$a_2 = b_1 \qquad b_2 = r_1 \qquad r_2 = a_2 - b_2 q_2$$

4. Why does the activity stop when $b_n = 0$?

5. Suppose d is a positive integer. Prove that $d \mid a_1$ and $d \mid b_1$ if and only if $d \mid a_2$ and $d \mid b_2$. There are four proofs here. Two of them are short. Two of them are longer.

6. Why does the previous result tell us $\gcd(a_1, b_1) = \gcd(a_2, b_2)$?

7. Suppose when trying to find $\gcd(a, b)$ using this algorithm, $b_n = 0$ but $b_{n-1} \neq 0$. Justify each of these equations.

$$\gcd(a, b) = \gcd(a_1, b_1) = \gcd(a_2, b_2) = \cdots = \gcd(a_n, b_n) = \gcd(a_n, 0) = a_n$$