Acting Permutation – Class Handout

Below are some things to consider as we explore permutations. Record your conjectures and observations. For homework, try to prove at least one of these conjectures.

Notations for Permutations: In two-line permutation notation, the top line always stays the same. Are there more efficient ways to record a permutation? What if instead of viewing a permutation as recording where all of the things move, we think of a permutation as a chain of events? For example, can we record what happens to the element 1 over time? How would we do this?

Commuting: Suppose we want to compose permutations like we do functions. What would it mean to do this and what would we get? Record two permutations from class and their composition. Does order matter in the composition of permutations? Provide an example that illustrates your answer. Will permutations ever commute? What conditions would be necessary for them to commute?

Inverses: What would it mean to "undo" a permutation? What does that mean in terms of the students who are acting out the permutation? Suppose I give you a random permutation. How can you find its inverse? Will your method always work?

Identity: Is there an identity permutation? What is it? How would you write it down? Write it down using all of the different notations we have discussed.

Order: Write down a permutation that consists of one cycle and act it out. What is the order of this cycle? In other words, how many times do you need to apply this permutation to get everyone back to their starting positions? Is there a general rule that will tell you the order of any permutation? Try your rule on permutations consisting of one cycle and many cycles.

Lack of Uniqueness: Record a permutation that the class acted out. Is this the only way to record what happened? If not, how many ways can you write it down? Can you develop a way that uses more or less cycles?

2-Cycles and Cycle Parity: A 2-cycle, or transposition, is a permutation that only switches two numbers. What is the inverse of a 2-cycle? Since we know that cycle notation is not unique, can you write the identity as a product of transpositions? Find three ways to do this. Can you write the identity using an odd number of 2-cycles? Why not?

More 2-Cycles: Consider the permutation (351426). Can you write this permutation using only 2-cycles? What is the fewest number of transpositions you can use? It may help to act this out. Is there a way to write this permutation with an even number of 2-cycles? What about an odd number? Why or why not?

Challenge: Find an algorithm for writing a permutation as the product of transpositions. Will a permutation only give you an odd or even number of transpositions? Why or why not?