

## Finding Groups in a (New) Color Cube Puzzle – Class Handout

### Part 1: $2 \times 2 \times 2$ Cubes

1. Choose four of your colors, and gather two cubes of each of these colors. Create a  $2 \times 2 \times 2$  cube such that each color appears exactly once on each face. Describe your setup (called the *original state*) by writing the color of each block in a table like the one that follows. Your original state should be unique to you; do not use the same color setup as anyone else!


Top Layer


Bottom Layer

2. The elements of the groups examined will be operations on the cube that lead to different arrangements of the colors that still satisfy the condition of each color appearing exactly once on each face. The group operation will be composition of the operations on the cube. The cube has six faces; we will consider each of these faces to be a “slice.” Orient the cube so that there is a left slice, a right slice, a front slice, a back slice, a top slice, and a bottom slice.
  - (a) Take the slice of blocks on the right and place it on the left maintaining the slice in its original orientation. Does this give a new arrangement of colors? This group element is labeled  $L$ .
  - (b) Return the cube to its original state. Similarly, take the slice of blocks in the back and move it to the front. Does this give a new arrangement of colors? This group element is labeled  $F$ .
  - (c) Now consider all color arrangements that are possible to obtain by composing  $L$  and  $F$ . Is this a group? If so, to which known group is it isomorphic? Give the Cayley table.
  - (d) Return the cube to its original state. Move the top slice of blocks to the bottom and call this group element  $B$ . Is  $B$  truly a new group element, or have you obtained a color arrangement that we already saw in part (c)? If so, write the equation for  $B$  in terms of  $L$  and  $F$ .

