Designing Round-Robin Tournaments Using Yarn – Class Handout

Part I: Divide into groups to do this activity, and obtain a set of yarn and pens. Designate at least one student as a record-keeper; you don't want your results to get lost in the shuffle. Remember that a *round-robin* tournament is a competition where every pair of players or teams will eventually play one another. An individual player plays at most once per round. Use the colored pens to distinguish between rounds as you draw pictures describing each tournament.

1. As a class, we designed a four-person tournament. Draw a color-coded picture describing the tournament. How many rounds were needed? How many total games were played?

2. Expand the tournament to include six people. Fill in the table and draw a picture. How many rounds were needed? How many total games were played? What difficulties did you run into, if any?

Round	Game 1	Game 2	Game 3
Round 1			
Round 2			
Round 3			
⋮	⋮	⋮	⋮

Part II: As the size of the tournament increases, the complexity of the problem quickly becomes unmanageable without an algorithm, a set of step-by-step instructions for solving a problem.

Now you will use an algorithm known as the "turning trick" to design a tournament. Number the participants from 1 through 6. Student 6 will stand on a chair to hold the final vertex above the others. Connect participants according to this pattern:

Round 1	$1 \to 6$	$2 \to 5$	$3 \to 4$
Round 2	$2 \to 6$	$3 \to 1$	$4 \to 5$
Round 3	$3 \to 6$	$4 \to 2$	$5 \to 1$
Round 4	$4 \to 6$	$5 \to 3$	$1 \to 2$
Round 5	$5 \to 6$	$1 \to 4$	$2 \to 3$

Now that you've seen a specific instance of the algorithm, here is a general version for any tournament with an even number of participants. Number the participants from 1 through $2n$. Connect participants according to this pattern:

Round 1	$1 \to 2n$	$2 \to 2n-1$	$3 \to 2n-2$	$4 \to 2n-3$	$\ldots$	$n \to n-1$
Round 2	$2 \to 2n$	$3 \to 1$	$4 \to 2n-1$	$5 \to 2n-2$	$\ldots$	$n+1 \to n$
Round 3	$3 \to 2n$	$4 \to 2$	$5 \to 1$	$6 \to 2n-1$	$\ldots$	$n+2 \to n+1$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

If your calculations go past $2n-1$, restart the count at 1. Why does this work?

For each of the following, make a color-coded picture of your solution.

3. Use the algorithm to design an eight-person tournament. What value should you give n?

4. Design a five-person tournament. How is this tournament different from the previous ones?

5. Design a seven-person tournament. (Hint: it may be easier than previous large tournaments!)