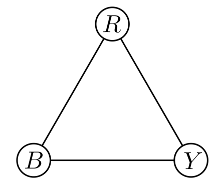
Symmetry and Group Theory with Plastic Triangles – Class Handout

Definition. A *symmetry* is an action which brings every point of an object to a position previously occupied either by itself or by some other point of the object.

1. Explain how you would solve the equation $5x + 8 = 21$ without using the words *subtract* or *divide*.

The following items deal with the symmetries of an equilateral triangle. The triangle shown to the right has colored vertices. These colors are there to help you keep track of what is going on as you complete the exploration that follows. Throughout the rest of the activity, please think of this configuration as the starting point.

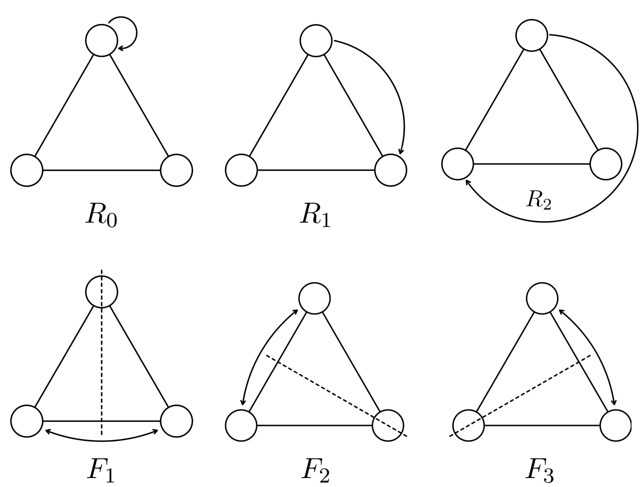


2. How many symmetries does an equilateral triangle have? That is, how many ways can you move the triangle around to get a triangle that matches the orientation of the starting triangle? (This means you want to start and end with a triangle that has a bottom edge and a top vertex.) Don't worry about notation, just use words to explain what you mean.

3. Can you think of any similar actions on an equilateral triangle that are not symmetries?

4. What happens if you perform one symmetry on an equilateral triangle followed by another one?

5. Using one of the triangles you have, determine and annotate the new locations of the colors after you apply the symmetry given in each picture below; record your answer on each picture by marking each vertex with R, Y, and B. Start with your triangle oriented as indicated in the image above and fill in the circles with the appropriate colors as they appear after you apply each symmetry.



6. Fill in the table below by performing the symmetry along the left column first, followed by the symmetry along the top row. As you begin, put the smaller triangle inside the larger one and then, as you carry out the symmetry actions, perform the first symmetry on the inside triangle only and then perform the second symmetry on both triangles at the same time. Record what happens to the inner triangle. We have filled in the box corresponding to $R_2 \bigotimes F_2$. (Each entry in the table should be one of the following: $R_0, R_1, R_2, F_1, F_2, F_3$.)

$\bigotimes$	R_0	R_1	R_2	F_1	F_2	F_3
R_0						
R_1						
R_2					F_1	
F_1						
F_2						
F_3						

7. Is there an identity symmetry? If so, what is it? Does each symmetry have an inverse?

8. Is the operation $\bigotimes$ commutative? If so, how can you tell? If not, why not?

9. Are there any interesting patterns in the table? What do these patterns tell you about the operation $\bigotimes$?

10. Explain how you would solve the equation $F_2 \bigotimes x = R_2$ using the ideas you developed in this activity. How would you represent the solution to this equation symbolically?